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Pressure losses in curved ducts: interaction factors for two bends in series

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THE PREPARATION OF THIS DATA ITEM

The work of the permanent professional staff of the Engineering Sciences Data Unit on this particular Item was monitored and guided by the Fluid Mechanics Steering Group which had the following constitution:

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The Item was accepted for inclusion in the Aerodynamics Sub-series by the Aerodynamics Committee which has the following constitution

(Continued on inside back cover)

PRESSURE LOSSES IN CURVED DUCTS: INTERACTION FACTORS FOR TWO BENDS IN SERIES

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PRESSURE LOSSES IN CURVED DUCTS: INTERACTION FACTORS FOR TWO BENDS IN SERIES

1. NOTATION AND DEFINITIONS

1.1 Notation

		SI Units
A	internal cross-sectional area* of duct	m ²
b	internal dimension* of rectangular cross-sectional duct normal to plane of bend (see Sketch 1.1)	m
D	internal diameter* of circular cross-sectional duct	m
D_E	equivalent diameter of duct of non-circular cross section (also known as hydraulic diameter), $4 \times$ cross-sectional area/internal perimeter	m
f	Fanning friction factor for fully-developed flow in a straight pipe, $\Delta p_s / [4(\frac{1}{2}\rho V^2)(L/D)]$; the value may be obtained from ESDU 66027 (Reference 3)	–
h	internal dimension* of duct cross section in plane of bend; duct width for rectangular section (see Sketch 1.1)	m
K_s	static-pressure loss coefficient for an individual bend in interference-free flow; the value is obtained from ESDU 83037 (Reference 4)	–
L	length of straight duct or pipe (measured along centre-line, see Sketches 1.1 to 1.3, and 4.1)	m
p	static pressure	Pa (N/m ²)
Δp_s	static-pressure loss in the absence of terms due to gravity	Pa (N/m ²)
Δp_t	total-pressure loss (mass averaged)	Pa (N/m ²)
m	mass flow rate	kg/s
n	number of individual bends comprising a composite mitre bend (see Sketch 1.3)	–
R	radius of curvature [†] of bend centre-line (see Sketches 1.1 and 1.3)	m
Re	Reynolds number, $\rho V D_E / \mu$	–
u	local absolute velocity	m/s

For footnotes see end of Notation Section.

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V	mean flow velocity, $m/\rho A$	m/s
α	kinetic energy coefficient (defined by Equation (4.3) in Section 4.2)	—
β	factor accounting for interference between bends in combination (defined by Equation (4.1) in Section 4.1)	—
γ	configuration angle, angle between direction of inlet and outlet flows (see Sketch 1.4)	degree
ε	effective roughness height of pipe surface; the value may be obtained from ESDU 66027 (Reference 3)	m
θ	overall turning angle of a bend (see Sketches 1.1, 1.2 and 1.3)	degree
λ	factor correcting for effect of downstream tangent length	—
μ	dynamic viscosity of fluid	N s/m ²
ρ	density of fluid	kg/m ³

Subscripts

a	refers to segment spacer length of composite mitre bend (see Sketch 1.3)
b	refers to total straight length of bend (see Sketch 4.1)
d	refers to tangent length downstream of second bend of combination (see Sketch 4.1)
fd	refers to position at which fully-developed straight pipe flow (as defined in Section 1.2) condition exists (see Sketch 4.1)
min	refers to minimum value
s	refers to spacer length between bends (see Sketches 1.4 and 4.1)
u	refers to tangent length upstream of first bend of combination (see Sketch 4.1)
1	refers to first bend of combination
2	refers to second bend of combination

