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Geometrical characteristics of typical bodies

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GEOMETRICAL CHARACTERISTICS OF TYPICAL BODIES

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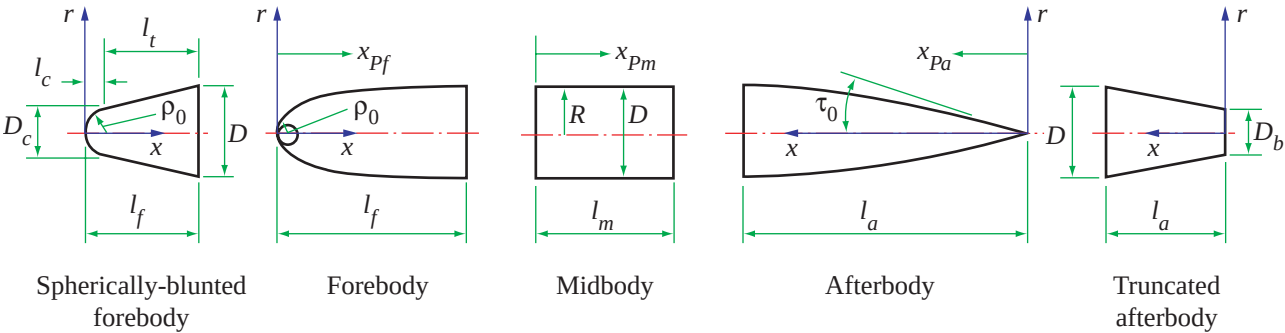
GEOMETRICAL CHARACTERISTICS OF TYPICAL BODIES

1. NOTATION AND UNITS (see Sketches 1.1 to 1.3)

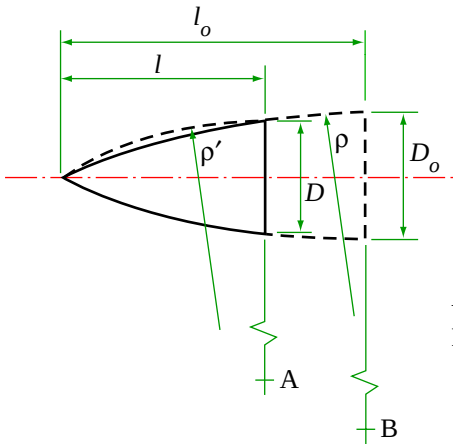
		SI	British
A	family parameter for Body 9 (see Table 8.1)		
a	area of maximum cross-section (see Section 5.2)	m ²	ft ²
a_c	area of maximum cross-section of spherical cap	m ²	ft ²
C_C	centroid of planform area coefficient, $2x_p/l$		
C_P	planform area coefficient, S_p/Dl		
C_S	surface area coefficient, S/pl		
C_V	volume (sometimes called "prismatic") coefficient, V/al		
D	maximum diameter of body	m	ft
D_c	maximum diameter of spherical cap	m	ft
D_o	maximum diameter of tangent ogive from which secant ogive is obtained	m	ft
E	secant ogive parameter, $\frac{1}{2}[4(l/D)^2 + 1]$		
E'	secant ogive parameter, $\frac{1}{2}[4(l'/D)^2 + 1]$		
F	tangent ogive parameter, $\frac{1}{2}[4(l/D)^2 - 1]$		
F_o	secant ogive parameter, $\frac{1}{2}[4(l_o/D_o)^2 - 1]$		
G	tangent ogive parameter, ρ/R		
H	maximum height of body component	m	ft
K_S	factor required to obtain D_{eff} , used in determining surface area of non-axisymmetric body (see Section 5.3)		
K_V	factor required to obtain D_{eff} , used in determining volume of non-axisymmetric body (see Section 5.2)		
k_1, k_2	family parameter for Bodies 7 and 8 (see Table 8.1)		
L	body overall length	m	ft
l	length of body component	m	ft

l_o	length of tangent ogive from which tangent ogive frustum, secant ogive, or secant ogive frustum are obtained	m	ft
l'	length of secant ogive from which frustum is obtained	m	ft
m	family parameter for Body 5 (see Table 8.1)		
n	family parameter for Body 6 (see Table 8.1)		
p	perimeter of maximum cross-section (see Section 5.3)	m	ft
R	maximum radius of body, $D/2$	m	ft
r	local radius of axisymmetric body	m	ft
S	surface area of body component (excluding base area for a truncated afterbody)	m ²	ft ²
S_p	planform area of body component	m ²	ft ²
V	volume of body component	m ³	ft ³
W	maximum width of body component	m	ft
x	axial distance measured from nose or tail	m	ft
x_p	axial distance measured from nose or tail of body component to its centroid of planform area	m	ft
α_1, α_2	function for Bodies 7 and 8 (see Table 8.1)		
δ	cone semi-apex angle	degree	degree
ρ	tangent or secant ogive radius of curvature	m	ft
ρ_0	radius of curvature at $x = 0$	m	ft
$\bar{\rho}_0$	dimensionless radius of curvature, $\rho_0 l / D^2$		
ρ'	radius of curvature of tangent ogive having same length and maximum diameter as a given secant ogive	m	ft
τ_0	half angle at $x = 0$	degree	degree
<i>Subscripts</i>			
a	relating to afterbody		
b	relating to afterbody base		
c	relating to spherical cap		
eff	denotes effective value		

