

LOST RANGE, FUEL AND TIME DUE TO CLIMB AND DESCENT: PROPELLER-DRIVEN AEROPLANES

1. NOTATION AND UNITS*

		SI	British
D	drag	N	lbf
F	total thrust of propellers and exhausts	N	lbf
g	acceleration due to gravity	m/s ²	ft/s ²
H	geopotential height	m	ft
h_e	energy height, $H + \left(\frac{rV}{3\,600} \right)^2 / (2g_0)$	m	ft
L	lift	N	lbf
m	aeroplane mass	kg	slug
m_F	mass of fuel	kg	slug
Q	total engine fuel consumption $\left\{ \begin{array}{l} \text{weight flow basis} \\ \text{mass flow basis} \end{array} \right.$	N/h kg/h	lbf/h slug/h
r	conversion factor (1 000 m/km, 1 852 m/n. mile, 6 076.12 ft/n.mile)	m/km, m/n.mile	ft/n.mile
t	time	h	h
V	true airspeed	km/h, kn	kn
V_e	equivalent airspeed, $V\sqrt{\sigma}$	km/h, kn	kn
W	aeroplane weight, mg	N	lbf
W_F	weight of fuel, m_Fg	N	lbf
Δh_e	change in energy height	m	ft
Δm_F	lost fuel (mass basis)	kg	slug
δm_F	contribution to lost fuel (mass basis) due to change of energy height	kg	slug

* It is customary British practice to work in terms of aeroplane weight and fuel weight flow; however, when using SI units aeroplane mass and fuel mass flow are used. Equations quoted in this Item have been derived in terms of aeroplane weight and fuel weight flow only but they are equally applicable if aeroplane mass and fuel mass flow are used, provided the substitutions $W = mg$, $\Delta W_F = \Delta m_F g$ are made. The SI units have not necessarily received universal acceptance for the particular applications, *i.e.* other multiples and sub-multiples may be used.

ΔR_a	lost range	km, n.mile	n.mile
δR_a	contribution to lost range due to change of energy height	km, n.mile	n.mile
Δt	lost time	h	h
δt	contribution to lost time due to change of energy height	h	h
ΔW_F	lost fuel (weight basis), $\Delta m_F g$	N	lbf
δW_F	contribution to lost fuel (weight basis) due to change of energy height, $\delta m_F g$	N	lbf
ρ	density of air	kg/m ³	slug/ft ³
σ	relative density, ρ/ρ_0		

Subscripts

cl	denotes value during climb
cr	denotes value during cruise
d	denotes value during descent
md	denotes value at minimum drag condition in 1 g level flight, see Appendix A of Item No. 73019
Δh_e	denotes value during a change in energy height
0	denotes value at sea level in International Standard Atmosphere (Item Nos 68046 and 72018 give numerical values)

2. INTRODUCTION

This Item extends to propeller-driven aeroplanes, powered by piston or turbine engines, the method of Item No. 74018 for estimating the “lost” range, “lost” fuel and “lost” time due to climb and descent. The use of these quantities permits estimates of still-air range, fuel and time based on cruise performance alone to be adjusted for the effects of climb and descent. The method is based on the concept of energy height which is itself introduced in Item No. EG 3/3. It is best suited to preliminary calculations at a stage when the available airframe, engine and propeller data are minimal and for cases where the distances flown in climb and descent are not a substantial part of the total flight distance.

Methods of estimating cruise range of propeller-driven aeroplanes are considered in Item No. 75018. The present Item can be used in conjunction either with such methods or with established cruise performance to estimate the following quantities.

The total flight distance for a given fuel load. Starting with an estimate of the distance that could be flown by burning the fuel under cruise conditions, the total flight distance is obtained by subtracting the “lost” range from that cruise distance.

The total fuel required to cover a given flight distance. Starting with an estimate of the fuel required to cover the flight distance under cruise conditions, the total fuel is obtained by adding the “lost” fuel to that cruise fuel.

The total flight time for a given flight distance or total fuel load. The total time is obtained by adding the “lost” time to the time taken under cruise conditions either to cover the total flight distance or to burn the given fuel load.

