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Flat panels in shear – Post-buckling analysis



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THE PREPARATION OF THIS DATA ITEM

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The Structures Committee, which first met in September 1940, has the co-operation of many engineers and scientists in industry, research establishments and universities from whom much assistance and unpublished information is being received.

FLAT PANELS IN SHEAR – POST-BUCKLING ANALYSIS

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FLAT PANELS IN SHEAR – POST-BUCKLING ANALYSIS

1. NOTATION

Both SI and British units are quoted but any coherent system of units may be used.

A_f	cross-sectional area of flange and associated skin excluding associated plate (see Table 2.3 of ESDU 02005)	m ²	in ²
A_{fe}	cross-sectional area of flange, associated skin and associated plate (see Table 2.3 of ESDU 02005)	m ²	in ²
A_s	cross-sectional area of stiffener excluding associated plate (but including any pad that is present)	m ²	in ²
A_{se}	cross-sectional area of stiffener and associated plate (including any pad that is present) (see Table 2.2 of ESDU 02005)	m ²	in ²
a	full depth of panel excluding skin thickness	m	in
a_p	depth of web used for buckling analysis (see Table 2.3 of ESDU 02005)	m	in
b	stiffener pitch or spacing	m	in
b_p	width of plate used for buckling analysis (see Table 2.2 of ESDU 02005)	m	in
d	flange or stiffener free (unattached) height	m	in
E	elastic modulus of component material	N/m ²	lbf/in ²
E_T	tangent modulus of component material	N/m ²	lbf/in ²
e_m	manufacturing eccentricity of stiffener	m	in
f	direct stress in component	N/m ²	lbf/in ²
f_{as}	allowable extreme fibre stress of stiffener material	N/m ²	lbf/in ²
f_{np}, f_{ns}	stress at which $E_T = E / 2$ for plate and stiffener materials respectively	N/m ²	lbf/in ²
$f_{o,x}$	overall flexural buckling stress of stiffener/plate combination	N/m ²	lbf/in ²
f_{p1}, f_{p2}	stresses at points on plate material stress-strain curve	N/m ²	lbf/in ²
f_{pir}	inter-rivet buckling stress of plate material acting with stiffener	N/m ²	lbf/in ²
f_{pr}	tensile stress in plate normal to flange cross-section at flange-to-plate rivet line	N/m ²	lbf/in ²

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f_s	average applied compressive stress in stiffener at centre of its length	N/m ²	lbf/in ²
f_{sd}	local buckling stress of stiffener outstand (stiffener element with dimension d_s)	N/m ²	lbf/in ²
f_{sfc}	value of f_s at which forced crippling of stiffener occurs	N/m ²	lbf/in ²
f_{sh}	maximum compressive stress at heel of stiffener	N/m ²	lbf/in ²
$f_{s,oT}$	torsional-flexural buckling stress of stiffener/plate combination	N/m ²	lbf/in ²
f_{sz}	local buckling stress of stiffener flange (stiffener element with dimension z_s)	N/m ²	lbf/in ²
f_T	pure torsional buckling stress of stiffener/plate combination	N/m ²	lbf/in ²
f_{tf}, f_{tp}	tensile strengths of flange and plate materials respectively	N/m ²	lbf/in ²
G	shear modulus of component material	N/m ²	lbf/in ²
G_S, G_T	secant and tangent moduli respectively of plate	N/m ²	lbf/in ²
h_e	distance between centroids of flanges (area A_{fe})	m	in
h_y	distance of the stiffener/plate interface from the stiffener/plate centroid along the y-axis	m	in
I_f	second moment of area of flange (area A_f) about axis through its centroid normal to plate	m ⁴	in ⁴
I_{fe}	second moment of area of flange including associated skin and associated plate (area A_{fe})	m ⁴	in ⁴
I_{pse}	polar moment of inertia of stiffener and associated plate about axis through stiffener/plate shear centre	m ⁴	in ⁴
$I_{x, s}$	second moment of area of stiffener (area A_s) about axis through its centroid parallel to plate	m ⁴	in ⁴
$I_{x, se}$	second moment of area of stiffener and associated plate (area A_{se}) about axis through stiffener/plate centroid parallel to plate	m ⁴	in ⁴
J_s	torsional constant of stiffener cross-section (area A_s)		
J_{se}	torsional constant of stiffener and associated plate cross-section (area A_{se})		
K	buckling stress coefficient defined by $q_b = KE_p(t_p/b_p)^2$		
k	diagonal tension factor		