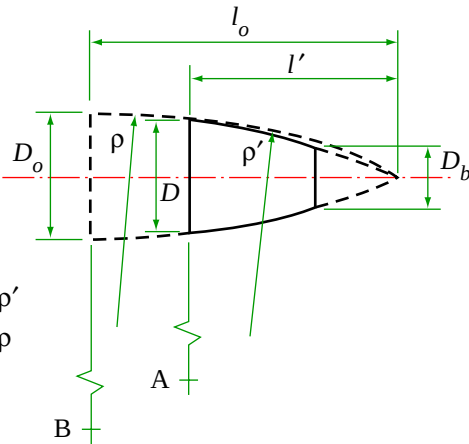
- f relating to forebody
- L relating to overall body
- m relating to midbody
- t relating to truncated section of forebody aft of any spherical blunting



Sketch 1.1 Geometry of typical axisymmetric body components



Sketch 1.2 Secant ogive



Sketch 1.3 Secant ogive frustum

2. INTRODUCTION

This Item provides data for the planform area, centroid of planform area, surface area, volume, radius of curvature and/or half-angle at $x = 0$ for a number of typical axisymmetric forebody and afterbody shapes, see Section 3. Planform area, centroid of planform area, surface area, and volume data for certain spherically-blunted forebodies and truncated afterbodies are also given, see Section 3. It is then a simple matter to calculate these geometrical parameters for a body consisting of any combination of these forebody and afterbody shapes, usually in combination with a cylindrical midbody, see Section 4.

The data in Tables 8.1 to 8.5, and Figures 1 to 44 relate to axisymmetric bodies. This Item also gives guidance, using Figures 45 to 48, on the effect of body cross-sections different from circular for all but the secant ogive and spherically-blunted bodies, see Section 5. The equations used to produce the data for the axisymmetric bodies are listed in Appendix A. This has been done to allow their implementation in computer programs and to extend the ranges of data available.

3. FOREBODY AND AFTERBODY SHAPES CONSIDERED

3.1 Presentation of Data for Axisymmetric Bodies

Seven families of axisymmetric forebody and afterbody shapes, in addition to two individual shapes and three families of spherically-blunted forebodies, are considered. Equations for generating these together with cross-references to Figures giving the shapes are presented in Table 8.1. Figure 1 illustrates selected shapes with specified D/l to give a better impression of the body profile. Table 8.2 provides cross-references to Figures giving data for planform area (C_P), centroid of planform area (C_C), surface area (C_S) and volume (C_V) for each body while Table 8.3 provides similar information for afterbodies formed by truncated cones, paraboloids, tangent ogives and secant ogives. Table 8.4 lists data for dimensionless radius of curvature (ρ_0) and/or half-angle parameter ($(l/D)\tan \tau_0$) for each of the bodies given in Table 8.1. Finally, Table 8.5 lists relevant geometrical relationships for spherically-blunted cones, tangent ogives and secant ogives based on continuity in surface slope across the junction formed by the spherical cap and the truncated ogive or conical section of the forebody.

3.2 Notes

In some applications the use of a particular family of shapes may be limited to those members exhibiting desirable qualities for that application. For example, a reflexed (or cusped) afterbody shape is usually avoided in practice. Where such peculiarities are present details are given in the Figures. It will also be seen from the notes on the Figures that certain members of one family may also be common to another family. While all the bodies (except Bodies 4, 5, 10 and 12) given in Table 8.1 exhibit continuity in surface slope across a junction with a cylindrical midbody, only Bodies 7 and 8, devised in Reference 9, exhibit continuity in surface curvature across such a junction.

4. CALCULATION OF VALUES OF C_P , C_C , C_S AND C_V

Many of the bodies considered in this Item can be used as either a forebody or an afterbody and so no attempt has been made to differentiate between these two uses in the notation used in the Figures. However, in order to define values of C_P , C_C , C_S and C_V for combinations of body components it is necessary to introduce appropriate subscripts for each component.

4.1 Calculation of Geometric Coefficients for Spherically-blunted Forebodies

For the case of spherically-blunted cone, tangent ogive and secant ogive forebodies the data for spherical caps are combined with the data for truncated cone, tangent ogive and secant ogive afterbodies to obtain overall forebody values of C_{Pf} , C_{Cf} , C_{Sf} and C_{Vf} . To distinguish between the spherical cap and truncated section of the forebody, the subscripts c and t have been used. Thus, for a blunted forebody of total length l_f ,

$$C_{Pf} = C_{Pc} \frac{D_c}{D} \frac{l_c}{l_f} + C_{Pt} \frac{l_t}{l_f}, \quad (4.1)$$

$$C_{Cf} = \frac{C_{Pc}}{C_{Pf}} C_{Cc} \frac{D_c}{D} \left(\frac{l_c}{l_f} \right)^2 + \frac{C_{Pt}}{C_{Pf}} \frac{l_t}{l_f} \left[2 \frac{l_c}{l_f} + C_{Ct} \frac{l_t}{l_f} \right], \quad (4.2)$$

$$C_{Sf} = C_{Sc} \frac{D_c}{D} \frac{l_c}{l_f} + C_{St} \frac{l_t}{l_f} \quad (4.3)$$

and

$$C_{Vf} = C_{Vc} \left(\frac{D_c}{D} \right)^2 \frac{l_c}{l_f} + C_{Vt} \frac{l_t}{l_f}. \quad (4.4)$$

4.2 Calculation of Geometric Coefficients for Complete Bodies

For a body of total length L , with any given cross-sectional shape, consisting of forebody, parallel midbody and afterbody of lengths l_f , l_m and l_a respectively,

$$C_{PL} = C_{Pf} \frac{l_f}{L} + \frac{l_m}{L} + C_{Pa} \frac{l_a}{L}, \quad (4.5)$$

$$C_{CL} = \frac{C_{Pf}}{C_{PL}} C_{Cf} \left(\frac{l_f}{L} \right)^2 + \frac{1}{C_{PL}} \frac{l_m}{L} \left[\frac{l_m}{L} + 2 \frac{l_f}{L} \right] + \frac{C_{Pa}}{C_{PL}} \frac{l_a}{L} \left[2 - C_{Ca} \frac{l_a}{L} \right], \quad (4.6)$$

$$C_{SL} = C_{Sf} \frac{l_f}{L} + \frac{l_m}{L} + C_{Sa} \frac{l_a}{L}, \quad (4.7)$$

and

$$C_{VL} = C_{Vf} \frac{l_f}{L} + \frac{l_m}{L} + C_{Va} \frac{l_a}{L}. \quad (4.8)$$

