

FLAT PANELS IN SHEAR. POST-BUCKLING ANALYSIS

1. NOTATION

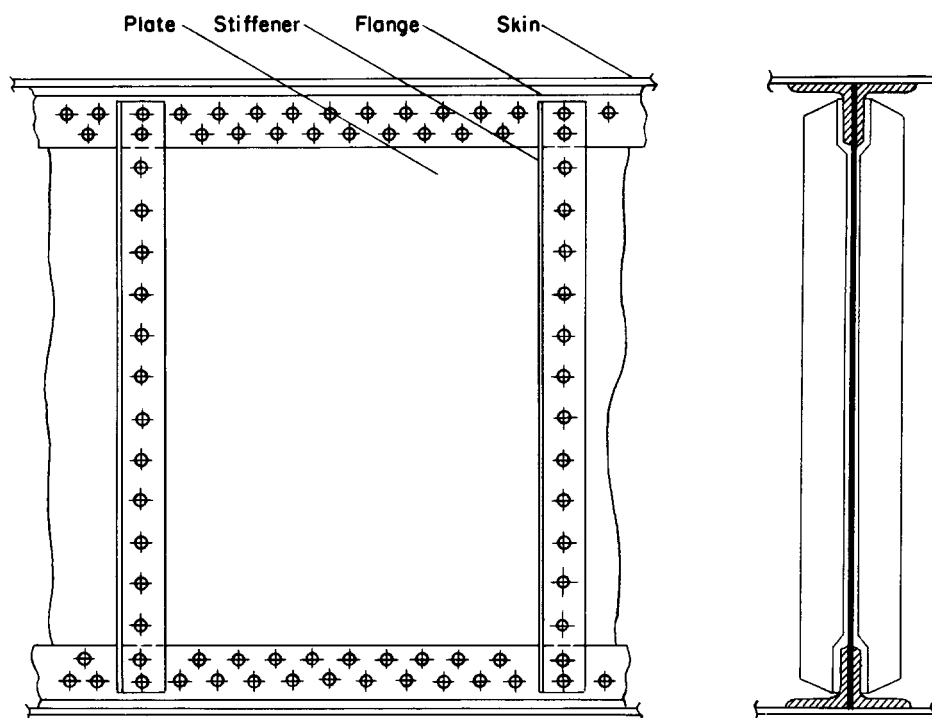
A_f	cross-sectional area of flange and associated skin excluding associated plate (see Sketch 1.1)	m^2	in^2
A_{fe}	cross-sectional area of flange, associated skin and associated plate	m^2	in^2
A_s	cross-sectional area of stiffener excluding associated plate (see Sketch 1.1)	m^2	in^2
a	flange spacing or length of plate	m	in
b	stiffener spacing or width of plate	m	in
C	parameter correcting f_s for plate aspect ratio		
F	compressive end load in flange of area A_{fe} due to tension field	N	lbf
f_{all}	allowable tensile stress for plate material in simple tension	N/m^2	lbf/in^2
f_{pr}	tensile stress in plate parallel to flange at flange to plate rivet line	N/m^2	lbf/in^2
f_s	average compressive stress in stiffener at centre of its length	N/m^2	lbf/in^2
f_{sc}	value of f_s at which forced crippling of stiffener occurs	N/m^2	lbf/in^2
f_{sh}	maximum compressive stress at heel of stiffener (equal to f_s in double stiffeners)	N/m^2	lbf/in^2
f_t	tensile strength of plate material	N/m^2	lbf/in^2
G	shear modulus of material of plate	N/m^2	lbf/in^2
G_S, G_T	secant and tangent moduli of panel, respectively (see Section 5)	N/m^2	lbf/in^2
h	distance between centroids of flanges (area A_f)	m	in
I_f	second moment of area of flange (area A_f) about an axis through its centroid normal to plate	m^4	in^4
I_s	second moment of area of stiffener (area A_s) about an axis through its centroid parallel to plate	m^4	in^4
k	diagonal tension factor		
l'	equivalent pin-ended column length of stiffener	m	in
M	maximum bending moment in flange due to tension field	N m	lbf in

