

DESIGN OF PARALLEL AXIS STRAIGHT SPUR AND HELICAL GEARS: GEOMETRIC DESIGN

1. NOTATION AND UNITS

In this notation, units are given for both SI and a familiar British system. (See BS 2519: Part 1: 1976 for definitions.)

		SI	British
a	centre distance	m	in
a'	unmodified centre distance [†]	m	in
b	facewidth	m	in
d	reference diameter (nominal pitch diameter)	m	in
d_a	tip diameter	m	in
h_a	addendum	m	in
m_n	normal module [‡]	mm	
P_n	normal diametral pitch [‡]		in ⁻¹
p_n	normal pitch [‡]		in
T	torque	Nm	lbf in
u	gear ratio, given by ω_1/ω_2 or z_2/z_1		
V	tip diameter factor ^{††}		
V_T	sum of tip diameter factors, given by $(V_1 + V_2)$		
v_p	pitch line speed	m/s	ft/min
z	number of teeth		
z_{1p}	trial number of teeth in pinion		
z_v	virtual number of teeth, given by $z \sec^3 \beta$		
α	pressure angle of standard gear tooth (20° in this Item)	degree	degree
α_w	working pressure angle	degree	degree

For footnotes, see end of Notation Section

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β	helix angle	degree	degree
ω	angular speed	rad/s	rev/min

Subscripts

1	appertains to pinion
2	appertains to wheel
max	indicates a maximum
min	indicates a minimum

[†] This is the centre distance appertaining to a standard gear pair.

[‡] Normal module, m_n , normal diametral pitch, P_n , and normal pitch, p_n , are alternative ways of expressing tooth pitch. Normal module is used when working in SI Units, and normal diametral pitch or normal pitch are used when working in British Units. In this Item equations are given in terms of m_n and P_n . The relationship between m_n , P_n and p_n is $m_n = 25.4/P_n = 25.4p_n/\pi$.

^{††} The relationship between the tip diameter factor and the British Standard addendum modification coefficient, x , is $V = 2(x + 1)$.

2. INTRODUCTION

This Item gives data for straight spur and helical gear design for gear ratios up to 10:1. The procedures given in this Item can only be used to design gears that are to be generated from a British Standard basic rack tooth profile having a pressure angle of 20°.

A design procedure that caters for both standard gears and those with modified addenda is given in Figures 1. Figure 1a is for use with designs in SI Units, where the tooth pitch is expressed in terms of the normal module. Figure 1b is for use with designs in familiar British Units, where the tooth pitch is expressed in terms of normal diametral pitch.

3. CHOICE BETWEEN STRAIGHT SPUR AND HELICAL GEARS

Figure 2 gives guidance only, on the basis of load and speed, on the regions in which experience has shown that the different types of gears are satisfactory. Other considerations that may influence the choice between straight spur or helical gears are given below.

Noise

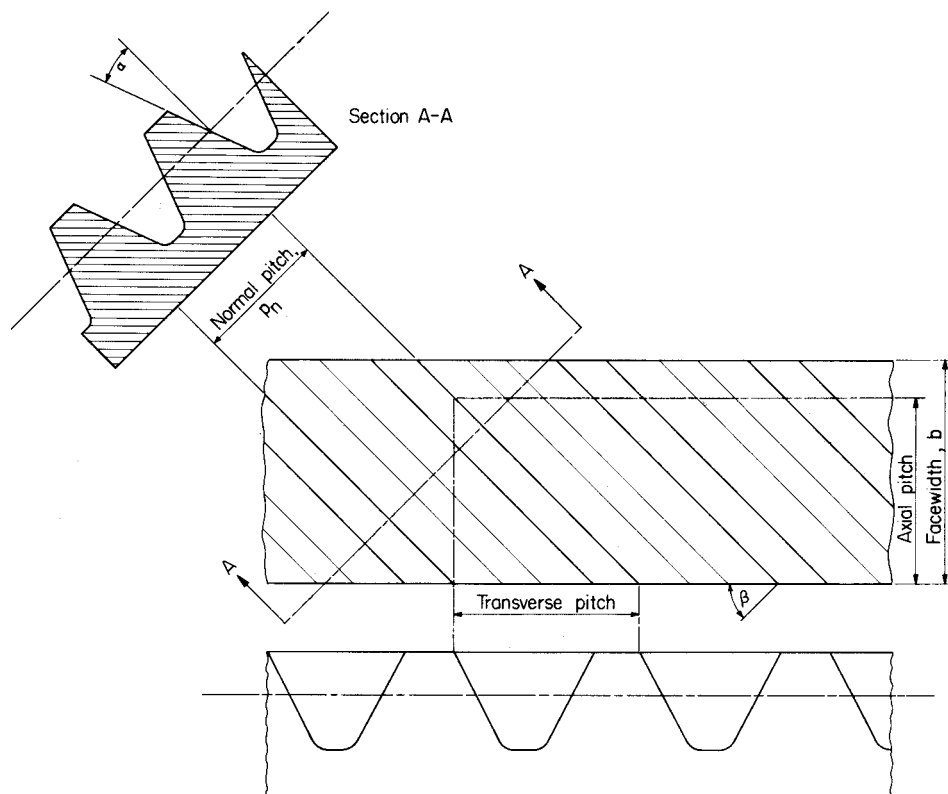
Helical gears are quieter than straight spur gears, particularly if full overlap of the teeth occurs, that is when the facewidth is greater than the axial pitch (see Sketch 3.1). Non-metallic gears (for example, fabric impregnated with synthetic resin) are quieter than steel gears and may be used to advantage when loading permits.