3. NOTES ON THE DERIVATION OF EXPRESSIONS FOR LOST RANGE, LOST FUEL AND LOST TIME

The derivation of the basic theoretical expressions for lost range, fuel and time is given in Appendix A of Item No. 74018 (see Equations (A1.4) (A2.3), and (A3.2)) and is not repeated here. To obtain more convenient forms of these expressions (*i.e.* suitable for use in a simple method of estimation) it is assumed that

- (i) the angles of climb and descent are small, *i.e.*

$$\left(\frac{W}{D}\right)_{cl} = \left(\frac{L}{D}\right)_{cl}, \quad \left(\frac{W}{D}\right)_d = \left(\frac{L}{D}\right)_d,$$

- (ii) the equivalent airspeed in the climb is similar to the initial-cruise equivalent airspeed and the equivalent airspeed in descent is similar to the end-of-cruise equivalent airspeed, *i.e.*,

$$\left(\frac{L}{D}\right)_{cl} \approx \left(\frac{L}{D}\right)_{cr} \quad \text{and} \quad \left(\frac{L}{D}\right)_d \approx \left(\frac{L}{D}\right)_{cr}.$$

While assumption (i) is valid for most operations where range performance is of importance, assumption (ii) is not always in accord with the methods of operation of propeller-driven aeroplanes. For these aeroplanes the operational climb speed is usually between $0.9 V_{e,md}$ and $1.1 V_{e,md}$ whereas typical cruise speeds are between $1.2 V_{e,md}$ and $1.5 V_{e,md}$. Assumption (ii) is more realistic for descent since typical values of $V_{e,cr}/V_{e,d}$ are between 1.1 and 0.9. In the derivation of the numerical expressions given in Sections 4, 5 and 6 the effects of the possible non-validity of assumption (ii) were considered. The effects on lost range and lost fuel are relatively small and have been disregarded but in the case of lost time it has been necessary to distinguish between cases where assumption (ii) is valid and other methods of operation.

For brevity, the equations in the text of Sections 4, 5 and 6 are quoted for the climb case only; with the subscript “cl” replaced by “d” they are applicable to descent.

4. THE LOST RANGE

The lost range is defined as

$$\Delta R_a = \Sigma \delta R_a = \Sigma W_{F, \Delta h_e} [(V/Q)_{cr} - (V/Q)_{\Delta h_e}] \quad (4.1)$$

where each contribution to lost range, δR_a , is the difference between the distance flown during a change of energy height, Δh_e , in which a quantity of fuel $W_{F, \Delta h_e}$ is burnt and the distance that could be flown in cruise at the cruise height and speed on that quantity of fuel. From Item No. 74018, Equation (A4) of Appendix A gives the following expression for the contribution to lost range due to a climb

$$\delta R_{a, cl} = \frac{\Delta h_{e, cl}}{r} \left(\frac{W}{F - D} \right)_{cl} \left[\frac{(V/Q)_{cr}}{(V/Q)_{cl}} - 1 \right] \quad (4.2)$$

Note that when using the expressions for lost range the value of Δh_e is negative for a descent.

When the assumptions (i) and (ii) of Section 3 are incorporated in Equation (4.2) the expression for $\delta R_{a, cl}$ can be written as

$$\delta R_{a, cl} = \frac{\Delta h_{e, cl}}{r} \left(\frac{L}{D} \right)_{cr} \left[\frac{\frac{(V/Q)_{cr}}{(V/Q)_{cl}} - 1}{\left(\frac{F}{W} \right)_{cl} \left(\frac{L}{D} \right)_{cr} - 1} \right] \quad (4.3)$$

and may be used in this form provided satisfactory estimates can be obtained for the mean values during climb (or descent) of the thrust to weight ratio and the specific air range. Derivation 2 presents the results of an examination of climb, cruise and descent performance in ISA conditions of a variety of propeller-driven aeroplane types (supplemented with computed data for some categories) and shows that these data may be represented sufficiently well by the relationships given in Table 4.1.