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Equations for calculation of International Standard Atmosphere and associated off-standard atmospheres

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EQUATIONS FOR CALCULATION OF INTERNATIONAL STANDARD ATMOSPHERE AND ASSOCIATED OFF-STANDARD ATMOSPHERES

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EQUATIONS FOR CALCULATION OF INTERNATIONAL STANDARD ATMOSPHERE AND ASSOCIATED OFF-STANDARD ATMOSPHERES

1. NOTATION AND DEFINITION

1.1 Notation

		<i>SI</i>	<i>British</i>
a	speed of sound	m/s, kn	ft/s, kn
g	acceleration due to gravity, see Section 8	m/s ²	ft/s ²
g_0	standard sea level value of g , see Table 11.1	m/s ²	ft/s ²
H	geopotential height	m	ft
H_b	geopotential height at base of layer in model atmosphere	m	ft
H_p	pressure height, (see Section 4.5)	m	ft
H_{p_b}	pressure height at base of layer in model atmosphere	m	ft
L	temperature gradient, dT/dH	K/m	K/ft
l	characteristic length	m	ft
M	Mach number, V/a	–	–
m	molar mass (see Section 1.2)	kg/kmol	(see Section 1.2)
p	pressure	N/m ²	lbf/ft ²
p_b	pressure at base of layer in model atmosphere	N/m ²	lbf/ft ²
q^*	kinetic pressure for $M = 1$ ($= \rho a^2/2$)	N/m ²	lbf/ft ²
$\Re$	universal gas constant (see Section 1.2)	N m/kmol K	(see Section 1.2)
R	gas constant for air, $\Re/m$	N m/kg K	lbf ft/slug K
Re^*	Reynolds number for $M = 1$ ($= \rho a l/\mu$)	–	–
r'_{e_0}	fictitious Earth radius used in relating H and Z , see Table 11.1 for value in ISA and Section 8 for general case	m	ft
S	Sutherland coefficient (see Section 6.2)	K	K

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T	temperature	K	K
T_b	temperature at base of layer in model atmosphere	K	K
V	true airspeed	m/s, kn	ft/s, kn
V_e	equivalent airspeed ($= V\sqrt{\sigma} = Ma\sqrt{\sigma}$)	m/s, kn	ft/s, kn
V_e^*	equivalent airspeed for $M = 1$ ($= a\sqrt{\sigma}$)	m/s, kn	ft/s, kn
Z	geometric height	m	ft
β_s	Sutherland coefficient (see Section 6.2)	N s/m ² K ^{1/2}	lbf s/ft ² K ^{1/2}
γ	ratio of specific heat capacities of air (= 1.4 in this Item)	—	—
ΔT	temperature increment	K	K
δ	relative pressure, p/p_0	—	—
θ	relative temperature, T/T_0	—	—
ϕ	geographic latitude, positive in northern hemisphere, negative in southern hemisphere	deg	deg
μ	dynamic viscosity	N s/m ²	lbf s/ft ²
ρ	density	kg/m ³	slug/ft ³
σ	relative density, ρ/ρ_0	—	—

Subscripts

H_p	denotes value at constant pressure height
sl	denotes value at sea level
std	denotes value in International Standard Atmosphere, ISA
Z	denotes value at geometric height Z
ϕ	denotes value at geographic latitude ϕ
0	denotes value at sea level in International Standard Atmosphere, ISA – see Table 11.1

1.2 Molar Mass and Gas Constants

In Derivation 1 the term “molar mass” is used to denote the quantity which used to be called “kilogram molecule”. The molar mass, m , is expressed in kilograms per kilo mole (kg/kmol). The mole is a base SI unit for the *amount of a substance* and is defined as the amount of substance of a system which contains as many elementary entities as there are carbon atoms in 0.012 kg of carbon 12. These elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles or specified groups of particles. In defining properties of model atmospheres, in particular the International Standard Atmosphere, a mixture of gases is taken which typifies the composition of clean dry air near sea level. The resulting mean air molar mass is assumed to remain constant throughout the height range considered. For consistency with the notation just described, the universal gas constant, $\mathfrak{R}$, takes units N m/kmol K while the gas constant for air, $R(= \mathfrak{R}/m)$ takes the familiar units of N m/kg K.

When working in British units, the method adopted in this Item is to accept that m and $\mathfrak{R}$ are defined in SI units only and to convert the gas constant for air, R , into the equivalent British units (lbf ft/slug K).

2. INTRODUCTION

This Item describes the “atmospheric model” and gives equations and numerical data required to evaluate properties of the International Standard Atmosphere (Derivation 1) and off-standard atmospheres which differ in a prescribed way from the Standard. The basic equations for temperature, T , pressure, p , and density ρ , are given in Sections 3 to 5 while the defining constants and temperature gradients are given in Tables 11.1 and 11.2. Table 11.3 gives numerical expressions for T , p and ρ while Table 11.4 gives the reverse expressions for determining pressure height for a given pressure, and the difference between pressure height and geopotential height for specified off-standard atmospheres. The relationships between pressure, pressure height and geopotential height are considered in Section 7 while the relationship between geopotential height and geometric height is presented in Section 8. Expressions for speed of sound and dynamic viscosity are given in Section 6.

For direct reference to tabulated values of T , θ , $\sqrt{\theta}$, p , δ , $\sqrt{\delta}$, ρ , σ , $\sqrt{\sigma}$, a , V_e^* , q^* , and Re^*/l for heights up to 50 km, see Item Nos 68046 (Reference 5) and 72018 (Reference 6). For information on the atmosphere for heights above 50 km, see Derivation 1 (up to 80 km), Reference 9 and Aerodynamics Data Item No. 77021 (Reference 10) (up to 1 000 km). For information on non-standard (or other) atmospheres, see Reference 11.