End-thrust

Spur gears, when accurately made and aligned, do not produce any end-thrust because their helix angle is zero. Helical gears develop end-thrust which must be resisted in some way, either by incorporating thrust bearings, or by using double helical gears which results in no net end-thrust.

Cost

Straight spur gears are generally cheaper to produce but, if dimensions are fixed by load capacity, the smaller size of helical gears required may make helical gears cheaper.



Sketch 3.1

4. CHOICE OF HELIX ANGLE (FOR HELICAL GEARS)

Large helix angles may produce excessive end-thrust while small angles may require excessive facewidth (or alternatively fine pitch) to achieve full overlap. In general, helix angles up to about 20° for single helical gears and 30° for double helical gears are satisfactory, but where a higher end-load can be tolerated the helix angle for a single helical gear can be increased to as much as 45° . For full overlap of the teeth, $\sin \beta$ must be greater than $\pi m_n \times 10^{-3} / b$ (or $\pi / P_n b$).

The exact choice of value of helix angle is also influenced by the type of machine used for cutting the gears, and this is discussed in Sections 4.1, 4.2 and 4.3. It is essential that the pitch and helix angle used for cutting the teeth of the pinion and the wheel are identical and that mating gears working on parallel shafts have helices of the opposite hand.

4.1 Rack-type Cutters (Single Helical Gears)

Any helix angle may be used, but setting of the machine is facilitated by choosing the angle such that the axial pitch (or for certain machines, the transverse pitch, see Sketch 3.1) is one of a range of pitches standard for that particular machine. In many cases it will not be known at the design stage what machine will be used and hence the range of axial pitches (or transverse pitches) will not be known. In these circumstances, choice of the exact helix angle may be left until the production stage as any small change in the nominal helix angle that may be required to ease production is unlikely to affect the design significantly.

4.2 Pinion-type Cutters

Gears and cutters must have a common axial pitch. The lead of any cutter must be equal to that of the helical guides of the machine on which it is used. Nominal helix angles are usually about 20° or 30° . The exact helix angle is determined by the leads of available guides and is normally expressed in terms of standard axial pitches.

4.3 Hobs

The helix angle is not restricted and there is no virtue in selecting a particular value except that it may be difficult to set up some hobbing machines for very long leads. To set up the machine an approximate helix angle suffices, but the axial pitch should preferably be one that is standard for the machine.

In some cases it may not be known at the design stage whether the pitch of the axial feed screw of the gear cutting machine is "millimetre size" or "inch size". In these circumstances, choice of the exact axial pitch may be left until the production stage as the small change in axial pitch need not affect any other dimensions of the gears.