The planform area, surface area and volume coefficients in Equations (4.1), (4.3) to (4.5), (4.7) and (4.8) are defined in terms of the diameter, D , perimeter, p , and area, a , respectively of the maximum cross-section of the body. The centroid of planform area coefficients in Equations (4.2) and (4.6) are defined in terms of the body length, l . For axisymmetric bodies $a = \pi D^2/4$ ($= \pi D_c^2/4$, for a spherical cap) and $p = \pi D$ ($= \pi D_c$, for a spherical cap); for non-axisymmetric bodies see Sections 5.2 and 5.3.

5. NON-AXISYMMETRIC BODIES

The data given in Tables 8.1 to 8.5 and Figures 1 to 44 relate to axisymmetric body components. This section is concerned with the case of non-axisymmetric bodies for all the shapes considered **except the secant ogive and spherically-blunted bodies**.

5.1 $\bar{\rho}_0, \tau_0$

The data for $\bar{\rho}_0$ and τ_0 given in Table 8.4 will apply to bodies with any cross-sectional shape provided that, where necessary, D is replaced by the relevant length, H or W for example.

5.2 C_V

The volume coefficient for a body component is independent of cross-sectional shape provided that the cross-section is symmetrical about the vertical and horizontal axes and that the non-dimensional equation representing the body profile is the same in both the horizontal and vertical plane passing through the longitudinal axis. Note that, of the profiles to be considered here (*i.e.* Bodies 1 to 3 and 5 to 9) the tangent ogive would be excluded by this proviso. Therefore, with the exception of the tangent ogive, the volume coefficient for a non-axisymmetric body with any of the profiles of Bodies 1, 2 and 5 to 9, whether truncated or not, may be obtained from the relevant Figures cross-referenced in Table 8.2 or 8.3.

For the case of the tangent ogive a good approximation to the volume coefficient can be obtained from Figure 28 for Body 3, for any cross-section that is symmetrical about the vertical and horizontal axes and whose area is a function of HW only, using a circular cross-section having an effective diameter

$$D_{eff} = K_V H, \quad (5.1)$$

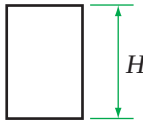
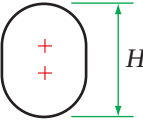
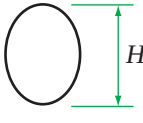
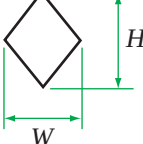
where K_V is given in Figure 45. Agreement with exact data for C_V is within ± 0.2 per cent.

For the case of the truncated tangent ogive afterbody a reasonable approximation to C_V may be obtained from Figure 43 with

$$D_{eff} = \sqrt{HW} \text{ and } D_{b,eff} = \sqrt{H_b W_b}. \quad (5.2)$$

This artifice leads to underestimates in C_V not greater than 1 per cent provided $W/H \geq 0.5$.

In order to calculate the volume from C_V it is necessary to know the volume, al , of the enclosing cylinder. The cross-sectional area, a , for a number of cross-sectional shapes is as follows.

Rectangular		$a = HW$
Flat-sided with semi-circular top and bottom		$a = HW + W^2 (\pi/4 - 1)$
Elliptical		$a = \pi HW/4$
Rhombic		$a = HW/2$

5.3 C_S

The data for surface area coefficient given in this Item are not so generally applicable to non-axisymmetric bodies as are the data for volume coefficient. In order to provide a guide to the effect of cross-sectional shape on C_S the cross-sections illustrated in Section 5.2 will be considered.

An analysis of exact data for the surface area coefficient of a number of bodies* with various profiles taken from those in Table 8.1 and having any of the four cross-sections illustrated in Section 5.2 has shown that a good approximation to C_S can be obtained using a circular cross-section with effective diameter

$$D_{eff} = K_S H \quad (5.3)$$

in conjunction with the data given for axisymmetric bodies (see Table 8.2). For a given cross-section, K_S (see Figure 46) is a function of W/H and is essentially independent of body profile and inverse fineness ratio, H/l . The independence with respect to body profile is subject to the proviso given in Section 5.2, so that, of those profiles to be considered here (*i.e.* Bodies 1 to 3 and 5 to 9) the tangent ogive is excluded (but see later). For bodies with the four cross-sections considered in Figure 46 agreement with exact data for C_S is within about ± 0.2 per cent.

For the case of bodies with a rectangular cross-section a simple, exact, expression for C_S has been derived and its use is recommended where better accuracy is required;

$$C_S = \frac{1}{1 + \frac{W}{H}} \left[(C_S)_{W/l} + \frac{W}{H} (C_S)_{H/l} \right], \quad (5.4)$$

* For the case of the general ellipsoid ESDU is indebted to Dr D. Martin and Dr A.R. Curtis of the Division of Numerical Analysis and Computing at NPL.

where $(C_S)_{W/l}$ and $(C_S)_{H/l}$ are the area coefficients for axisymmetric bodies with inverse fineness ratios W/l and H/l respectively. Here again the use of Equation (5.4) is subject to the proviso given in Section 5.2 and therefore is not strictly applicable to bodies with a tangent ogive profile, for which, even with $W/H \geq 0.5$, underestimates of up to 2 per cent would result. For better accuracy (within ± 0.2 per cent over the whole range of W/H from 0 to 1) Equation (5.3) should be used in conjunction with the data for Body 3 in Figure 19 and K_S obtained from Figure 47.