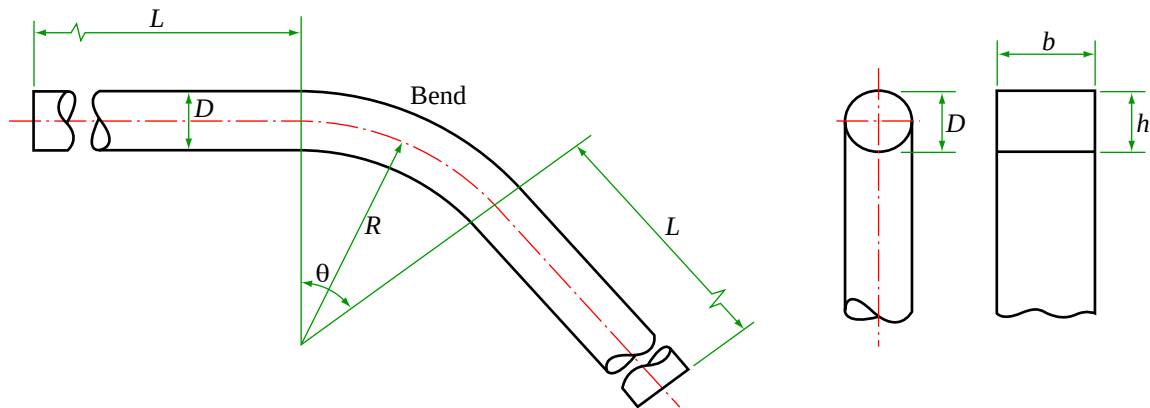
* The internal cross-sectional dimensions to be used in this Item must be actual values and not the nominal values quoted by manufacturers which may be considerably different.

† For composite mitre bends R is defined as $(L_d/2)/\cot(\theta/2n)$.

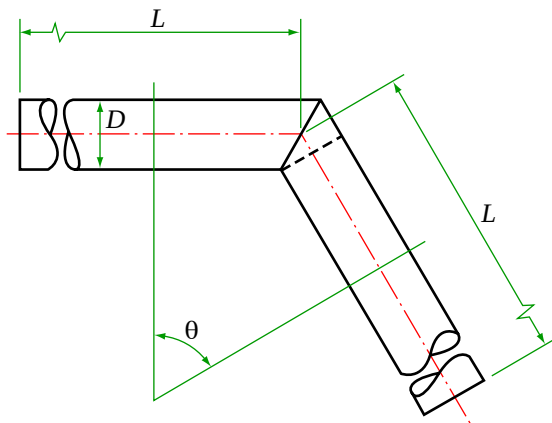
1.2 Definition

Fully-developed flow, for the purposes of this Item, is defined as flow having a constant pipe friction gradient; the appropriate value of pipe friction gradient may be calculated from ESDU 66027 (Reference 3).