5. NUMBER OF TEETH IN PINION AND WHEEL

Figures 3, 4 and 5 give guidance on the choice of the number of teeth in the pinion and wheel. The minimum number of teeth is limited by the precision of the tooth form required for satisfactory quiet meshing. Figure 3 shows the minimum number of teeth that experience has shown to be satisfactory for various cutting processes and tooth speeds. Any point to the left of a cutting process line indicates that a gear can be cut by that process. In addition, the desirability of avoiding an undercut tooth profile provides a further limitation represented by the horizontal cut off for various helix angles. The minimum numbers of teeth in a gear for zero undercut are given in Table 9.3. Figure 7 shows the shapes of tooth profiles along with their limiting values of tip diameter factor to avoid undercut profiles and narrow tooth tip widths (see Section 7.2).

Figure 4 shows the number of teeth in the pinion that experience has shown to be generally satisfactory for the materials and gear ratios given. If the number of teeth chosen is more than about 30 per cent greater than that given in Figure 4, the bending stresses should be checked before proceeding further (see Derivation 13.)

Figure 5 shows the area to be avoided when choosing tooth numbers. The shaded area represents the tooth numbers, speeds and materials at which resonance may occur. Analysis of this phenomenon is complex but it can usually be avoided by choosing a number outside the shaded area. Other resonance effects, such as those caused by external inputs (for example, torsional vibration, or variation of input speed) are not included in Figure 5.

For ease of manufacture, tooth numbers, z , with prime factors exceeding 100 should be avoided. When using the flowcharts, Figures 1, the initial value of the number of teeth in the pinion may have to be changed. The effects of increasing or decreasing the trial number of teeth in the pinion, z_{1p} , on values of the normal module, m_n , or the normal diametral pitch, P_n , the helix angle, β , and the tip diameter factors, V_1 , V_2 , and V_T , are summarised in Table 9.2.

6. STANDARD NORMAL MODULES, NORMAL DIAMETRAL PITCHES AND NORMAL PITCHES

It is possible in very nearly all designs to use a standard cutter. Normal modules and diametral pitches for standard cutters are listed in Table 9.1 along with equivalent normal pitches. Three systems of standard pitch, based on normal module, m_n , normal diametral pitch P_n , and normal pitch, p_n , are used. The preferred values of modules and diametral pitches given in Table 9.1 are the same as those recommended in Derivations 7, 10, 11 and 15.

7. ADDENDUM MODIFICATION

Addendum modification leads to an increase or decrease in the blank diameter of a standard gear. This change results from the displacement of a standard cutting tool from its nominal position and hence the gear is cut to a standard tooth depth on an oversize or undersize blank (see Sketch 7.1). The nominal pitch circles of a meshing gear pair so produced are no longer tangential, and because of this are instead termed reference circles. The actual pitch circles are tangential at the pitch point (see Sketch 7.1). The profile of the tooth is changed when addendum modification is used, and these changes can be seen in Figure 7.

