

CURVED PANELS IN SHEAR. POST-BUCKLING ANALYSIS

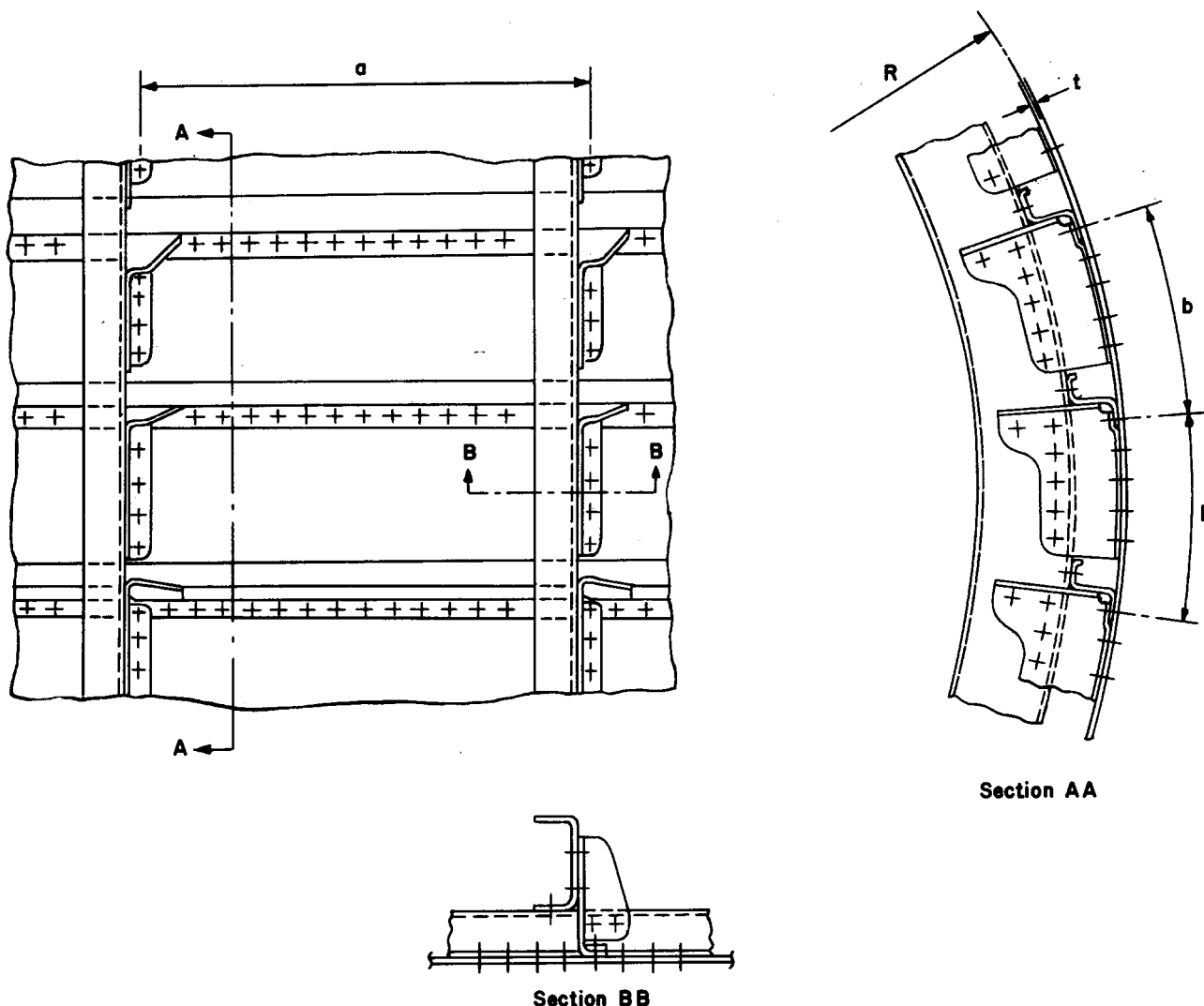
1. NOTATION

A_f, A_s	cross-sectional area of frame and stiffener respectively (excluding any contribution from plate)	m^2	in^2
A_{fe}	cross-sectional area of frame and associated plate	m^2	in^2
A_{se}	cross-sectional area of stiffener and associated plate	m^2	in^2
a	distance between frames	m	in
b	circumferential spacing of stiffeners at radius R	m	in
E	modulus of elasticity of material	N/m^2	lbf/in^2
f	maximum tensile stress in plate	N/m^2	lbf/in^2
f_f, f_s	uniform direct compressive stress acting in frames and stiffeners respectively (see Section 4)	N/m^2	lbf/in^2
k	diagonal tension factor		
q	nominal shear stress in plate	N/m^2	lbf/in^2
q_b	shear stress at which plate first buckles	N/m^2	lbf/in^2
R	radius of curvature of plate	m	in
T_α	tangent of angle between direction of diagonal tensile stress and a generator of cylinder of which plate forms a part		
T_θ	tangent of angle between direction of f and a generator of cylinder of which plate forms a part		
t	plate thickness	m	in
W_s	total inward radial loading acting on stiffener	N	lbf
w_s	uniform inward radial loading per unit length acting on central portion of stiffener (see Section 4)	N/m	lbf/in
w_f	uniform inward radial loading per unit length of circumference acting on frame	N/m	lbf/in

Both SI and British units are quoted but any coherent system of units may be used.

