

ELASTIC STRESSES AND DEFLECTIONS UNDER UNIFORM PRESSURE OF FLAT RECTANGULAR SANDWICH PANELS; ALL EDGES CLAMPED (ISOTROPIC FACE PLATES AND ORTHOTROPIC CORES OF ZERO FLEXURAL STIFFNESS)

1. NOTATION

a	length of panel in x -direction	m	in
B	face stiffness parameter given by $B = \left[\frac{1}{E_1 t_1} + \frac{1}{E_2 t_2} \right]^{-1}$	N m/m ²	lbf in/in ²
b	width of panel in y -direction	m	in
d	distance between centre-lines of face plates	m	in
E	modulus of elasticity of face plate material	N/m ²	lbf/in ²
f_{Ex}	maximum direct stress in x -direction at short edge of face plate	N/m ²	lbf/in ²
f_{Ey}	maximum direct stress in y -direction at long edge of face plate	N/m ²	lbf/in ²
f_x, f_y	maximum direct stresses in face plate at centre of panel in x - and y -directions, respectively	N/m ²	lbf/in ²
G_{cx}, G_{cy}	shear moduli of core in zx - and yz -planes, respectively	N/m ²	lbf/in ²
h	depth of core	m	in
p	normal pressure on panel	N/m ²	lbf/in ²
q_{cx}, q_{cy}	maximum shear stresses in core in zx - and yz -planes, respectively	N/m ²	lbf/in ²
q_{xy}	maximum face plate shear stresses	N/m ²	lbf/in ²
t	thickness of face plate	m	in
w	maximum normal deflection of panel	m	in
x, y, z	panel reference axes (see Sketch 2.1)		
<i>Suffixes</i>			
1, 2	identification numbers for thicknesses, stresses and moduli of face plates		

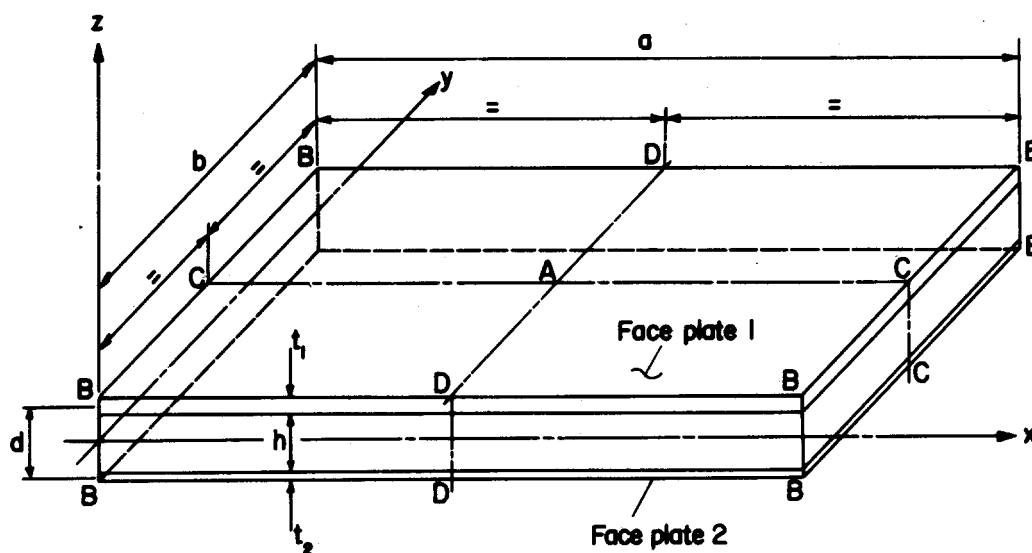
Both SI and British Units are quoted but any coherent system of units may be used.

2. NOTES

Curves are given for the maximum deflection and the maximum direct and shear stresses in initially flat rectangular sandwich panels of uniform thickness and various length to width ratios with all edges clamped under uniform normal pressure. The face plates are isotropic and have the same value of Poisson's ratio; however, they can be of different thicknesses and have different moduli of elasticity. The core material may be either isotropic or orthotropic. It is assumed that while the core provides a shear path between the faces its own flexural stiffness makes no significant contribution to that of the panel.