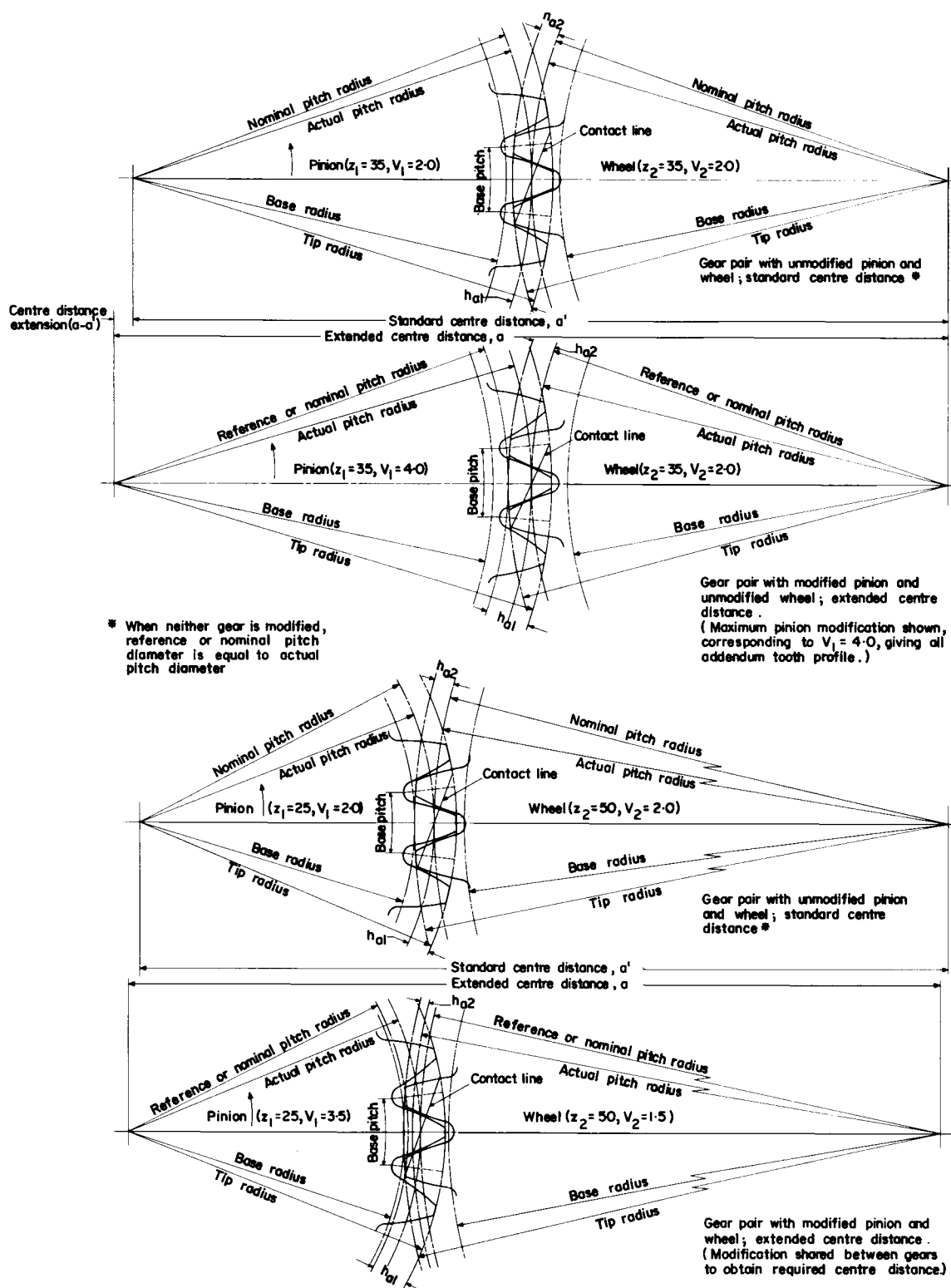
The advantages of using addendum modification are that

- (a) a gear pair can be designed to work at a predetermined centre distance without recourse to non-standard pitches,
- (b) undercut tooth profiles can be avoided,
- (c) tooth proportions can be optimised,
- (d) designs with a working pressure angle, α_w , other than 20° can be obtained even though generated from the British Standard basic rack form with a 20° pressure angle.

It should be noted that an increase or decrease in standard centre distance will automatically result in a corresponding change in pressure angle (see Section 7.4), and an increase in backlash.

7.1 Increased, Decreased and Standard Centre Distances

When addendum modification is used, the pinion and/or the wheel diameters are increased or decreased to accommodate the increase or decrease in centre distance (see Sketch 7.1). A standard centre distance may still be obtained if the amount of addendum modification applied to one gear (usually the pinion) is removed from the other gear in the pair (usually the wheel). These methods of modification are described in Sections 7.1.1 to 7.1.4, and are summarised in Table 9.4.



Sketch 7.1

7.1.1 Designs using standard centre distances with non-standard pinions and wheels

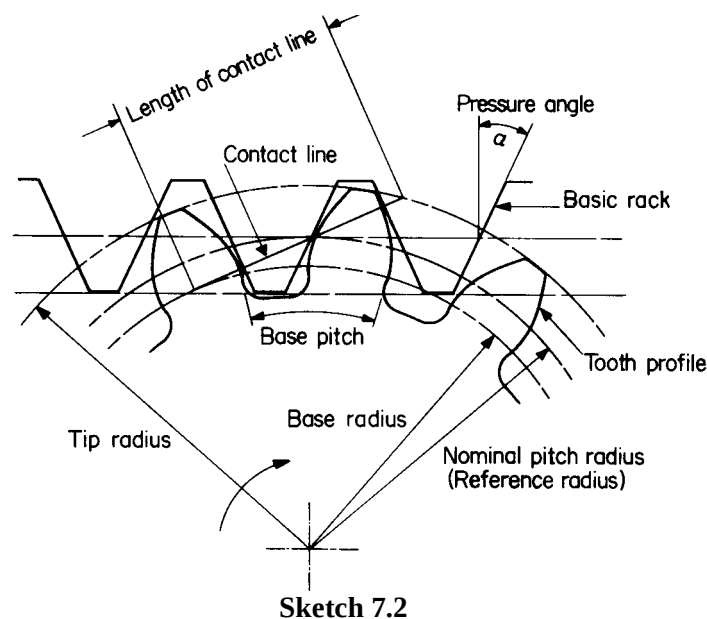
With these designs the pinion diameter may be increased to alleviate weakening of the gear caused by an undercut tooth profile, whilst the wheel diameter is reduced by the same amount. Although dependent on the tooth numbers, the wheel will not usually have an undercut tooth profile when this modification method is applied. This results in an unchanged standard centre distance, and an unchanged pressure angle of 20° . There are the following advantages of using this method of modification.