2. INTRODUCTION

This Item provides data for the analysis of curved panels carrying shear loads in which the plate has buckled. A panel is defined as a thin plate, or sheet, reinforced by a system of circumferential frames and straight stiffeners, as shown in Sketch 2.1.



Sketch 2.1

The data provided have been derived on the assumption that the panels are uniformly loaded in pure shear. It has been assumed that a significant system of buckles is developed which is contained between the stiffeners and frames. The data are semi-empirical and apply to cases where a is at least twice b .

The general method of analysis applies only in the elastic range, although some of the other Items referred to herein include data on the effects of going beyond the elastic range. It has been assumed that the stiffeners and frames are manufactured from materials having the same values of the modulus of elasticity as the plate. In the event of significantly differing moduli appropriate adjustments should be made. (For example, see Section 7.6.1 of Item No. 77014 "Flat panels in shear. Post-buckling analysis".)

3. THE DIAGONAL TENSION FIELD

A curved panel buckled in shear deforms into a set of inclined waves and carries a tensile stress, approximately parallel to the waves, together with a small compressive stress across them. These stresses result in loadings on the panel edge members. Furthermore, the tensile stress reduces the plate curvature and leads to inward radial loading on the frames and stiffeners. The proportion of the total shear reacted by diagonal tension is termed the “diagonal tension factor”, k , and Figure 1 provides curves of k plotted against q/q_b for various values of t/R . Values of q_b may be obtained from Item No. Struct. 02.03.18*. When using that Item frames that are not attached to the plate should be ignored. The tangent of the angle of the diagonal tensile stress, T_α , is given by Figures 2 to 6 inclusive: it should be noted that this is not the angle of the buckle waves, due primarily to the influence of the plate corners. In practice the value of T_α may vary slightly from the value given by Figures 2 to 6 as a result of detail design factors, such as rivet pitch, stiffener/frame overlap design, *et cetera*. Thus, the values of stresses and loads that are dependent upon T_α should be treated as estimates only.

4. FRAME AND STIFFENER STRESSES

The uniform direct compressive stresses acting in the frames and stiffeners, respectively, are

$$f_f = \frac{kqtaT_\alpha}{A_{fe}} \quad (4.1)$$

and
$$f_s = \frac{kqtb}{A_{se}T_\alpha}. \quad (4.2)$$

Experiment indicates that reasonable values of A_{fe} and A_{se} are given by

$$A_{fe} = A_f + 0.5(1 - k)at \quad (4.3)$$

and
$$A_{se} = A_s + 0.5(1 - k)bt. \quad (4.4)$$

Note that in the case where the frames are not attached directly to the skin $A_{fe} = A_f$.

In addition to its direct action the diagonal tension field tends to flatten the plates and thereby exerts an inward radial loading on the frames and stiffeners. This loading is uniform over the central portion of the stiffeners at

$$w_s = \frac{kqtbT_\alpha}{R}. \quad (4.5)$$

Owing to the proximity of the frames this loading is reduced towards the ends of the stiffeners. Figure 7 gives the total inward radial load acting on the stiffener, W_s , in terms of $W_s R / (kqtb a)$ plotted against T_α for various values of a/b . Note that the line for $a/b = \infty$ corresponds to $W_s = w_s a$. The inward radial loading on the frames, in addition to the stiffener support loads W_s , is uniform at the value of

$$w_f = \frac{kqtb}{2R}. \quad (4.6)$$

* Item No. Struct. 02.03.18 “Buckling stress coefficients for curved plates in shear. (Axial length exceeding circumferential length, edges simply-supported)”.

Values of W_s , w_s and w_f have been derived for panels having equally spaced stiffeners on the assumptions that the plate surface takes the form given by a series of straight lines connecting the boundary members at the angle given by T_α to the generator of the cylinder. The values given by Equations (4.5) and (4.6) and Figure 7 are therefore conservative upper limit values.

5. PLATE STRESS

Figure 8 gives the maximum tensile stress in the plate, f , in terms of T_α and k . The stress f is the result of the tension field combined with that portion, $(1 - k)$, of the loading reacted in pure shear. Bending stresses in the plate due to the buckles have been neglected. The tangent of the angle of the maximum tensile stress, T_θ , is given by Figure 9.