k_r	stiffener-to-plate attachment factor		
k_s	radius of gyration of stiffener (area A_s)	m	in
k_{se}	radius of gyration of stiffener-plate combination (area A_{se})	m	in
l'	equivalent pin-ended column length of stiffener	m	in
l_{sr}	rivet pitch along stiffener	m	in
M_b	overall bending moment on beam in plane of plate	N m	lbf in
M_f	maximum local bending moment in flange	N m	lbf in
m_p, m_s	material characteristics of plate and stiffener materials respectively		
P	allowable load in component (as denoted by appropriate suffixes)	N	lbf
P_{fm}	applied direct (end) load in flange due to overall bending moment on beam, $P_{fm} = M_b/h_e$	N	lbf
P_{fq}	flange compressive load component due to diagonal tension field	N	lbf
P_{fpr}	maximum resultant shear flow on attachments of flange (in stiffener region)	N/m	lbf/in
$P_{s ult}$	failing end load of stiffener and associated plate $P_{s ult} = f_{o,x} A_{se}$	N	lbf
Q_a	applied shear load on panel parallel to stiffeners	N	lbf
q	allowable shear stress in plate (as denoted by appropriate suffixes)	N/m ²	lbf/in ²
q_a	nominal applied shear stress in plate $q_a = Q_a/h_e t_p$	N/m ²	lbf/in ²
q_b	shear stress at which plate first buckles	N/m ²	lbf/in ²
R_k	polar radius of gyration of stiffener and associated plate (Area A_{se}) section about the shear centre, $R_k = \sqrt{\frac{I_{pse}}{A_{se}}}$	m	in
t	component thickness	m	in
t_{2f}, t_{2p}, t_{2s}	0.2 per cent proof stresses in tension of flange, plate and stiffener materials respectively	N/m ²	lbf/in ²
w	dimension of flange, stiffener or pad	m	in
w_p	effective width of plate associated with stiffener in resisting instability (see Section 4.3.1)	m	in
x_1	distance of shear centre from centroid of stiffener and associated plate in x-direction	m	in