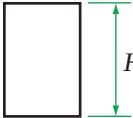
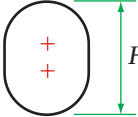
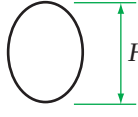
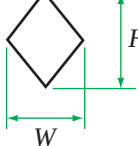
For truncated afterbodies having conical or parabolic profiles with any of the four cross-sections considered, good agreement with exact data for C_S (within about ± 0.2 per cent) is obtained from Figure 38 or 39 with

$$D_{eff} = K_S H \text{ and } D_{b, eff} = K_S H_b \quad (5.5)$$

in which Figure 46 is used for K_S .

For the truncated tangent ogive afterbody with a rectangular cross-section a reasonable approximation to C_S may be obtained from Figure 40 in conjunction with Equations (5.2). This procedure leads to underestimates no greater than 1 per cent provided $H_b/H \geq 0.2$ and $W/H \geq 0.5$.

In order to calculate the surface area from C_S it is necessary to know the surface area, pl , of the enclosing cylinder. The perimeter, p , for a number of cross-sectional shapes is as follows.

Rectangular		$p = 2(H + W)$
Flat-sided with semi-circular top and bottom		$p = 2H + W(\pi - 2)$
Elliptical		$p = 2H \int_0^{\pi/2} \sqrt{1 - [1 - (W/H)^2] \sin^2 \theta} d\theta$
Rhombic		$p = 2\sqrt{H^2 + W^2}$

For the ellipse, Figure 48 (plotted from data on elliptic functions given in Derivation 1) gives p/H plotted against W/H for $W/H \leq 0.5$. For $0.5 \leq W/H \leq 1$ a good approximation to p/H (within ± 0.02 per cent) is given by the formula

$$\frac{p}{H} \approx \frac{\pi}{2} \left[1 + \frac{W}{H} \right] + 0.3 \left[1 - \frac{W}{H} \right]^{2.2} \quad (5.6)$$

5.4 C_P, C_C

The data for C_P and C_C given in the Figures will apply to bodies with any cross-sectional shape provided that, where necessary, D is replaced by the relevant length, H or W , accordingly.

6. DERIVATION AND REFERENCES

6.1 Derivation

The Derivation lists selected sources that have assisted in the preparation of this Item.

1. ABRAMOWITZ, M. Handbook of Mathematical Functions. National Bureau of Standards,
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6.2 References

The References lists selected sources of information supplementary to that given in this Item.

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7. EXAMPLES

7.1 Example 1

Find the surface area and volume of a three-section axisymmetric body of overall length 2 m and diameter 0.2 m. The forebody, of length 0.4 m, utilises Body 7 with $k_1 = 1$ and the afterbody, of length 0.6 m, utilises Body 9 with $A = 0.75$. The parallel midbody is therefore of length 1 m.

For the forebody, $\frac{D}{l_f} \equiv \frac{D}{l} = \frac{0.2}{0.4} = 0.5.$

Thus, from Figure 23 with $D/l = 0.5$ and $1/(1 + k_1) = 0.5$,

$$C_S \equiv C_{Sf} = 0.818.$$

From Figure 29 for Body 7, with $1/(1 + k_1) = 0.5$,

$$C_V \equiv C_{Vf} = 0.688.$$

For the afterbody, $\frac{D}{l_a} \equiv \frac{D}{l} = \frac{0.2}{0.6} = 0.333.$

Thus, from Figure 25 with $D/l = 0.333$ and $A = 0.75$,

$$C_S \equiv C_{Sa} = 0.632.$$

From Figure 28 for Body 9, with $A = 0.75$,

$$C_V \equiv C_{Va} = 0.486.$$

$$\begin{aligned} \text{Therefore, from Equation (4.7), } C_{SL} &= C_{Sf} \frac{l_f}{L} + \frac{l_m}{L} + C_{Sa} \frac{l_a}{L} \\ &= 0.818 \times 0.2 + 0.5 + 0.632 \times 0.3 \\ &= 0.853. \end{aligned}$$

$$\text{Therefore, } S = C_{SL} \times \pi \times D \times L = 0.853 \times \pi \times 0.2 \times 2 = 1.07 \text{ m}^2.$$

$$\begin{aligned} \text{From Equation (4.8), } C_{VL} &= C_{Vf} \frac{l_f}{L} + \frac{l_m}{L} + C_{Va} \frac{l_a}{L} \\ &= 0.688 \times 0.2 + 0.5 + 0.486 \times 0.3 \\ &= 0.783. \end{aligned}$$

$$\text{Therefore, } V = C_{VL} \times \frac{\pi}{4} \times D^2 \times L = 0.783 \times \frac{\pi}{4} \times (0.2)^2 \times 2 = 0.492 \text{ m}^3.$$

7.2 Example 2

In the estimation of the aerodynamic force characteristics of a missile configuration it is required to know the body planform area and the position of the centroid of this area relative to the nose of the body. The axisymmetric body consists of a spherically-blunted tangent ogive forebody of length 0.6 m with blunting ratio, $\rho_0/R = 0.4$, a parallel midbody of length 1.8 m, diameter, $D = 0.3$ m and a truncated conical afterbody of length 0.6 m and base diameter, $D_b = 0.2$ m.