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Q	net panel shear load parallel to stiffeners	N	lbf
q	nominal shear stress in plate $Q/(ht)$	N/m^2	lbf/in^2
q_a	allowable nominal shear stress in plate	N/m^2	lbf/in^2
q_b	shear stress at which plate first buckles	N/m^2	lbf/in^2
R	maximum resultant loading per unit run along plate to flange joint	N/m	lbf/in
t	plate thickness	m	in
t_{so}	thickness of outstand of simple angle stiffener	m	in
t_{2s}	0.2 per cent proof stress in tension of stiffener material	N/m^2	lbf/in^2
$\bar{y}_s$	distance between centroid of stiffener (area A_s) and mid-plane of plate	m	in
βb	average width of plate assumed to act with each stiffener in resisting instability under end load due to tension field	m	in

Both SI and British units are quoted but any coherent system of units may be used.



Sketch 1.1

2. INTRODUCTION

This Item provides data for the analysis of flat panels carrying shear loads in which the plate has buckled. A panel is defined as a thin plate, or sheet, reinforced by a system of longitudinal and transverse members, as shown in Sketch 1.1. The panels are assumed to be part of a beam, as in an aircraft spar. It is possible, with suitable modification, to use the data provided to analyse a continuous array of panels formed by orthogonal stiffeners on a large sheet.

A panel buckled in shear deforms into a set of inclined waves and carries a tensile stress, approximately parallel to the waves, together with a smaller perpendicular compressive stress. This leads to loadings on the panel edge members. Thus, the stress distribution can be conveniently described as a combination of a pure shear stress and a “diagonal tension field” acting in the general direction of the buckles. The proportion of the total shear reacted by diagonal tension is termed the “diagonal tension factor”, k , and curves of k plotted against q/q_b are provided in Item No. 77018*. This Item deals with the analysis of this complex post-buckled stress system and the following sections provide the data listed below.

Section 4.2 Maximum nominal plate shear stress at (a) failure of the plate at a buckle and (b) failure of the plate at the rivet line.

Section 4.3 Compressive stress in the stiffener; also (a) the effective length of the stiffener together with the skin which may be associated with it in resisting stiffener overall buckling and (b) the value of the compressive stress in the stiffener at which forced crippling will occur.

Section 4.4 End load and bending moment acting on the flanges as a result of the diagonal tension field.

Section 4.5 Load per unit run along the flange to plate connection.

Section 5 Secant and tangent moduli of the panel.

A fully worked example employing these data is given in Section 7.

3. GENERAL CONSIDERATIONS AND LIMITATIONS

The data have been derived on the assumption that the panels are loaded uniformly in pure shear but will usually be applied to beams in which the flanges carry tensile or compressive loads. The majority of the data are based upon experiment and therefore include the effects of any non-uniformity in the stress pattern in the panel. The data apply to panels with $0.2 \leq b/a \leq 1.0$ with a well developed tension field, q/q_b greater than about 2, in which case the effect of initial irregularities of manufacture is not significant. For loads less than this initial irregularities of the plate may become significant and the discontinuity shown in the figures with an abscissa of q/q_b at a value of $q/q_b = 1.0$ is unlikely to occur. However, in the region below q/q_b of about 2 it may be more efficient to design a panel in which the stiffeners are of reduced stiffness and the plate of increased thickness so that failure occurs by simultaneous buckling of the plate and stiffeners.

* Data Item No. 77018 “Curved panels in shear. Post-buckling analysis”. The value of k may be obtained from Figure 1 by taking the radius of curvature as infinite.

The general method of analysis applies only in the elastic range, although some of the other Items referred to herein include data on the effects of exceeding the elastic range. It has been assumed that the stiffeners and flanges are manufactured from materials having the same value of modulus of elasticity as the plate. In the event of significantly differing moduli appropriate adjustments should be made, as in the example in Section 7.

When applying the data to a beam in bending it is necessary to determine an effective width of skin acting with the area A_f . For the compression flange this is likely to be governed by considerations of compressive instability and reference should be made to Data Items dealing with compressive buckling of plates. As a result of this the effective area of the compression flange is likely to be lower than that of the tension flange. Use of the compression flange value of A_f will result in slightly conservative stresses if $A_f/(ht)$ is less than about 2; above this value the plate and stiffener stresses and flange loads are relatively insensitive to variations in A_f .

