

ELASTIC STRESSES AND DEFLECTIONS UNDER UNIFORM PRESSURE OF FLAT RECTANGULAR SANDWICH PANELS; LONG EDGES CLAMPED, SHORT EDGES SIMPLY-SUPPORTED (ISOTROPIC FACE PLATES AND ORTHOTROPIC CORES OF ZERO FLEXURAL STIFFNESS)

1. NOTATION

a	length of panel in x-direction	m	in
B	face stiffness parameter given by $B = \left[\frac{1}{E_1 t_1} + \frac{1}{E_2 t_2} \right]^{-1}$	N m/m ²	lbf in/in ²
b	width of panel in y-direction	m	in
d	distance between centre-lines of face plates	m	in
E	modulus of elasticity of face plate material	N/m ²	lbf/in ²
f_{Ey}	maximum direct stress in y-direction at long edge of face plate	N/m ²	lbf/in ²
f_x, f_y	maximum direct stresses in face plate at centre of panel in x- and y-directions, respectively	N/m ²	lbf/in ²
G_{cx}, G_{cy}	shear moduli of core in zx- and yz-planes, respectively	N/m ²	lbf/in ²
h	depth of core	m	in
p	resultant normal pressure on panel	N/m ²	lbf/in ²
q_{cx}, q_{cy}	maximum shear stresses in core in zx- and yz-planes, respectively	N/m ²	lbf/in ²
q_{xy}	maximum shear stresses in face plates	N/m ²	lbf/in ²
t	thickness of face plate	m	in
w	maximum normal deflection of panel	m	in
x, y, z	panel reference axes (see Sketch 2.1)		
<i>Suffixes</i>			
1, 2	identification numbers for thicknesses, stresses and moduli of face plates		

Both SI and British Units are quoted but any coherent system of units may be used.

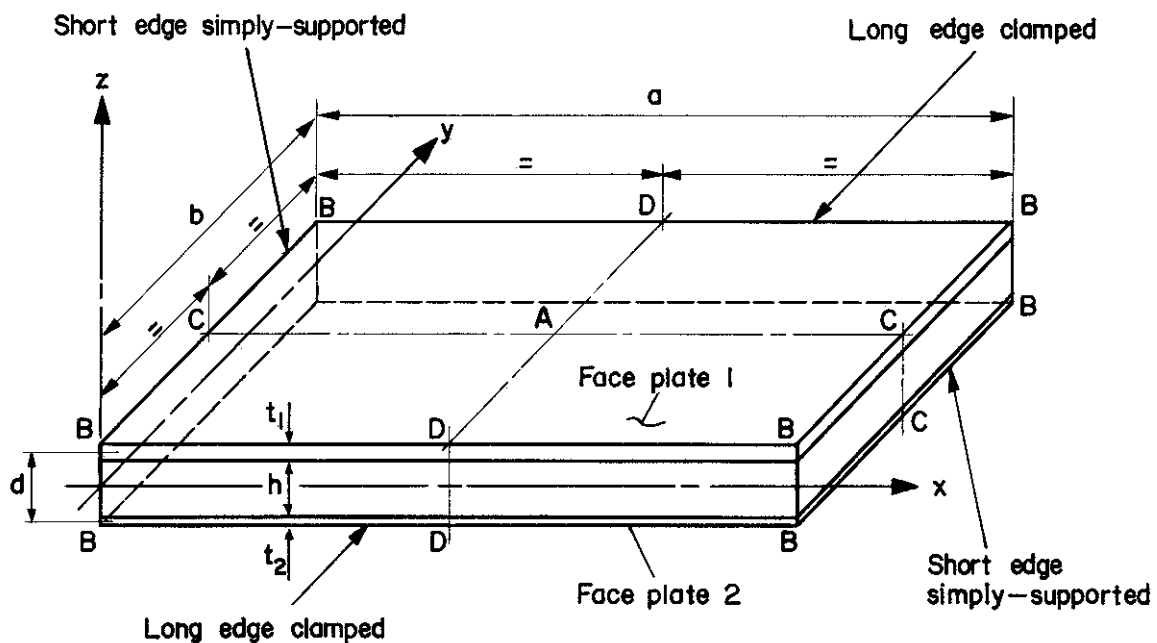
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2. NOTES

Curves are given for the maximum deflection and the maximum direct and shear stresses in initially flat rectangular sandwich panels of uniform thickness with long edges clamped and short edges simply-supported of various length to width ratios under uniform normal pressure. The face plates are isotropic and have the same value of Poisson's ratio; however, they can be of different thicknesses and have different moduli of elasticity. The core material may be either isotropic or orthotropic. It is assumed that while the core provides a shear path between the faces its own flexural stiffness makes no significant contribution to that of the panel.