In Figure 1 the maximum deflection of the panel is plotted in the form wBd^2/pb^4 against $Bh/G_{cy}b^2$ for various values of the ratio G_{cy}/G_{cx} grouped for particular values of a/b . Similarly the maximum direct stresses in the face plates at the centre of the panel in the x - and y -directions are plotted in the form fdt/pb^2 in Figures 2 and 3. In a similar form, the maximum direct stresses in the face plates at the edges of the panel and the maximum shear stresses in the face plates are given in Figures 4a, 4b, 5 and 6. The maximum shear stresses in the core plotted in the form qd/pb are given in Figures 7 and 8.

The positions at which the maximum deflection and stresses occur are shown in Sketch 2.1. The stresses f_x and f_y and the maximum deflection occur at Point A in the centre of the panel. These stresses are compressive in the face plate on which the higher positive pressure is applied and tensile in the other plate. The maximum direct stresses at the edges, f_{Ex} and f_{Ey} , occur at points C and D, respectively, and are tensile in the face plate on which the higher positive pressure acts and compressive in the other plate. The maximum shear stress in the face plate, q_{xy} , occurs on the diagonals between the centre point A and the corner points B. The maximum shear stresses in the core, q_{cx} and q_{cy} , occur at points C and D in the short and long edges, respectively.



Sketch 2.1 Panel geometry

The curves are based on elastic small deflection theory applied to sandwich panels and have been calculated for a value of Poisson's ratio of 0.3. The face plates are assumed to be thin compared to the depth of the core and to resist the bending moments developed in the panel by acting differentially. The core is assumed to be infinitely stiff in the transverse (z) direction, the shear stresses being constant throughout its depth. These assumptions will be valid provided that $d/t \geq 6$, $b/d \geq 10$ and that w/d is greater than 0.5.

3. DERIVATION

The derivation lists sources that have assisted in the preparation of this Item.

1. BRUCE, J.
ROBINSON, J.R. The buckling and bending of orthotropic sandwich panels in compression and shear. Unpublished work at Handley Page Ltd, 1956.
2. PLANTEMA, F.J. *Sandwich construction. The bending and buckling of sandwich beams, plates, and shells.* Chapter 3, pp. 55-77, John Wiley and Sons, Inc., New York, 1966.
3. ALLEN, H.G. *Analysis and design of structural sandwich panels.* Pergamon Press, London, 1969.
4. FOLIE, G.M. Bending of clamped orthotropic sandwich plates. *Journal of the engineering mechanics division, Proceedings of the American Society of Civil Engineers*, EM3, pp. 243-265, June 1970.

4. EXAMPLE

It is required to find the maximum deflection of and stresses in a flat rectangular sandwich panel of width 520 mm and length 900 mm having all its edges clamped, when subjected to a pressure of 75 kN/m² that acts on face 1. The panel has an orthotropic honeycomb core of depth 50 mm aligned so that the maximum shear modulus of 329 MN/m² is along the length of the panel (the zx-plane) and the minimum shear modulus of 128 MN/m² is across the width of the panel (the yz-plane). The face plate thicknesses are $t_1 = 0.6$ mm and $t_2 = 0.4$ mm and both are made of aluminium alloy with $E = 69\,000$ MN/m².

From the given data it follows that

$$\frac{a}{b} = 1.73,$$

$$d = 50 + \left(\frac{0.6 + 0.4}{2} \right) = 50.5 \text{ mm},$$

$$B = \left[\frac{1}{69\,000 \times 10^6} \left(\frac{1}{0.6 \times 10^{-3}} + \frac{1}{0.4 \times 10^{-3}} \right) \right]^{-1} = 16.5 \text{ MN m/m}^2,$$

so that

$$\frac{Bh}{G_{cy}b^2} = \frac{16.5 \times 10^6 \times 50 \times 10^{-3}}{128 \times 10^6 \times (520 \times 10^{-3})^2} = 0.0238$$

and

$$\frac{G_{cy}}{G_{cx}} = \frac{128}{329} = 0.389.$$

Also

$$\frac{d}{t_1} = 84 > 6 \text{ and } \frac{b}{d} = 10.3 > 10$$

and therefore the curves may be used for the analysis of this panel.