The stiffeners may be attached either to one side of the plate only, in which case they are referred to as “single stiffeners”, or else to both sides, back to back, as in Sketch 1.1, when they are termed “double stiffeners”. Both single and double stiffeners are considered.

4. ANALYSIS OF PANEL STRENGTH

4.1 Initial Buckling of the Plate

Figures 4 to 10 of Item No. Struct. 02.03.02* provide data from which the elastic buckling stress of flat panels that form a continuous beam of the type illustrated in Sketch 1.1 may be calculated. (Note that that Item requires the calculation of the second moment of area of the stiffener to include a contribution from the plate.) In the application of Item No. Struct. 02.03.02 care must be taken when choosing the unsupported values for the “length” and “width” of the plate, see the example, Section 7.

4.2 Strength of the Plate

4.2.1 Permanent deformation and failure of the plate away from its edges

The buckled plate is subjected to a combination of shear, tensile and bending stresses, the effects of which are most severe at the crests of the waves. Therefore, yielding first occurs at these crests, and shows as slight waviness of the plate after unloading without appreciable change of form of the whole panel. With increased loading the yielding progresses through the thickness of the plate with the waviness becoming sufficient to cause appreciable permanent deformation of the whole panel. For ductile materials fracture of the whole plate, by tearing in a direction normal to the principal tensile stress, will not occur until, over some region of the plate, the average stress through the thickness is sufficient to cause failure. Figure 1 presents the allowable value of nominal shear stress, q_a , plotted as q_a/f_{all} against q_b/f_{all} for a range of values $A_s/(bt)$ for both permanent deformation and failure of the plate; for convenience in use lines of constant q_a/q_b are also given. The permanent deformation curves are provided as a guide to the onset of local waviness of the plate and have been calculated according to the shear strain energy criterion of von Mises and Hencky. When using these curves f_{all} should be taken to be the proof stress of the plate material. An estimate of the value of q_a at which permanent shear deformation of the whole panel occurs may be made by using the proof stress of the plate material in conjunction with the failure curves. The failure curves are based on the principal stress criterion and when used for strength estimation f_{all} may be taken as the tensile strength of the material of the plate.

* Item No. Struct. 02.03.02 “Flat panels in shear. Buckling of long panels with transverse stiffeners”.

Figure 1 has been derived on the assumption that the panel boundary members are rigid and therefore takes no account of any non-uniformity of the stress pattern in the plate or of the effects of stress concentrations such as those that occur at rivet holes. Therefore, in practice, the plate may fail at stresses slightly below the values obtained from Figure 1.

4.2.2 Failure of the plate at the plate-to-flange joint

In practice the stress distribution in the plate along the joint with the flange is different from that prevailing nearer the centre of the plate (see Section 4.2.1). Figure 2 gives the nominal shear stress, q_a , at which, in the absence of stress concentration effects, the combined stress field acting at the tensile flange produces failure of the plate. In Figure 2 f_{all} should be taken as the tensile strength of the plate material. The stresses have been combined using either the shear strain energy criterion of von Mises and Hencky or the principal stress criterion, depending upon which predicts failure at the lower stress. The small effect of the variation of the diagonal tension factor has been eliminated by taking the lower envelope to a family of curves. Stress concentrations due to flexibility of the flanges or to the presence of rivet holes have not been allowed for and may lead to failure at stresses below the value obtained from Figure 2.

In the presence of a stress concentration a more detailed calculation of the maximum stress can be made by taking account of the three individual components of the stress field, used in constructing Figure 2, and applying the appropriate stress concentration factor. The three components are a diagonal tensile stress k_q , a pure shear stress $(1 - k)q$ and a tensile stress f_{pr} . The value of k and the angle of the diagonal tension stress may be obtained from Item No. 77018.