1.3 Geometry of Bends

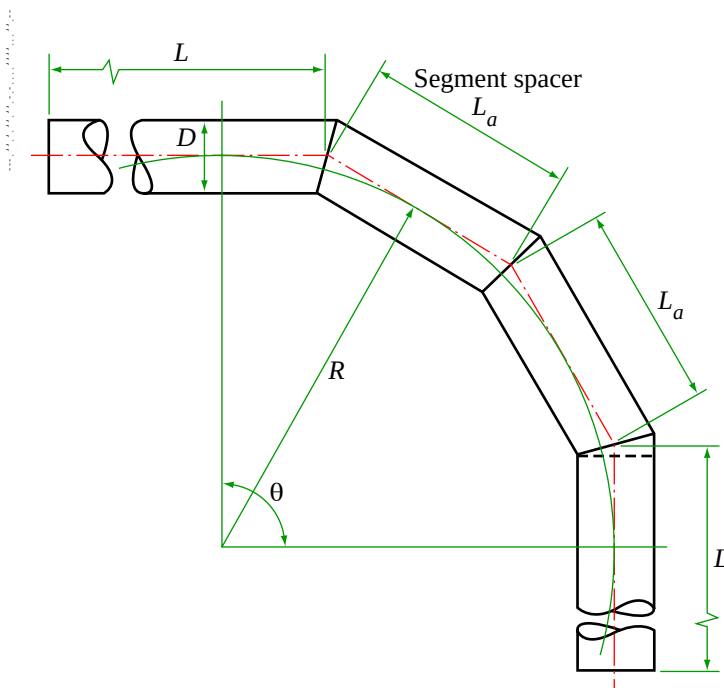


Sketch 1.1 Circular-arc bend



Sketch 1.2 Single mitre bend
Minimum downstream tangent shown dotted

$$L_{d,min} = \frac{D}{2} \tan \frac{\theta}{2}$$



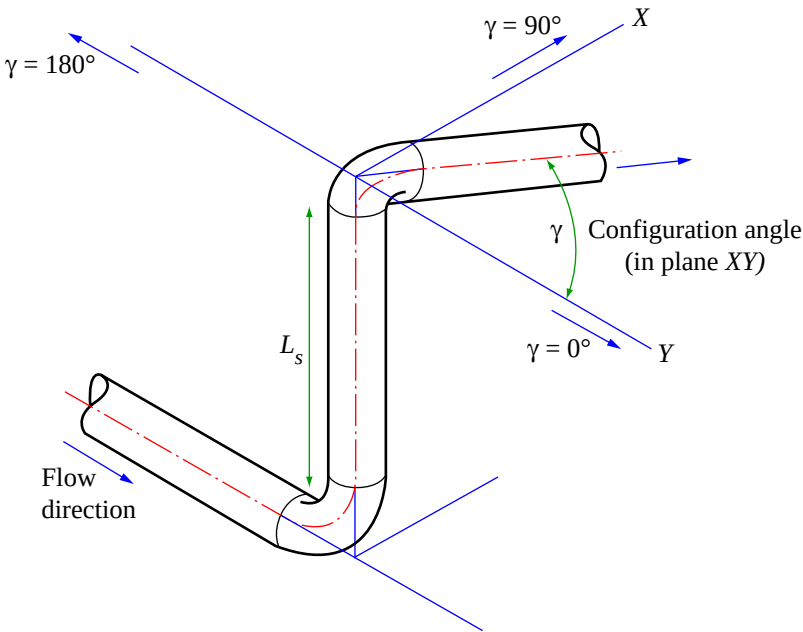
Sketch 1.3
Composite mitre bend

$$R = \frac{D}{2} \cot \frac{\theta}{2n}$$

(n is number of individual bends $n = 3$ in example shown)

Minimum downstream tangent shown dotted

$$L_{d,min} = \frac{D}{2} \tan \frac{\theta}{2n}$$



Sketch 1.4
Configuration angles for two 90° ($\theta_1 = \theta_2 = 90^\circ$) bends in combination

2. INTRODUCTION

With two bends in series the overall pressure loss is not obtained by summing the individual bend losses unless the spacing between the bends is sufficiently large to eliminate interference effects (see Table 11.2).

This Item gives values of a correction or interaction factor, β , that relate the overall loss of combinations of two bends in series to the sum of the losses for the individual bends in interference-free flow.

The types of bends considered in this Item are circular arc and mitre, both of constant cross-sectional area. A further subdivision of mitre bends into either single mitre or composite types is made. The geometries of the three types of bends are shown in Sketches 1.1 to 1.3.

For a particular combination of bends the interaction factor, β , is given as a function of the following geometrical parameters.

- | | | |
|-----|---------------------------|---|
| (a) | θ_1 and θ_2 | the overall turning angles of the individual bends. |
| (b) | γ | the configuration angle (see Sketch 1.4) of the two bends (angle between directions of inlet and outlet flows). |
| (c) | L_s/D | the non-dimensional spacer length between bends. |
| (d) | L_d/D | the non-dimensional downstream tangent length (for circular-arc bends only). |
| (e) | $(R/D)_1$ and $(R/D)_2$ | the radius ratios of individual bends (where applicable). |

3. APPLICABILITY OF THE DATA

The data in this Item are only applicable to:

- (i) single-phase fluids,
- (ii) fluids which can be regarded as incompressible (see Table 11.1) and Newtonian,
- (iii) duct or pipe systems running full,
- (iv) flows with negligible heat transfer.

In addition, the data are strictly only applicable for bend-tangent connections that produce flush internal surfaces between these components, as for example carefully aligned flanged couplings (Section 5.8 gives some guidance for other conditions).

The limited data available are mainly for smooth-surfaced, carefully manufactured (see Section 7) bends of circular cross section at Reynolds numbers of the order of 10^5 . It is assumed that the effects of surface roughness and Reynolds number are accounted for in the loss coefficient, K_s , of each individual bend, and that the interaction factor, β , is independent of those parameters. For bends of non-circular cross section, see Section 6.

The ranges of applicability of the parameters relevant to the data given in this Item are summarised in Table 11.1.