The maximum deflection is obtained by interpolation from Figure 1 which yields

$$\frac{wBd^2}{pb^4} = 0.0045$$

so that

$$w = \frac{0.0045 \times 75 \times 10^3 \times (520 \times 10^{-3})^4}{16.5 \times 10^6 \times (50.5 \times 10^{-3})^2} = 0.000586 \text{ m or } 0.586 \text{ mm.}$$

It can be seen that $w/d = 0.0116 < 0.5$.

The maximum direct stresses at the centre of the face plates are obtained from Figures 2 and 3.

From Figure 2 by interpolation

$$\frac{f_{x1}dt_1}{pb^2} = \frac{f_{x2}dt_2}{pb^2} = 0.026$$

so that

$$f_{x1} = \frac{0.026 \times 75 \times 10^3 \times (520 \times 10^{-3})^2}{50.5 \times 10^{-3} \times 0.6 \times 10^{-3}} = \frac{0.026 \times 402 \times 10^6}{0.6} = 17.4 \text{ MN/m}^2$$

and

$$f_{x2} = \frac{0.026 \times 402 \times 10^6}{0.4} = 26.1 \text{ MN/m}^2.$$

Similarly, from Figure 3, $\frac{f_{y1}dt_1}{pb^2} = \frac{f_{y2}dt_2}{pb^2} = 0.039$

so that

$$f_{y1} = \frac{0.039 \times 402 \times 10^6}{0.6} = 26.1 \text{ MN/m}^2$$

and

$$f_{y2} = \frac{0.039 \times 402 \times 10^6}{0.4} = 39.2 \text{ MN/m}^2.$$

The maximum direct stresses at the edges of the face plates are obtained from Figures 4a, 4b and 5.

From Figures 4a and 4b by interpolation

$$\frac{f_{Ex1}dt_1}{pb^2} = \frac{f_{Ex2}dt_2}{pb^2} = 0.060$$

so that

$$f_{Ex1} = \frac{0.06 \times 402 \times 10^6}{0.6} = 40.2 \text{ MN/m}^2$$

and

$$f_{Ex2} = \frac{0.06 \times 402 \times 10^6}{0.4} = 60.3 \text{ MN/m}^2.$$

Similarly, from Figure 5, $\frac{f_{Ey1}dt_1}{pb^2} = \frac{f_{Ey2}dt_2}{pb^2} = 0.067$

so that

$$f_{Ey1} = \frac{0.067 \times 402 \times 10^6}{0.6} = 44.9 \text{ MN/m}^2$$

and

$$f_{Ey2} = \frac{0.067 \times 402 \times 10^6}{0.4} = 67.3 \text{ MN/m}^2.$$

The maximum shear stress in the face plates is obtained in a similar manner from Figure 6 which gives

$$\frac{q_{xy1} dt_1}{pb^2} = \frac{q_{xy2} dt_2}{pb^2} = 0.010$$

so that

$$q_{xy1} = \frac{0.010 \times 402 \times 10^6}{0.6} = 6.7 \text{ MN/m}^2$$

and

$$q_{xy2} = \frac{0.010 \times 402 \times 10^6}{0.4} = 10.1 \text{ MN/m}^2.$$

The maximum shear stresses in the core are obtained from Figures 7 and 8.

From Figure 7 by interpolation

$$\frac{q_{cx}}{p} \frac{d}{b} = 0.420$$

so that

$$\begin{aligned} q_{cx} &= \frac{0.420 \times 75 \times 10^3 \times 520 \times 10^{-3}}{50.5 \times 10^{-3}} = 0.420 \times 0.772 \times 10^6 \\ &= 0.324 \text{ MN/m}^2. \end{aligned}$$

From Figure 8

$$\frac{q_{cy}}{p} \frac{d}{b} = 0.525$$

so that

$$q_{cy} = 0.525 \times 0.772 \times 10^6 = 0.405 \text{ MN/m}^2.$$

This example may be compared with the similar examples of Item Nos 76013 and 76022 in which panels having all edges simply-supported, and panels with long edges clamped and short edges simply-supported, are considered.