$$\text{For the forebody, } \frac{l_f}{D} = \frac{l}{D} = \frac{0.6}{0.3} = 2.$$

$$\text{From Table 8.5, } \frac{D_c}{D} = \frac{\rho_0}{R} \left[\frac{\rho}{R} - 1 \right] / \left[\frac{\rho}{R} - \frac{\rho_0}{R} \right]$$

$$\begin{aligned} \text{and } \frac{\rho}{R} &= \frac{1}{2} \left\{ \left[4 \left(\frac{l}{D} \right)^2 - 4 \frac{\rho_0}{R} \frac{l}{D} + 1 \right] / \left[1 - \frac{\rho_0}{R} \right] \right\} \\ &= \frac{1}{2} [(4 \times 2^2 - 4 \times 0.4 \times 2 + 1) / (1 - 0.4)] = 11.5. \end{aligned}$$

Therefore,
$$\frac{D_c}{D} = 0.4 [(11.5 - 1)/(11.5 - 0.4)]$$
$$= 0.378.$$

Again, from Table 8.5,
$$\frac{l_c}{D} = \frac{\rho_0}{R} \left[\frac{1}{2} \frac{\rho}{R} - \frac{l}{D} \right] / \left[\frac{\rho}{R} - \frac{\rho_0}{R} \right]$$
$$= 0.4 \left[\frac{11.5}{2} - 2 \right] / (11.5 - 0.4)$$
$$= 0.135.$$

Therefore,
$$\frac{l_c}{D_c} = \frac{l_c D}{D D_c} = \frac{0.135}{0.378} = 0.357.$$

Thus, from Figure 30 with $l_c/D_c = 0.357$,

$$C_P \equiv C_{Pc} = 0.731 \text{ and}$$
$$C_C \equiv C_{Cc} = 1.171.$$

For the truncated ogive forebody section,

$$\frac{l_t}{D} = \frac{l_f}{D} - \frac{l_c}{D} = 2 - 0.135$$
$$= 1.865,$$

$$\frac{D_c}{D} \equiv \frac{D_b}{D} = 0.378 \text{ and}$$

$$\frac{D - D_c}{l_t} \equiv \frac{D - D_b}{l_a} = \frac{D}{l_t} - \frac{D_c}{D} \frac{D}{l_t}$$
$$= \frac{1}{1.865} - \frac{0.378}{1.865}$$
$$= 0.334.$$

Thus, from Figures 33 and 36 with $D_b/D = 0.378$ and $(D - D_b)/l_a = 0.334$,

$$C_P \equiv C_{Pt} = 0.795 \text{ and}$$
$$C_C \equiv C_{Ct} = 1.130.$$

Now,
$$\frac{l_c}{l_f} = \frac{l_c D}{D l_f} = \frac{0.135}{2} = 0.0675,$$

and
$$\frac{l_t}{l_f} = \frac{l_t}{D} \frac{D}{l_f} = \frac{1.865}{2} = 0.9325.$$

Therefore, from Equation (4.1),

$$\begin{aligned} C_{Pf} &= C_{Pc} \frac{D_c}{D} \frac{l_c}{l_f} + C_{Pt} \frac{l_t}{l_f} \\ &= 0.731 \times 0.378 \times 0.0675 + 0.795 \times 0.9325 \\ &= 0.76, \end{aligned}$$

and from Equation (4.2),

$$\begin{aligned} C_{Cf} &= \frac{C_{Pc}}{C_{Pf}} C_{Cc} \frac{D_c}{D} \left(\frac{l_c}{l_f} \right)^2 + \frac{C_{Pt}}{C_{Pf}} \frac{l_t}{l_f} \left[2 \frac{l_c}{l_f} + C_{Ct} \frac{l_t}{l_f} \right] \\ &= \frac{0.731}{0.76} \times 1.171 \times 0.378 \times (0.0675)^2 \\ &\quad + \frac{0.795}{0.76} \times 0.9325 [2 \times 0.0675 + 1.13 \times 0.932] \\ &= 1.161. \end{aligned}$$

For the afterbody, $\frac{D_b}{D} = \frac{0.2}{0.3} = 0.667.$

Thus, from Figures 32 and 35 with $D_b/D = 0.667$,

$$\begin{aligned} C_P &\equiv C_{Pa} = 0.832 \text{ and} \\ C_C &\equiv C_{Ca} = 1.067. \end{aligned}$$

Therefore, from Equation (4.5),

$$\begin{aligned} C_{PL} &= C_{Pf} \frac{l_f}{L} + \frac{l_m}{L} + C_{Pa} \frac{l_a}{L} \\ &= 0.76 \times \frac{0.6}{3} + \frac{1.8}{3} + 0.832 \times \frac{0.6}{3} \\ &= 0.918. \end{aligned}$$

Giving, $S_p = C_{PL} \times D \times L = 0.918 \times 0.3 \times 3 = 0.826 \text{ m}^2.$

From Equation (4.6),

$$\begin{aligned}
 C_{CL} &= \frac{C_{Pf}}{C_{PL}} C_{Cf} \left(\frac{l_f}{L} \right)^2 + \frac{1}{C_{PL}} \frac{l_m}{L} \left[\frac{l_m}{L} + 2 \frac{l_f}{L} \right] + \frac{C_{Pa} l_a}{C_{PL} L} \left[2 - C_{Ca} \frac{l_a}{L} \right] \\
 &= \frac{0.76}{0.918} \times 1.161 \times \left(\frac{0.6}{3} \right)^2 + \frac{1}{0.918} \times \frac{1.8}{3} \times \left[\frac{1.8}{3} + \frac{2 \times 0.6}{3} \right] \\
 &\quad + \frac{0.832}{0.918} \times \frac{0.6}{3} \times \left[2 - 1.067 \times \frac{0.6}{3} \right] \\
 &= 1.016.
 \end{aligned}$$

Therefore, $x_p = C_{CL} \times \frac{L}{2} = 1.016 \times \frac{3}{2} = 1.524 \text{ m}.$