4.3 Strength of the Stiffener

After the plate has buckled the stiffeners are required to support compressive loads due to the diagonal tension field and to resist the out-of-plane forces associated with the plate buckles. The stiffeners are therefore prone to failure by buckling, failure by forced crippling associated with the plate buckle, and failure of their attachments. Figures 3, 4 and 5 give the average compressive stress, f_s , acting on double section stiffeners. Note that Figure 5, for $A_f/(ht) = 0$, is provided as a guide to assist interpolation and that the parameter C , used to account for panel aspect ratio, is given by Figure 6. In the case of single stiffeners the reduced stiffness implied by the offset of the stiffener centroid from the plate is to some extent counteracted by the tendency of the diagonal tension field to keep the stiffener straight. Thus, the average stress is similar to that for a double stiffener of the same area. A conservative estimate of the stress likely to approximate to the stress at the heel of a single stiffener is given by using Figures 3, 4 and 5 but replacing A_s by $A_s/\{1 + \bar{y}_s^2 A_s/I_s\}$ and reading f_s as f_{sh} . The stresses f_s or f_{sh} , together with any stresses arising from sources other than the shear loading, should be compared with the appropriate failure stress in forced crippling and the various buckling modes as described in the following sub-sections.

4.3.1 Stability of the stiffener

The overall buckling stress of the stiffeners out of the plane of the plate should be estimated by means of Item No. Struct. 01.01.01*. When the stiffeners are resisting buckling an effective width βb of the adjacent plate acts with them and Figure 7 gives the coefficient β . The effective width of plate should be included in the calculation of the stiffener radius of gyration when using Item No. Struct. 01.01.01. In the case of a stiffener attached to the plate by more than one row of rivets the effective width of plate should be taken to extend $0.5\beta b$ beyond the outer rivet lines. The tension field in the plate gives some restraint to the stiffener against overall buckling. This may be accounted for by replacing the stiffener length by the reduced equivalent pin-ended strut length, l' , given by Figure 8, when using Item No. Struct. 01.01.01. Figure 8 does not include an allowance for constraints due to the attachment of the stiffener to the flanges. Therefore, l' may be further reduced by the rotational restraint at the flanges. Since the stiffener load arises from the

* Item No. Struct. 01.01.01 "The strength of struts".

plate it should be considered as acting through the mid-plane of the plate. Consequently, single stiffeners are eccentrically loaded and, as Item No. Struct. 01.01.01 shows, this considerably reduces their strength. For both single and double stiffeners an allowance should be made for the effects of manufacturing inaccuracies when applying Data Item No. Struct. 01.01.01.

With open section stiffeners the possibility of failure in a torsional-flexural buckling mode should be investigated using the method given in either Item No. 89007 or Item No. Struct. 01.01.10^{*}. The plate will provide restraint against torsional deformation because of the diagonal tension stress so that simple estimates of the type set out in Section 7.8.2.2 of the Example are likely to give values below the actual torsional-flexural buckling strength. Local buckling of the stiffener outstand should also be considered and Data Item No. Struct. 01.01.08[†] should be used for this purpose.

Failure of the stiffener by forced crippling occurs when the buckled plate, assisted by the compressive load in the stiffener, forces the stiffener to deform locally with a plate buckle. Figure 9 gives the value of f_{sc} at which this occurs. The figure is derived from limited data for unlipped and lipped angle and Z-section stiffeners.

4.3.2 Stiffener riveting

Where the plate and stiffener are riveted together the possibility of buckling between rivets at stress f_{sh} should be checked using Item No. Struct. 02.01.08[‡]. The tension caused by the buckled plate tends to force the stiffener and plate apart and may cause the rivets to fail. This effect will depend upon the degree of development of the tension field and the thickness of the plate. Limited data suggest that, provided the tensile load capacity per unit run of the riveted assembly exceeds $0.22t_f$ for single stiffeners and $0.15t_f$ for double stiffeners, failure in this mode is unlikely.

The stiffener-to-flange joints are required to transmit the compressive load in the stiffener. Figures 3, 4 and 5 give f_s , the average stress in the stiffener at its centre, but the stress is reduced towards the ends due to the increase in the effective width of plate. This reduction in end load is about 25 per cent at $q/q_b = 2$ falling to about 10 per cent at $q/q_b = 20$.