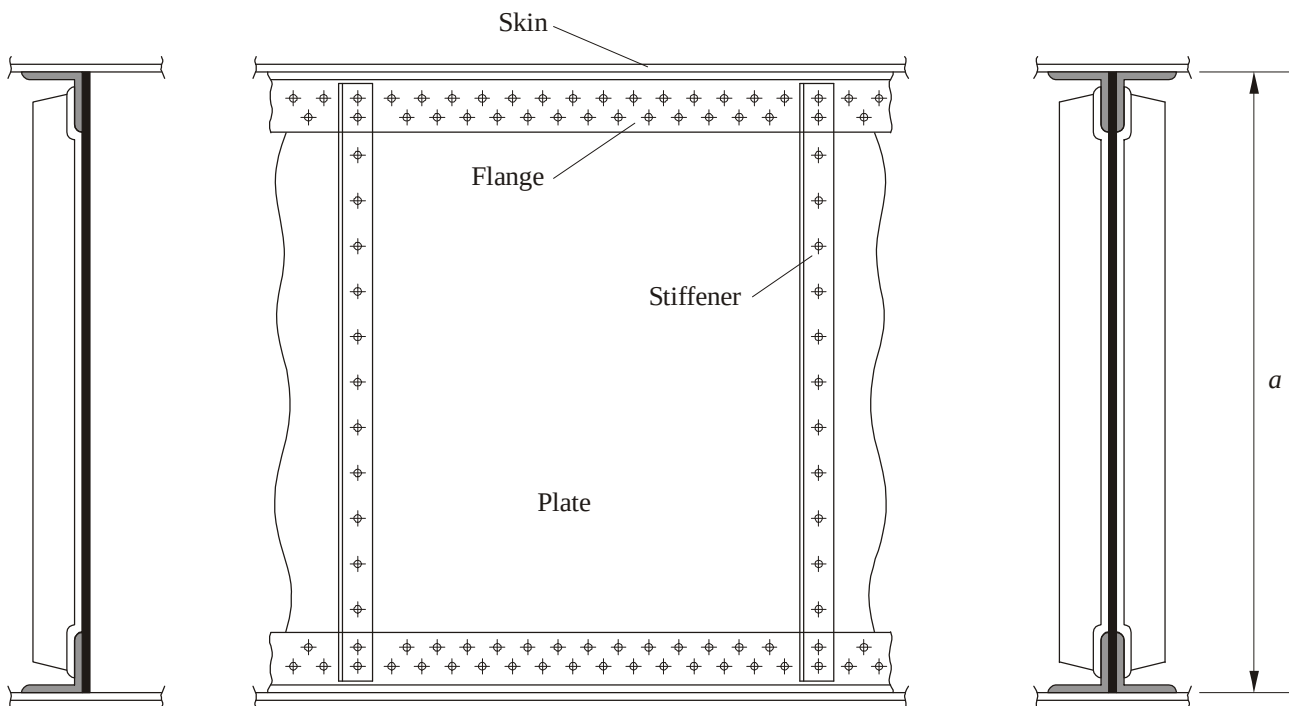
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$\bar{y}_f$	distance between centroid of flange (area A_f) and mid-plane of skin	m	in
$\bar{y}_{fe}$	distance between centroid of flange (area A_{fe}) and mid-plane of skin	m	in
$\bar{y}_{fie}$	distance between centroid of flange (area A_{fe}) and innermost fibre of flange	m	in
y_{fr}	distance of innermost rivet/bolt row from heel of flange	m	in
$\bar{y}_{fre}$	distance between centroid of flange (area A_{fe}) and innermost rivet line	m	in
y_{pe}, y_{ne}	algebraic distances from neutral axis to extreme fibres above and below axis	m	in
$\bar{y}_s$	distance between centroid of stiffener (area A_s) and mid-plane of plate in y -direction	m	in
$\bar{y}_{se}$	offset of neutral axis of stiffener and associated plate from mid-plane of plate	m	in
y_1	distance of shear centre (see Table 2.2 of ESDU 02005) from stiffener/plate centroid along y -axis	m	in
z_s	width of stiffener lip	m	in
z_{sr}	distance between first and second rows of stiffener fasteners	m	in
β	coefficient for effective width of plate acting with stiffener in resisting instability (see Section 4.3.1), $\beta = 0.5(1 - k)$		
Γ_s	total warping constant* of stiffener about its shear centre ($\Gamma_{s1} + \Gamma_{s2}$)	m ⁶	in ⁶
Γ_{s1}	primary warping constant* of stiffener about its shear centre	m ⁶	in ⁶
Γ_{s2}	secondary warping constant* of stiffener about its shear centre	m ⁶	in ⁶
$\epsilon_{p1}, \epsilon_{p2}$	strains at points on plate material stress-strain curve		
$\epsilon_{s1}, \epsilon_{s2}$	strains at points on stiffener material stress-strain curve		
ν_s	Poisson's ratio for stiffener material		
ρ	density of component material	kg/m ³	lb/in ³

* The warping constant can be obtained using the ESDU Struct 00.07 series of Data Items or ESDU 77023.