- (a) The contact ratio is higher than that obtained with a gear pair working at an extended centre distance. (The contact ratio is the ratio of the length of the path of contact to the base pitch, see Sketch 7.1 or 7.2.) Higher values of contact ratio give rise to lower tooth bending stresses and improved load sharing between teeth.
- (b) A standard centre distance and pressure angle are used.

There is the following disadvantage.

- (a) Neither the pinion nor the wheel can be of standard proportions.

If the number of teeth in the pinion is greater than that given in Table 9.3 an undercut tooth profile will not exist and, providing that there is no restriction on the centre distance, it may be possible to design a standard gear pair to work at a standard centre distance (see Section 7.1.4 and Table 9.4).



7.1.2 Designs using extended centre distances with standard wheels

With these designs the pinion diameter may be increased to alleviate weakening of the gear caused by an undercut tooth profile whilst the wheel remains at standard proportions. This results in an increased centre distance, but with a standard wheel. There are the following advantages of using this method of modification.

- (a) When low gear ratios with small numbers of teeth in the pinion and wheel are used (and the wheel would be undercut if reduced in size) undercutting of the wheel is avoided.

- (b) The load capacity of the gear pair is improved.
- (c) Only the pinion is changed from standard proportions.

There are the following disadvantages.

- (a) The centre distance is non-standard and hence the working pressure angle, α_w , is greater than the nominal value of 20° , which gives rise to increased backlash.
- (b) The contact ratio is less than that obtained with a gear pair working at a standard centre distance. (The contact ratio is the ratio of the length of the path of contact to the base pitch, see Sketch 7.1 or 7.2.) Lower values of contact ratio give rise to higher tooth bending stresses and worse load sharing between teeth.

If the numbers of teeth in the pinion and wheel are greater than those given in Table 9.3 an undercut tooth profile will not exist and, providing that there is no restriction on the centre distance, it may be possible to design a standard gear pair to work at a standard centre distance (see Section 7.1.4 and Table 9.4).

7.1.3 Designs using extended centre distances with non-standard pinions and wheels

With these designs the pinion diameter may be increased to alleviate weakening of the gear caused by an undercut tooth profile whilst the wheel diameter is also increased. This results in an increased centre distance with both the pinion and wheel of non-standard proportions. There are the following advantages of using this method of modification.

- (a) It allows a gear pair to be designed to a predetermined centre distance.
- (b) It removes undercutting from the pinion (and the wheel when low gear ratios and small numbers of teeth are used).
- (c) The load capacity of the gear pair is improved.
- (d) It ensures that good meshing of the teeth occurs when low gear ratios and small tooth numbers are used.

There are the following disadvantages.

- (a) The centre distance is non-standard and hence the working pressure angle, α_w , is greater than the nominal value of 20° , which gives rise to increased backlash.
- (b) Both the pinion and the wheel require modification.
- (c) The contact ratio is less than that obtained with a gear pair working at a standard centre distance. (The contact ratio is the ratio of the length of the path of contact to the base pitch, see Sketch 7.1 or 7.2.) Lower values of contact ratio give rise to higher tooth bending stresses and worse load sharing between teeth.

If the numbers of teeth in the pinion and wheel are greater than those given in Table 9.3, an undercut tooth profile will not exist and, providing that there is no restriction on the centre distance, it may be possible to design a standard gear pair to work at a standard centre distance (see Section 7.1.4 and Table 9.4).