In Figure 1 the maximum deflection of the panel is plotted in the form wBd^2/pb^4 against $Bh/G_{cy}b^2$ for various values of the ratio G_{cy}/G_{cx} grouped for particular values of a/b . Similarly the maximum direct stresses in the face plates at the centre of the panel in the x - and y -directions are plotted in the form fdt/pb^2 in Figures 2 and 3. In a similar form, the maximum direct and shear stresses in the face plates at the edge of the panel are given in Figures 4, 5 and 6, respectively. The maximum shear stresses in the core plotted in the form qd/pb are given in Figures 7 and 8.

The position at which the maximum deflection and stresses occur is shown in Sketch 2.1. The maximum deflection and stresses f_x and f_y occur at point A in the centre of the panel. These stresses are compressive in the face plate on which the resultant pressure p is considered to act and tensile on the other plate. The maximum direct stresses in the edge, f_{Ey} , occur at point D and are tensile in the face plate on which the resultant pressure acts and compressive in the other plate. The maximum shear stresses q_{xy} occur in the face plates at either side of, and immediately adjacent to, the corner points B. The maximum shear stresses in the core, q_{cx} and q_{cy} , occur at points C and D in the short and long edges, respectively.



Sketch 2.1 Panel Geometry

The curves are based on elastic small deflection theory applied to sandwich panels and have been calculated for a value of Poisson's ratio of 0.3. The face plates are assumed to be thin compared to the depth of the core and to resist the bending moments developed in the panel by acting differentially. The core is assumed to be infinitely stiff in the transverse (z) direction, the shear stresses being constant throughout its depth. These assumptions will be valid provided that $d/t \geq 6$, $b/d \geq 10$ and that w/d is not greater than 0.5. The

edge condition clamped assumes there is no rotation of the edge due to bending effects.

3. DERIVATION

1. RAVILLE, M.E. Deflection and stresses in a uniformly loaded, simply-supported, rectangular sandwich plate. Forest Products Laboratory, Madison 5, Wis., USA, Rep. No. 1847, September 1962.
2. FOLIE, G.M. The theory of sandwich panels subjected to transverse loads under various edge conditions. University of Southampton, Department of Civil Engineering Rep. No. CE/37/67, 1967.
3. ALLEN, H.G.
FOLIE, G.M. Graphs for the analysis of rectangular sandwich plates under uniform transverse pressure. University of Southampton, Department of Civil Engineering Rep. No. CE/22/68, 1968.
4. ALLEN, H.G. *Analysis and design of structural sandwich panels.* Pergamon Press, London, 1969.

4. EXAMPLE

It is required to find the maximum deflection and stresses of a flat rectangular sandwich panel of width 520 mm and length 900 mm having its long edges clamped and its short edges simply-supported, when subjected to a resultant pressure of 75 kN/m² that acts on face 1. The panel has an orthotropic honeycomb core of depth 50 mm aligned so that the maximum shear modulus of 329 MN/m² is along the length of the panel (the *zx*-plane) and the minimum shear modulus of 128 MN/m² is across the width of the panel (the *yz*-plane). The face plate thicknesses are $t_1 = 0.6$ mm and $t_2 = 0.4$ mm and both are made of aluminium alloy with $E = 69\,000$ MN/m².