8. TABLES

TABLE 8.1
Axisymmetric Forebody and Afterbody Shapes Considered

Body No. (Forebody or Afterbody)	Generating Equation	Common Name	Fig. No. for Shape (see also Fig. 1)
1	$\frac{r}{R} = \frac{1}{\pi^{1/2}} \left\{ \cos^{-1} \left[1 - 2 \frac{x}{l} \right] - 2 \left[1 - 2 \frac{x}{l} \right] \left[\frac{x}{l} - \left[\frac{x}{l} \right]^2 \right]^{1/2} \right\}^{1/2}$	von Kármán Ogive (Reference 2)	2
2	$\frac{r}{R} = \frac{x}{l} \left[3 - 2 \frac{x}{l} \right]^{1/2}$	Lighthill Profile (Reference 4)	2
3	$\frac{r}{R} = \left\{ F^2 + (2F + 1) \left[1 - \left[1 - \frac{x}{l} \right]^2 \right] \right\}^{1/2} - F$ where $F = \frac{1}{2} \left[4 \left[\frac{l}{D} \right]^2 - 1 \right]$	Tangent Ogive	3
4	$\frac{r}{R} = \left\{ \left[\frac{\rho}{\rho'} E \right]^2 - 4 \left[\frac{l_o}{D} - \frac{l x}{D l} \right]^2 \right\}^{1/2} - \left\{ \left[\frac{\rho}{\rho'} E \right]^2 - 4 \left[\frac{l_o}{D} \right]^2 \right\}^{1/2}$ where $E = \frac{1}{2} \left[4 \left[\frac{l}{D} \right]^2 + 1 \right]$ and, given l/D , l_o/D is obtained from the quadratic, $8E \left[\frac{l_o}{D} \right]^2 - 8 \frac{l}{D} E \frac{l_o}{D} + E^2 \left[1 - \left[\frac{\rho}{\rho'} \right]^2 \right] = 0$	Secant Ogive	*
5	$\frac{r}{R} = \left[\frac{x}{l} \right]^m$ ditto, $m = 1$	Power Law (Reference 6) Cone	4 4
6	$\frac{r}{R} = \left[1 - \left[1 - \frac{x}{l} \right]^2 \right]^n$ ditto, $n = \frac{1}{2}$ ditto, $n = \frac{3}{4}$ ditto, $n = 1$	Modified Ellipsoid Ellipsoid Sears-Haack (References 3, 5) Paraboloid	5 5 5 5
7	$\frac{r}{R} = \frac{\alpha_1^{1/2}}{8} (15 - 10\alpha_1 + 3\alpha_1^2)$ where $\alpha_1 = \left[\frac{x}{l} + k_1 \left[\frac{x}{l} \right]^2 \right] / (1 + k_1)$	ESDU I	6
8	$\frac{r}{R} = \alpha_2 (3 - 3\alpha_2 + \alpha_2^2)$ where $\alpha_2 = \left[\frac{x}{l} + k_2 \left[\frac{x}{l} \right]^2 \right] / (1 + k_2)$	ESDU II	7
9	$\frac{r}{R} = 1 - (3 - 2A) \left[1 - \frac{x}{l} \right]^2 + 2(1 - A) \left[1 - \frac{x}{l} \right]^3$	Myring (Cubic) (Reference 8)	8

* As these body shapes require more than one defining parameter, sketches for these shapes are not included.

TABLE 8.1 (Concluded)

Body No. (Forebody or Afterbody)	Generating Equation	Common Name	Fig. No. for Shape (see also Fig. 1)
10	$\frac{r}{R} = 2 \left[\frac{x}{l} \frac{l}{D} \left[\frac{\rho_0}{R} - \frac{x}{l} \frac{l}{D} \right] \right]^{\frac{1}{2}}, \quad \text{for } \frac{x}{l} \leq \frac{1}{2} \frac{\cos \delta}{l/D} - \sin \delta$ where $\frac{\rho_0}{R} = \sec \delta (1 + \sin \delta) [1 - 2(l/D) \tan \delta]$ and $\frac{r}{R} = 1 - 2 \frac{l}{D} \tan \delta \left[1 - \frac{x}{l} \right], \quad \text{for } \frac{x}{l} \geq \frac{1}{2} \frac{\cos \delta}{l/D} - \sin \delta$	Spherically-blunted Cone	*
11	$\frac{r}{R} = 2 \left[\frac{x}{l} \frac{l}{D} \left[\frac{\rho_0}{R} - \frac{x}{l} \frac{l}{D} \right] \right]^{\frac{1}{2}}, \quad \text{for } \frac{x}{l} \leq \frac{1}{2l/D} \frac{\rho_0}{R} \left[\frac{G - 2l/D}{G - \rho_0/R} \right]$ and $\frac{r}{R} = 1 - G + [G^2 - 4(l/D)^2 (x/l - 1)^2]^{\frac{1}{2}},$ for $\frac{x}{l} \geq \frac{1}{2l/D} \frac{\rho_0}{R} \left[\frac{G - 2l/D}{G - \rho_0/R} \right]$ where $G = \frac{\rho}{R} = \frac{1}{2} \left[\frac{4(l/D)^2 - 4(\rho_0/R)(l/D) + 1}{1 - \rho_0/R} \right]$	Spherically- blunted Tangent Ogive	*
12	$\frac{r}{R} = 2 \left[\frac{x}{l} \frac{l}{D} \left[\frac{\rho_0}{R} - \frac{x}{l} \frac{l}{D} \right] \right]^{\frac{1}{2}},$ for $\frac{x}{l} \leq \frac{\rho_0}{R} \frac{1}{l/D} \left[\frac{(\rho/\rho')E' - 2(l_0/D - l'/D + l/D)}{2[(\rho/\rho')E' - \rho_0/R]} \right]$ and $\frac{r}{R} = \left\{ \left[\frac{\rho}{\rho'} E' \right]^2 - 4 \left[\frac{l}{D} \right]^2 \left[\frac{1}{2l/D} \left\{ \left[\frac{\rho_0}{R} \right]^2 - 2 \frac{\rho_0}{R} \frac{\rho}{\rho'} E' + 4 \left[\frac{l_0}{D} \right]^2 \right\} \right. \right. \right. \\ \left. \left. \left. + \frac{1}{2l/D} \frac{\rho_0}{R} - \frac{x}{l} \right]^2 \right\}^{\frac{1}{2}} - \left\{ \left[\frac{\rho}{\rho'} E' \right]^2 - 4 \left[\frac{l_0}{D} \right]^2 \right\}^{\frac{1}{2}} \right\}^{\frac{1}{2}}$ for $\frac{x}{l} \geq \frac{\rho_0}{R} \frac{1}{l/D} \left[\frac{(\rho/\rho')E' - 2(l_0/D - l'/D + l/D)}{2[(\rho/\rho')E' - \rho_0/R]} \right]$ where $E' = \frac{1}{2} \left[4 \left[\frac{l'}{D} \right]^2 + 1 \right]$ and, given $l'/D, l_0/D$ is obtained from the quadratic $8E' \left[\frac{l_0}{D} \right]^2 - 8 \frac{l'}{D} E' \frac{l_0}{D} + E'^2 \left[1 - \left[\frac{\rho}{\rho'} \right]^2 \right] = 0$	Spherically-blunted Secant Ogive	*