Suffixes

<i>c</i>	compression
<i>d</i>	permanent deformation
<i>e</i>	effective value calculated including contribution from plate
<i>f</i>	flange
<i>h</i>	heel of flange of stiffener
<i>l</i>	local
<i>m</i>	moment loading
<i>p</i>	plate
<i>r</i>	rivet or fastener row
<i>s</i>	stiffener
<i>sk</i>	skin
<i>t</i>	tension
<i>ult</i>	ultimate stress
<i>x, y</i>	local axes parallel and perpendicular to plate respectively (see Sketch 4.1)



Sketch 1.1

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2. INTRODUCTION

This Item provides data for the analysis of a flat, stiffened panel forming the web of a beam, the web having buckled under shear. A panel is defined as a thin plate, or sheet, reinforced by a system of longitudinal and transverse members referred to as flanges and stiffeners respectively, as shown in Sketch 1.1. The panels are assumed to be part of a beam, as in an aircraft spar. It is possible, with suitable modification, to use the data provided to analyse a continuous array of panels formed by orthogonal stiffeners on a large sheet.

The computerised form of the analysis of this Item is provided in ESDU 02005*. The action of a web in shear may be explained with the aid of the simple structure as shown in Sketch 2.1a, consisting of an initially rectangular frame of stiff bars, pinned at the corners and braced internally by two slender diagonals of equal size. As long as the applied shear load, Q_a , is small, the two diagonals will carry equal and opposite loads. At a certain value of Q_a , the compression diagonal will buckle (see Sketch 2.1a) and thus lose its ability to take any additional load. Consequently, if Q_a is further increased, the additional diagonal bracing force must be furnished by the tension diagonal; at high applied loads, the load in the tension diagonal will be so large that the load in the compression diagonal will be negligible by comparison. An analogous change in the state of stress will occur if the internal bracing is replaced by a thin plate (see Sketch 2.1c).

Thus, a panel buckled in shear deforms into a set of inclined waves and carries, when these are fully developed, a tensile stress, approximately parallel to the waves, together with a smaller, perpendicular compressive stress (see Sketches 2.1c and 2.1d). This leads to a compressive load and a bending moment in the panel flanges and a compressive load in the stiffeners. Thus the stress distribution can be described as a combination of pure shear stress and a "diagonal tension field" acting in the general direction of the buckles (see Sketches 2.1b and 2.1d). The proportion of the total shear reacted by diagonal tension is termed the diagonal tension factor, k , and values for this factor, for flat panels, are given by the formula $\tanh[0.5 \log_{10}(q_a / q_b)]$ and are also given in ESDU 77018† as curves of k plotted against q_a / q_b .

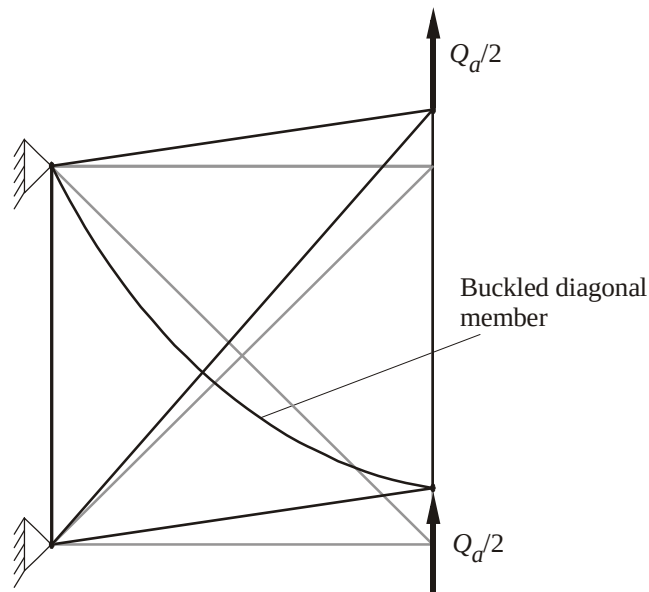
The initial plate buckling stress, q_a , given by the analysis applies only to the elastic range but the post-buckling analysis takes account of inelastic behaviour in the plate and stiffeners. The analysis is only applicable to panels fabricated from isotropic materials. This Item presents an analysis of the post-buckled stress system in the panel as a beam element subjected to an applied transverse shear load, Q_a . In addition to this, there will be an end load, P_{fm} , in the flanges due to the overall bending moment, M_b , on the beam in the plane of the plate. The end load, P_{fm} , is assumed to be constant along the flanges; however, in reality, P_{fm} varies along the flanges. The applied transverse shear load, Q_a , and the overall bending moment on the beam in the plane of the plate, M_b , are assumed to be in a fixed ratio to one another. This Item deals with the analysis of this complex post-buckled stress system and the following sections provide the data listed below.