From the given data it follows that

$$\frac{a}{b} = 1.73,$$

$$d = 50 + \left(\frac{0.6 + 0.4}{2} \right) = 50.5 \text{ mm},$$

so that

$$B = \left[\frac{1}{69\,000 \times 10^6} \left(\frac{1}{0.6 \times 10^{-3}} + \frac{1}{0.4 \times 10^{-3}} \right) \right]^{-1} = 16.5 \text{ MN m/m}^2,$$

$$\frac{Bh}{G_{cy}b^2} = \frac{16.5 \times 10^6 \times 50 \times 10^{-3}}{128 \times 10^6 \times (520 \times 10^{-3})^2} = 0.0238$$

and

$$\frac{G_{cy}}{G_{cx}} = \frac{128}{329} = 0.389.$$

Also $\frac{d}{t_1} = 84 > 6$ and $\frac{b}{d} = 10.3 > 10$

and therefore the curves may be used for the analysis of this panel.

The maximum deflection is obtained by interpolation from Figure 1 which yields

$$\frac{wBd^2}{pb^4} = 0.0054$$

so that
$$w = \frac{0.0054 \times 75 \times 10^3 \times (520 \times 10^{-3})^4}{16.5 \times 10^6 \times (50.5 \times 10^{-3})^2} = 0.000704 \text{ m or } 0.704 \text{ mm}.$$

It can be seen that $w/d = 0.0139 < 0.5$.

The maximum direct stresses at the centre of the face plates are obtained from Figures 2 and 3.

From Figure 2 by interpolation

$$\frac{f_{x1}dt_1}{pb^2} = \frac{f_{x2}dt_2}{pb^2} = 0.023$$

so that
$$f_{x1} = \frac{0.023 \times 75 \times 10^3 \times (520 \times 10^{-3})^2}{50.5 \times 10^{-3} \times 0.6 \times 10^{-3}} = \frac{0.023 \times 402 \times 10^6}{0.6} = 15.4 \text{ MN/m}^2$$

and
$$f_{x2} = \frac{0.023 \times 402 \times 10^6}{0.4} = 23.1 \text{ MN/m}^2.$$

Similarly
$$\frac{f_{y1}dt_1}{pb^2} = \frac{f_{y2}dt_2}{pb^2} = 0.042$$

so that
$$f_{y1} = \frac{0.042 \times 402 \times 10^6}{0.6} = 28.1 \text{ MN/m}^2$$

and
$$f_{y2} = \frac{0.042 \times 402 \times 10^6}{0.4} = 42.2 \text{ MN/m}^2.$$

The maximum direct stresses in the long edge of the face plates are obtained from Figure 4 which yields

$$\frac{f_{Ey1}dt_1}{pb^2} = \frac{f_{Ey2}dt_2}{pb^2} = 0.081$$

so that
$$f_{Ey1} = \frac{0.081 \times 402 \times 10^6}{0.6} = 54.3 \text{ MN/m}^2$$

and
$$f_{Ey2} = \frac{0.081 \times 402 \times 10^6}{0.4} = 81.4 \text{ MN/m}^2.$$

The maximum shear stress in the face plates is obtained in a similar manner from Figures 5 and 6 and they give

$$\frac{q_{xy1} dt_1}{pb^2} = \frac{q_{xy2} dt_2}{pb^2} = 0.0192$$

so that
$$q_{xy1} = \frac{0.0192 \times 402 \times 10^6}{0.6} = 12.9 \text{ MN/m}^2$$

and
$$q_{xy2} = \frac{0.0192 \times 402 \times 10^6}{0.4} = 19.3 \text{ MN/m}^2.$$

The maximum shear stresses in the core are obtained from Figures 7 and 8.

From Figure 7 by interpolation

$$\frac{q_{cx}}{p} \frac{d}{b} = 0.247$$

so that
$$q_{cx} = \frac{0.247 \times 75 \times 10^3 \times 520 \times 10^{-3}}{50.5 \times 10^{-3}} = 0.247 \times 0.772 \times 10^6 = 0.191 \text{ MN/m}^2.$$

From Figure 8

$$\frac{q_{cy}}{p} \frac{d}{b} = 0.52$$

so that
$$q_{cy} = 0.52 \times 0.772 \times 10^6 = 0.401 \text{ MN/m}^2.$$

This example may be compared with that of Item No. 76013 in which a similar panel having all edges simply-supported is considered.