* As these body shapes require more than one defining parameter, sketches for these shapes are not included.

PLANFORM AREA, CENTROID OF PLANFORM AREA, SURFACE AREA AND VOLUME COEFFICIENTS

TABLE 8.2 Axisymmetric Forebodies and Afterbodies

Body No. (Forebody or Afterbody)	Figure No. for:			
	Planform Area Coefficient C_P	Centroid of Planform Area Coefficient C_C	Surface Area Coefficient C_S	Volume Coefficient C_V
1	9	14	19	26
2	9	14	19	26
3	9	14	19	28
4	10	15	20	27
5	11	16	21	26
6	11	16	22	26
7	12	17	23	29
8	12	17	24	29
9	13	18	25	28
10	30, 32	30, 35	31, 38	31, 42
11	30, 33	30, 36	31, 40	31, 43
12	30, 34	30, 37	31, 41	31, 44

TABLE 8.3 Truncated Axisymmetric Afterbodies

Afterbody	Figure No. for:			
	Planform Area Coefficient C_P	Centroid of Planform Area Coefficient C_C	Surface Area Coefficient C_S	Volume Coefficient C_V
Cone	32	35	38	42
Paraboloid	32	35	39	42
Tangent Ogive	33	36	40	43
Secant Ogive	34	37	41	44

DIMENSIONLESS RADIUS OF CURVATURE AND HALF-ANGLE PARAMETER

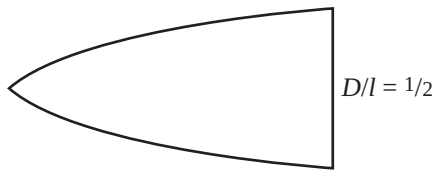
TABLE 8.4 Axisymmetric Forebodies and Afterbodies

Body No. (Forebody or Afterbody)	$\bar{\rho}_0$	$\frac{l}{D} \tan \tau_0$
1	0	∞
2	—	$\sqrt{3}/2$
3	—	$1 + (\frac{1}{2}F)$
4	—	$1 + (\frac{1}{2}F_o)^*$
5	∞ (for $m < \frac{1}{2}$) $1/8$ (for $m = \frac{1}{2}$) 0 (for $m > \frac{1}{2}$)	∞ (for $m < 1$) $\frac{1}{2}$ (for $m = 1$) 0 (for $m > 1$)
6	∞ (for $n < \frac{1}{2}$) $\frac{1}{4}$ (for $n = \frac{1}{2}$) 0 (for $n > \frac{1}{2}$)	∞ (for $n < 1$) 1 (for $n = 1$) 0 (for $n > 1$)
7	$0.439/(1 + k_1)$	∞
8	—	$1.5/(1 + k_2)$
9	—	A
10	$\rho_0 l/D^2$	∞
11	$\rho_0 l/D^2$	∞
12	$\rho_0 l/D^2$	∞

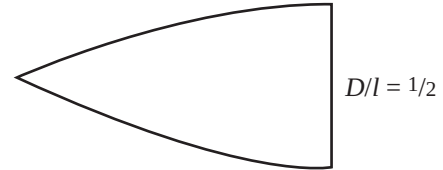
* Where $F_o = \frac{1}{2} \left[4 \left(\frac{l_o}{D_o} \right)^2 - 1 \right]$ and $\frac{l_o}{D_o} = \frac{l_o}{D} \left\{ \frac{\rho}{\rho^*} E - \left[\left(\frac{\rho}{\rho^*} E \right)^2 - 4 \left(\frac{l_o}{D} \right)^2 \right]^{1/2} \right\}$