- Section 4.2 Maximum nominal plate shear stress at (a) failure of the plate at a buckle and (b) failure of the plate at the rivet line.
- Section 4.3 Compressive stress in the stiffener; also (a) the effective length of the stiffener together with the skin that may be associated with it in resisting stiffener overall buckling and (b) the value of the compressive stress in the stiffener at which forced crippling will occur.
- Section 4.4 End load and bending moment acting on the flanges as a result of the diagonal tension field. Load per unit run along the flange to plate connection.
- Section 5 Secant and tangent moduli of the panel.

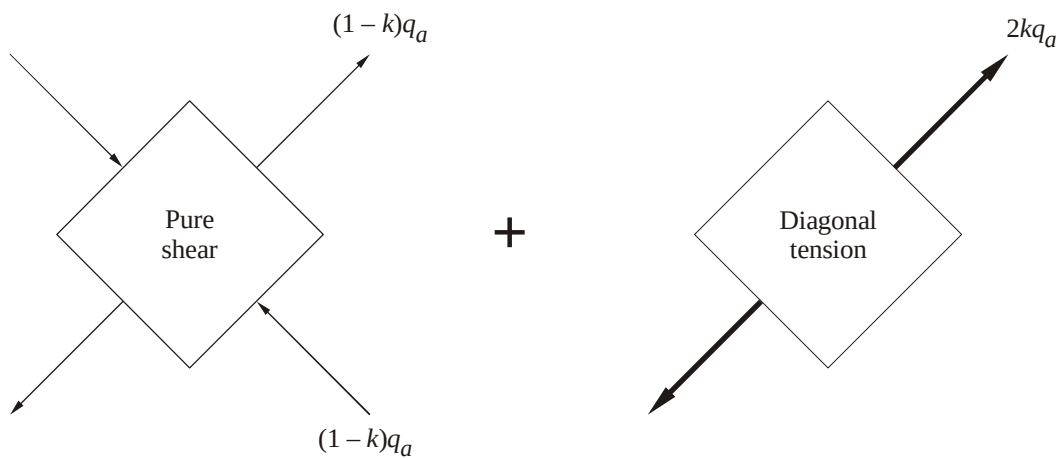
A fully worked example is given in Section 7.

* ESDU 02005 "Flat panels in shear – post-buckling analysis".