4.4 Strength of the Flanges

The components of the diagonal tension field parallel to and normal to the flanges introduce a direct compressive load and a bending moment, respectively, into the flanges. The direct compressive load, F , may be determined from Figures 10, 11 and 12 which give F/Q plotted against q/q_b for various values of $A_s/(bt)$ and $A_f/(ht)$; the set of curves for $A_f/(ht) = 0$ is provided as a guide to assist interpolation. Figure 13 gives the maximum bending moment in the flange, M , as $M/(qtb^2)$ plotted against q/q_b . The curve of Figure 13 is largely based on experimental evidence and, owing to the limited amount of data available, it is not possible to distinguish where the maximum value of bending moment occurs. In the absence of other information it is suggested that the maximum bending moment M should be assumed to occur both at the stiffeners and, with opposite sign, midway between them.

The rivets joining the plate to the flanges must transmit the shear complementary to that portion of Q carried by the plate in pure shear together with the components of the diagonal tension field parallel to and normal to the flange. The resultant loading has a maximum value, R , in the region of the stiffeners and Figure 14 gives $R/(qt)$ plotted against $I_f/(b^3t)$ for a range of values of q/q_b . Note that the rivets joining the stiffener to the flange must additionally transmit the compressive end load in the stiffener, see Section 4.3.2. Figure

* Item No. 89007 "Flexural and torsional-flexural buckling of thin-walled open section struts".

Item No. Struct. 01.01.10 "Torsional instability of stringers and struts of angle section".

† Item No. Struct. 01.01.08 "Local instability of struts with flat sides".

‡ Item No. Struct. 02.01.08 "Buckling in compression of sheet between rivets".

14 may also be used to obtain values of R for plate joints that are parallel to the stiffeners by taking $I_f/(b^3t)$ as infinite.

5. PANEL STIFFNESS

With the development of the diagonal tension field the overall in-plane stiffness of the panel is reduced. The reduction is represented by the secant modulus, G_S , which is the ratio of the nominal shear stress in the plate to the total shear strain of the panel, and by the tangent modulus, G_T , which is the rate of increase of nominal shear stress in the plate with total shear strain of the panel. Figures 15 to 17 and 18 to 20 give the ratios G_S/G and G_T/G , respectively, plotted against q/q_b for various values of $A_s/(ht)$ and $A_f/(ht)$. The curves of Figures 17 and 20 give a theoretical limit only and are presented as a guide to interpolation.

Figures 15 to 20 are derived on the assumption that the flanges do not bend. The effect of flange flexibility in the plane of the panel is to reduce the value of both G_S/G and G_T/G below that shown by the curves. The reduction is less than 10 per cent if $(A_s + \beta bt)b^3/(360I_f h)$ is less than 0.5, and less than 5 per cent if it is less than 0.2. For further information on the effect of flange flexibility, reference should be made to Derivation 10.

6. DERIVATION

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7. EXAMPLE

7.1 Additional Notation for this Example

It is convenient to declare the following additional symbols for use in the example described in Section 7.2. Note that where other Data Items are employed their notation will be used without definition.

E	modulus of elasticity of material	N/m ²	lbf/in ²
E_T	tangent modulus of material	N/m ²	lbf/in ²
f_{fc}	compressive stress in flange	N/m ²	lbf/in ²
f_{ft}	tensile stress in flange	N/m ²	lbf/in ²
f_n	stress at which $E_T = \frac{1}{2}E$ (see Item No. 76016)	N/m ²	lbf/in ²
f_{fr}	tensile stress acting along flange at flange-to-plate rivet line	N/m ²	lbf/in ²
f_{tp}	tensile strength of plate material	N/m ²	lbf/in ²
J_s	torsion constant of cross-sectional area of stiffener (area A_s)	m ⁴	in ⁴
k_s	radius of gyration of stiffener (area A_s) about an axis through its centroid parallel to plate	m	in
m	material characteristic (see Item No. 76016)		
P_f	direct load acting on flange (area A_{fe}) due to bending moment applied to panel in plane of plate	N	lbf
$\bar{y}_f$	distance between centroid of flange (area A_f) and mid-plane of skin	m	in
$\bar{y}_{fie}$	distance between centroid of flange (area A_{fe}) and innermost fibre of flange	m	in
$\bar{y}_{fre}$	distance between centroid of flange (area A_{fe}) and innermost rivet line	m	in

Suffixes

The following suffixes may be applied to the notation of Section 1 as well as to the above notation.

e	effective value calculated including a contribution from plate
f	relates to flange
s	relates to stiffener

Both SI and British units are quoted although only SI units are used in this example.