For various combinations of bends the maximum spacer length, L_s , for negligible interference effects, *i.e.* $\beta = 1$, between two bends in series is given in Table 11.2. For larger values of L_s this Item is not required since the pressure loss due to each bend can be calculated separately.

4.2 Total-pressure Loss

The total-pressure loss corresponding to the static-pressure loss between two sections which span a combination of two bends in series may be expressed as:

$$\Delta p_t = \Delta p_s + \frac{1}{2} \rho V^2 (\alpha_u - \alpha_d) \quad (4.2)$$

where u and d refer to sections in the upstream and downstream tangents respectively, and α , the kinetic energy coefficient which allows for the effects of a non-uniform velocity profile, is given by:

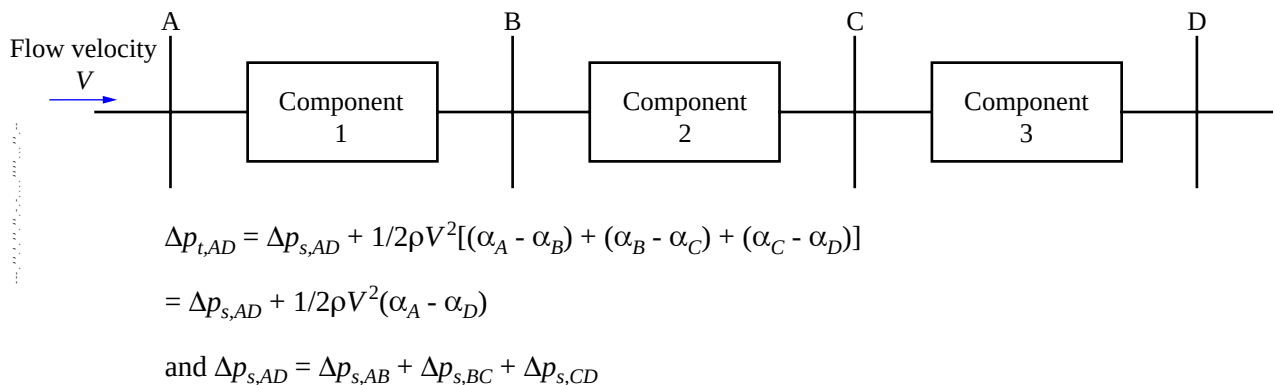
$$\alpha = \left(\frac{1}{m} \int_A \frac{1}{2} \rho u^2 dm \right) / \left(\frac{1}{2} \rho V^2 \right). \quad (4.3)$$

Between sections where fully-developed flow exists, i.e. $L_u/D = (L_u/D)_{fd}$ and $L_d/D \geq (L_d/D)_{fd, min}$ (see Table 11.2), α_d will approximate to α_u (which is equal to α_{fd}) and Equation (4.2) reduces to:

$$\Delta p_t = \Delta p_s. \quad (4.4)$$

For downstream tangent lengths insufficiently long to allow fully-developed flow (as defined in Section 1.2) to recover, i.e. $L_d/D < (L_d/D)_{fd, min}$, little information on velocity profiles is available and generally no evaluation of α_d is possible. However, the velocity profile at exit from a two-bend combination is usually distorted considerably from the velocity profile at inlet to the first bend. As a result, α_d at $L_d/D = 0$ is always greater than α_u . With increasing L_d/D , the value of α_d rapidly falls to the fully-developed flow condition, i.e. $\alpha_d = \alpha_{fd}$ at $L_d/D = (L_d/D)_{fd}$ (in turbulent flow, for $Re = 10^5$, $\alpha_{fd} \approx 1.06$). Hence for an estimate of maximum total-pressure loss the minimum value of α_d (equal to α_{fd}) is substituted into Equation (4.2) which, when fully-developed flow occurs upstream of the first bend ($\alpha_u = \alpha_{fd}$), reduces to $\Delta p_t = \Delta p_s$.

Note that when the overall pressure losses for a system of several components are required, knowledge of the kinetic pressure terms, $(\frac{1}{2} \rho V^2) \alpha$, of intermediate components is not required since each term must occur twice, once as a negative quantity and once as a positive quantity, and will cancel out in the overall calculation (see the Example in Section 9 and Sketch 4.2).



Sketch 4.2 System pressure losses for several components in series