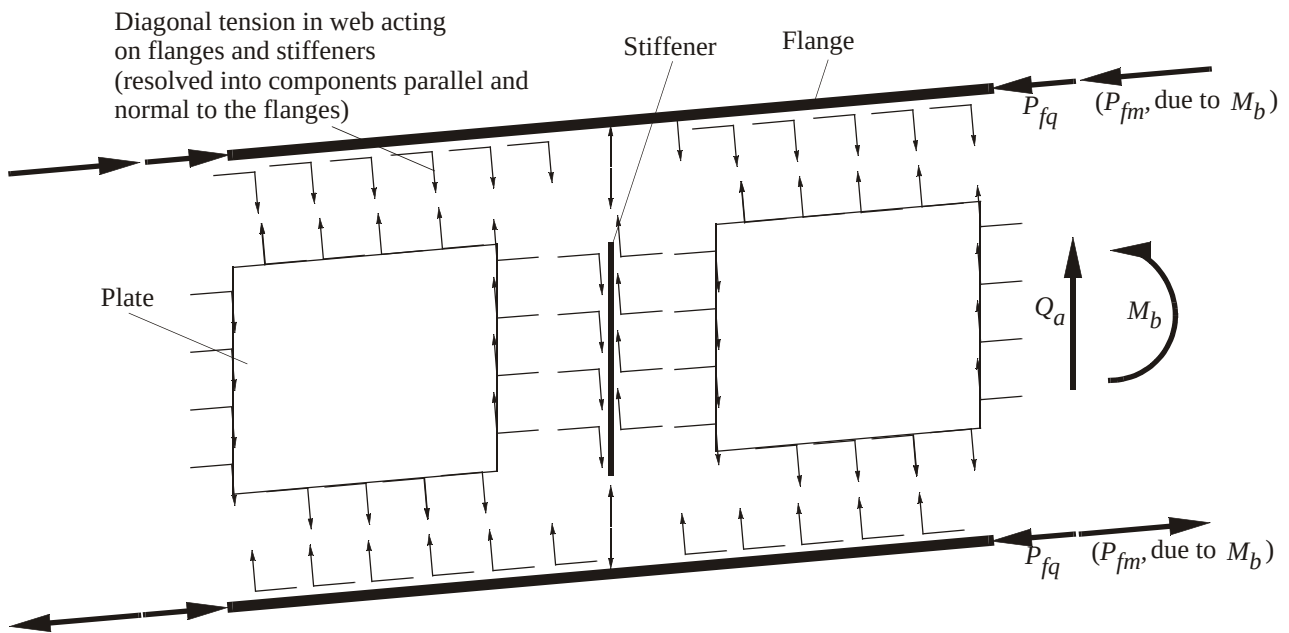
† ESDU 77018 "Curved panels in shear – Post-buckling analysis". The value of k may be obtained from Figure 1 of ESDU 77018 by taking the radius of curvature as infinite.



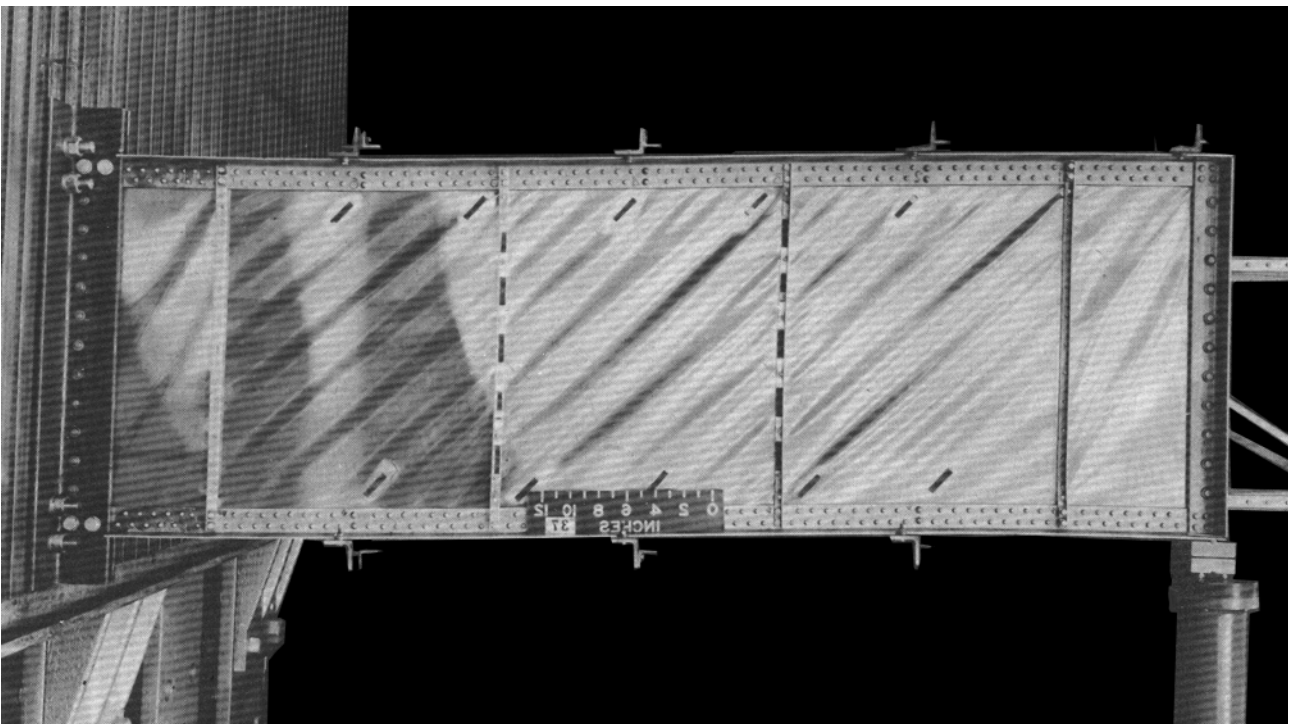
Sketch 2.1a Frame in shear braced by diagonal links