TABLE 8.5 Selected Geometric Relationships for Spherically-blunted Forebodies

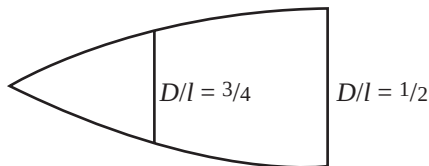
<i>Cone</i>	<i>Tangent Ogive</i>	<i>Secant ogive</i>
Given l/D and ρ_0/R	Given l/D and ρ_0/R	Given l'/D , ρ_0/R and ρ/ρ'
$\frac{l_c}{D} = \frac{1}{2} \frac{\rho_0}{R} \left\{ 1 - \frac{[1 + 4(l/D)^2 - 4(\rho_0/R)(l/D)]^{1/2} - (2l/D - \rho_0/R)(\rho_0/R)}{1 + (2l/D - \rho_0/R)^2} \right\}$ $\frac{D_c}{D} = \frac{\rho_0}{R} \left\{ \frac{\rho_0/R + (2l/D - \rho_0/R)[1 + 4(l/D)^2 - 4(\rho_0/R)(l/D)]^{1/2}}{1 + (2l/D - \rho_0/R)^2} \right\}$ $\frac{l_t}{D} = \frac{l}{D} - \frac{l_c}{D}$	$\frac{\rho}{R} = \frac{1}{2} \left\{ \frac{4(l/D)^2 - 4(\rho_0/R)(l/D) + 1}{1 - \rho_0/R} \right\}$ $\frac{D_c}{D} = \frac{\rho_0}{R} \left\{ \frac{\rho/R - 1}{\rho/R - \rho_0/R} \right\}$ $\frac{l_c}{D} = \frac{\rho_0}{R} \left\{ \frac{1/2 \rho/R - l/D}{\rho/R - \rho_0/R} \right\}$ $\frac{l}{D} = \frac{l_f}{D} = \frac{l_c}{D} + \frac{l_t}{D}$	$\frac{\rho}{R} = \frac{\rho}{\rho'} \frac{1}{2} \left[4 \left[\frac{l'}{D} \right]^2 + 1 \right] = \frac{\rho}{\rho'} E'$ $\frac{l_o}{D} \text{ from quadratic for Body 12 in Table 8.1.}$ $\frac{l}{D} = \frac{1}{2} \left[\left[\frac{\rho_0}{R} \right]^2 - 2 \frac{\rho_0}{R} \frac{\rho}{R} + \left[\frac{l_o}{R} \right]^2 \right]^{1/2} - \frac{l_o}{D} + \frac{l'}{D} + \frac{1}{2} \frac{\rho_0}{R}$ $\frac{l_c}{D} = \frac{\rho_0}{R} \left[\frac{\rho/R - 2(l_o/D - l'/D + l/D)}{2(\rho/R - \rho_0/R)} \right]$ $\frac{D_c}{D} = \left[\left[\frac{\rho}{R} \right]^2 - 4 \left[\frac{l_o}{D} - \frac{l'}{D} + \frac{l}{D} - \frac{l_c}{D} \right]^2 \right]^{1/2}$ $- \left[\left[\frac{\rho}{R} \right]^2 - 4 \left[\frac{l_o}{D} \right]^2 \right]^{1/2}$



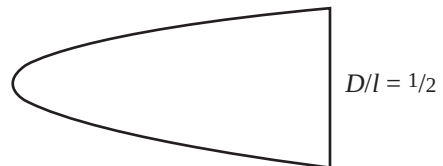
Body 1. Von Kármán ogive



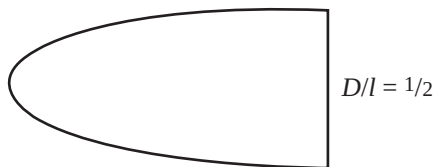
Body 2. Lighthill profile



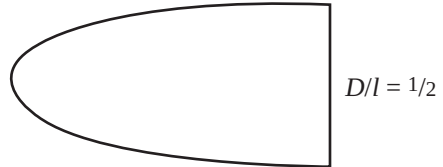
Bodies 3 and 4. Tangent ogive ($F = 7.50$)
and secant ogive* ($E = 4.06$)



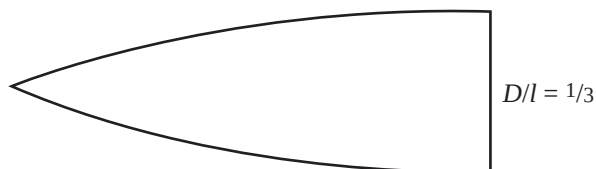
Body 5. Power law ($m = 1/2$)



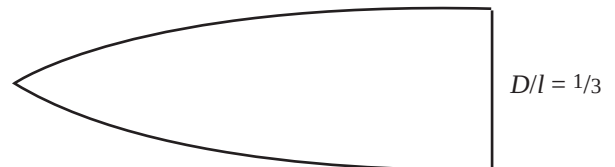
Body 6. Ellipsoid ($n = 1/2$)



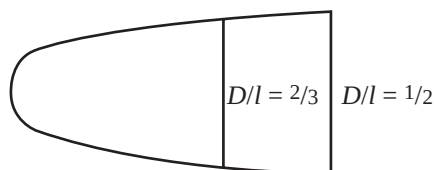
Body 7. ESDU I ($k_1 = 1/2$)



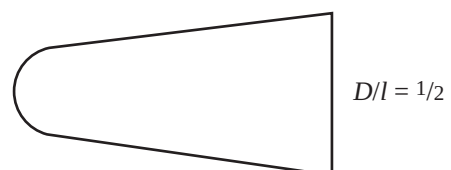
Body 8. ESDU II ($k_2 = 1/2$)



Body 9. Myring ($A = 1.5$)



Bodies 11 and 12.
Spherically-blunted tangent ogive ($\rho_0/R = 0.6$)
and secant ogive* ($\rho_0/R = 0.63$)



Body 10.
Spherically-blunted cone ($\rho_0/R = 0.6$)

* The secant ogive is a tangent ogive with a truncated base

FIGURE 1 TYPICAL AXISYMMETRIC FOREBODIES AND AFTERBODIES