3. THE ATMOSPHERIC MODEL

The atmospheric model described here has been widely adopted. The atmosphere consists of air which is considered to be a perfect gas and thus local values of pressure, density and temperature are related by the perfect gas equation

$$p = \frac{\rho \mathfrak{R} T}{m} = \rho R T. \quad (3.1)$$

The atmosphere is assumed to be static with respect to the earth and so the hydrostatic equation must be satisfied,

$$\frac{dp}{dz} = -\rho g. \quad (3.2)$$

Equation (3.2) provides a relationship between pressure, density and geometric height, Z , but in considering pressure distribution in the atmosphere, it is convenient to work in terms of geopotential height, H .

Geopotential height is the (geometric) height in a uniform gravitational field which gives the same potential energy as exists at the point under consideration in the actual, variable gravity field. Consideration of the work done in moving between two geopotential surfaces separated by a distance Z leads to the relationship

$$H = \frac{1}{g_{sl}} \int_0^Z g \, dZ. \quad (3.3)$$

Equation (3.3) relates the geopotential height in a uniform gravitational field in which the acceleration due to gravity is taken as equal to the sea-level value, g_{sl} , to the actual variation of g with Z . For the purposes of atmospheric modelling and aircraft performance work, g_{sl} in Equation (3.3) is taken as the standard value, g_0 , and the resulting measurement of H is called the standard geopotential height (throughout this Item, “geopotential height” implies, “standard geopotential height”). The accepted value of g_0 is given in Table 11.1 and reference to the Lambert equation for the variation of g_{sl} with latitude (see Section 8, Equation (8.2)) shows that this value of g_0 corresponds to a geographic latitude of 45.5425 degrees. It follows from Equation (3.3) that a given value of geopotential height corresponds to slightly different geometric heights at different latitudes – Section 8 gives information on this relationship.

To obtain a relationship between standard geopotential height and pressure, Equations (3.1), (3.2) and (3.3) are combined to give,

$$\frac{dp}{dH} = -g_0 \frac{mp}{\Re T}. \quad (3.4)$$

To define a particular model atmosphere (as in Section 3.1) the usual procedure is to substitute into Equation (3.4) a particular variation of temperature with height and to deduce the corresponding variation of pressure with height. The temperature-height profile is derived from experimental results, frequently obtained over large geographic areas and/or time periods, and may consist of numerical data or be an analytical expression. It should however be noted that the variety of temperature profiles that may be used in conjunction with Equation (3.4) is restricted to those that produce a vertically stable atmosphere (see Reference 2).

The variation of temperature with height may be expressed as

$$T = f(H) \quad (3.5)$$

and if Equations (3.4) and (3.5) are combined the following relationship between pressure and height is obtained

$$\frac{dp}{p} = -\frac{g_0}{\Re} \int \frac{m \, dH}{f(H)}. \quad (3.6)$$

The integration of Equation (3.6) depends on the form of $f(H)$ and the variation, if any, of m with H .

3.1 Atmospheres with Linear or Zero Change of T with H and Constant Molar Mass

If a model atmosphere is assumed to consist of a number of layers within each of which temperature varies linearly with height or is constant, Equation (3.5) can be expressed as,

$$T = T_b + L(H - H_b). \quad (3.7)$$

In Equation (3.7), T_b is the temperature at height H_b and L is the temperature gradient at heights between H_b and H_{b+1} .

For heights up to about 80 km the mean molar mass of air can be assumed constant (see Section 1.2 and References 1, 9, 13) and equal to the sea-level value m_0 . When Equation (3.7) is substituted into Equation (3.6) together with the assumed constant relationship $R = \mathfrak{R}/m_0$ two solution for pressure ratio are obtained,

$$\frac{p}{p_b} = \left(\frac{T}{T_b} \right)^{-g_0/RL} \quad (3.8)$$

for a layer where $L \neq 0$ and

$$\frac{p}{p_b} = \exp \left(- \frac{g_0(H - H_b)}{RT_b} \right), \quad (3.9)$$

for a layer where $L = 0$.

Values of p at any height may be derived from these ratios by use of multipliers which relate conditions at the bases of successive layers. The general expression for the pressure at any height is then of the form,

$$p = \left(\frac{p}{p_b} \right) \left(\frac{p_b}{p_{b-1}} \right) \dots \dots \dots \left(\frac{p_2}{p_1} \right) p_1, \quad (3.10)$$

where p_1 is the pressure at the base of the first layer – usually taken as mean sea level. Note that for heights below sea level, properties should be calculated as for the first layer above sea level.

Atmospheric density, ρ , is calculated most readily using Equation (3.1) with values of T and p from Equations (3.7) to (3.10). However, comparable expressions to Equations (3.8) and (3.9) can be derived[†], to give

$$\frac{\rho}{\rho_b} = \frac{p}{p_b} \frac{T_b}{T} = \left(\frac{T}{T_b} \right)^{-(1 + g_0/RL)} \quad (3.11)$$

for a layer where $L \neq 0$ and

$$\frac{\rho}{\rho_b} = \frac{p}{p_b} = \exp \left(- \frac{g_0(H - H_b)}{RT_b} \right) \quad (3.12)$$

for a layer where $L = 0$.

[†] Equations (3.11) and (3.12) do not apply to the “off-standard” atmospheres of Section 5, see footnote to Section 5.1.