Sketch 2.1b Stress systems in diagonal tension panel
(assuming the angle of the diagonal tension stress to be 45°)



Sketch 2.1c Internal and external loads in diagonal tension panel (exploded view)



**Sketch 2.1d Typical diagonal tension beam
(from NACA Technical Note No. NACA TN-2661)**

3. GENERAL CONSIDERATIONS AND LIMITATIONS

The data have been derived on the assumption that the panels are loaded uniformly in pure shear but will usually be applied to beams in which the flanges carry tensile or compressive loads. The majority of the data are based upon experiment and therefore include the effects of any non-uniformity in the stress pattern in the panel. The data apply to panels with $0.2 \leq b_p / a_p \leq 1.0$ with a well developed tension field, q_a / q_b greater than about 2, in which case the effect of initial irregularities of manufacture is not significant. For loads less than this initial irregularities of the plate may become significant and the discontinuity shown in the figures with an abscissa of q_a / q_b at a value of $q_a / q_b = 1.0$ is unlikely to occur. However, in the region below q_a / q_b of about 2 it may be more efficient to design a panel in which the stiffeners are of reduced stiffness and the plate of increased thickness so that failure occurs by simultaneous buckling of the plate and stiffeners.

The general method of analysis applies only in the elastic range, although some of the other Items referred to herein include data on the effects of exceeding the elastic range. It has been assumed that the stiffeners and flanges are manufactured from materials having the same value of modulus of elasticity as the plate. In the event of significantly differing moduli appropriate adjustments should be made, as in the example in Section 7.

When applying the data to a beam in bending it is necessary to determine an effective width of skin acting with the area A_f . For the compression flange this is likely to be governed by considerations of compressive instability and reference should be made to Data Items dealing with compressive buckling of plates. As a result of this the effective area of the compression flange is likely to be lower than that of the tension flange. Use of the compression flange value of A_f will result in slightly conservative stresses if $A_f / (ht_p)$ is less than about 2; above this value the plate and stiffener stresses and flange loads are relatively insensitive to variations in A_f .

The stiffeners may be attached either to one side of the plate only, in which case they are referred to as "single stiffeners", or else to both sides, back to back, as in Sketch 1.1, when they are termed "double stiffeners". Both single and double stiffeners are considered.

4. ANALYSIS OF PANEL STRENGTH

4.1 Initial Buckling of the Plate

Figures 4 to 10 of ESDU Struct 02.03.02* provide data from which the elastic buckling stress of flat panels that form a continuous beam of the type illustrated in Sketch 1.1 may be calculated. (Note that that Item requires the calculation of the second moment of area of the stiffener to include a contribution from the plate.). In the application of ESDU Struct 02.03.02 care must be taken when choosing the unsupported values for the "length" and "width" of the plate, see the example in Section 7.

* ESDU Struct 02.03.02 "Flat panels in shear. Buckling of long panels with transverse stiffeners".