6. DERIVATION

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|----|---|--|
| 1. | KUHN, P.
PETERSON, J.P.
LEVIN, L.R. | A summary of diagonal tension. Part I - Method of analysis. Technical Note No. NACA-TN-2661, National Advisory Committee on Aeronautics, Langley Field, Va, USA, October 1951. |
| 2. | KUHN, P.
PETERSON, J.P.
LEVIN, L.R. | A summary of diagonal tension. Part II - Experimental evidence. Technical Note No. NACA-TN-2662, National Advisory Committee on Aeronautics, Langley Field, Va, USA, January 1952. |
| 3. | GRAYLEY, M.E. | Radial loads on frames and stiffeners of buckled circular cylinders. ESDU Memorandum No. 24, September 1977. |

7. EXAMPLE

A panel of the type shown in Sketch 2.1 is subjected to a shear stress of 200 MN/m^2 . The plate, frames and stiffeners are all manufactured from a material for which $E = 68\,500 \text{ MN/m}^2$. The major dimensions are $t = 1.6 \text{ mm}$, $R = 1700 \text{ mm}$, $a = 500 \text{ mm}$ and $b = 150 \text{ mm}$. The areas of the boundary members of the panels are $A_s = 174 \text{ mm}^2$ and $A_f = 178 \text{ mm}^2$.

7.1 Plate Buckling Stress

This is calculated using Item No. Struct. 02.03.18 and, in the notation of that Item,

$$\frac{a}{b} = \frac{500}{150} = 3.33 \quad \text{and} \quad \frac{b}{\sqrt{Rt}} = \frac{150}{\sqrt{1700 \times 1.6}} = 2.88$$

so that, from the figure,

$$K = 6.2.$$

$$\begin{aligned} \text{Thus,} \quad q_b &= KE(t/b)^2 \\ &= 6.2 \times 68\,500 \times 10^6 \times (1.6 \times 10^{-3}/150 \times 10^{-3})^2 \text{ N/m}^2 \text{ or } 48.3 \text{ MN/m}^2. \end{aligned}$$

$$\text{Hence,} \quad q/q_b = 200/48.3 = 4.14.$$

7.2 Diagonal Tension Factor k and Tangent of Angle of Tension Field T_α

From Figure 1, at

$$\left(10^3 \times \frac{t}{R}\right) = 10^3 \times \frac{1.6}{1700} = 0.94$$

and $q/q_b = 4.14$, interpolation gives $k = 0.58$.

From Figures 2 to 6 with

$$A_s/(bt) = 174/(150 \times 1.6) = 0.725,$$

$$A_f/(at) = 178/(500 \times 1.6) = 0.223,$$

$$\frac{E}{q} \left(\frac{b}{R}\right)^2 = \frac{68\,500 \times 10^6}{200 \times 10^6} \left(\frac{150 \times 10^{-3}}{1700 \times 10^{-3}}\right)^2 = 2.66$$

and $k = 0.58$, interpolation gives $T_\alpha = 0.90$.

7.3 Maximum Plate Stress

From Figure 8 at $k = 0.58$ and $T_\alpha = 0.90$, $f/q = 1.58$.

Thus, $f = 1.58 \times 200 \times 10^6 \text{ N/m}^2$ or $f = 316 \text{ MN/m}^2$.

Similarly, from Figure 9, $T_\theta = 0.93$.

7.4 Stiffener Loads and Stresses

The direct compressive stress is calculated using Equations (4.4) and (4.2).

$$A_{se} = A_s + 0.5(1 - k)bt = 174 + 0.5(1 - 0.58) \times 150 \times 1.6 = 224 \text{ mm}^2.$$

$$\begin{aligned} f_s &= \frac{kqtb}{A_{se}T_\alpha} = \frac{0.58 \times 200 \times 10^6 \times 1.6 \times 10^{-3} \times 150 \times 10^{-3}}{224 \times 10^{-6} \times 0.9} \\ &= \frac{186 \times 10^3 \times 150 \times 10^{-3}}{224 \times 10^{-6} \times 0.9} = 138 \times 10^6 \text{ N/m}^2 \text{ or } f_s = 138 \text{ MN/m}^2. \end{aligned}$$

The inward radial loading acting over the central portion of the stiffener is given by Equation (4.5).

$$w_s = \frac{kqtbT_\alpha}{R} = \frac{186 \times 10^3 \times 150 \times 10^{-3} \times 0.9}{1700 \times 10^{-3}} = 14\,800 \text{ N/m or } w_s = 14.8 \text{ kN/m}.$$

Other loadings acting on the stiffener should be suitably combined with f_s and w_s . When checking stiffener stability it is suggested that w_s should be taken to act along the entire length of the stiffener since the reduction of loading towards the ends of the stiffener is unlikely to influence its stability significantly.