



# ESDU 78001

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## **The momentum-change component of pressure change in two-phase flow and other non-equilibrium effects**

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## THE PREPARATION OF THIS DATA ITEM

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## THE MOMENTUM-CHANGE COMPONENT OF PRESSURE CHANGE IN TWO-PHASE FLOW AND OTHER NON-EQUILIBRIUM EFFECTS

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## THE MOMENTUM-CHANGE COMPONENT OF PRESSURE CHANGE IN TWO-PHASE FLOW AND OTHER NON-EQUILIBRIUM EFFECTS

### 1. NOTATION AND UNITS

|                 |                                                                                             | SI                           |
|-----------------|---------------------------------------------------------------------------------------------|------------------------------|
| $A$             | cross-sectional area of flow                                                                | $\text{m}^2$                 |
| $D$             | pipe inside diameter                                                                        | $\text{m}$                   |
| $G$             | mass velocity of whole mixture $= 4(\dot{m}_G + \dot{m}_L)/\pi D^2$                         | $\text{kg/s m}^2$            |
| $\dot{m}_G$     | mass flow rate of gas or vapour component of mixture                                        | $\text{kg/s}$                |
| $\dot{m}_L$     | mass flow rate of liquid component of mixture                                               | $\text{kg/s}$                |
| $p$             | absolute static pressure                                                                    | $\text{Pa (N/m}^2\text{)}$   |
| $dp/dz$         | pressure gradient                                                                           | $\text{Pa/m (N/m}^3\text{)}$ |
| $dp_a/dz$       | component of pressure gradient due to momentum change                                       | $\text{Pa/m (N/m}^3\text{)}$ |
| $dp_f/dz$       | component of pressure gradient due to friction                                              | $\text{Pa/m (N/m}^3\text{)}$ |
| $dp_g/dz$       | component of pressure gradient due to gravity                                               | $\text{Pa/m (N/m}^3\text{)}$ |
| $u$             | velocity (scalar)                                                                           | $\text{m/s}$                 |
| $VR$            | ratio of mean gas or vapour velocity to mean liquid velocity                                | —                            |
| $x$             | mass quality of gas or vapour component given by<br>$x = \dot{m}_G/(\dot{m}_G + \dot{m}_L)$ | —                            |
| $z$             | pipe axial length, positive in direction of flow                                            | $\text{m}$                   |
| $\rho_G$        | density of gas or vapour phase                                                              | $\text{kg/m}^3$              |
| $\rho_H$        | density defined by Equation (P1.1) for use with homogeneous theory                          | $\text{kg/m}^3$              |
| $\rho_L$        | density of liquid phase                                                                     | $\text{kg/m}^3$              |
| $\omega_{LO}^2$ | momentum flux multiplier                                                                    | —                            |

### Subscripts

- 1 refers to conditions at an upstream station
- 2 refers to conditions at a downstream station

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## 2. INTRODUCTION

This Item is one of a group that provides data enabling pressure losses in the flow of two-phase mixtures to be calculated.

Because conditions may change rapidly in the direction of flow with two-phase mixtures, it is often necessary, when calculating finite changes in pressure, to carry out an integration along the flow path of locally evaluated pressure gradients. Thus each Item in this series contributes towards the calculation of local pressure gradient which can itself be treated as the sum of three separate components, *i.e.*

$$\underbrace{\frac{dp}{dz}}_{\text{overall local gradient}} = \underbrace{\frac{dp_f}{dz}}_{\text{frictional component}} + \underbrace{\frac{dp_g}{dz}}_{\text{gravitational component}} + \underbrace{\frac{dp_a}{dz}}_{\text{momentum-change component}} \quad (2.1)$$

The component,  $dp_f/dz$ , due to friction is dealt with for straight pipes by ESDU 76018<sup>1</sup> and the component,  $dp_g/dz$ , due to gravity, is dealt with for straight pipes by ESDU 77016<sup>2</sup>. The effects of changing momentum for all configurations of equal inlet and outlet flow areas can be dealt with by this Item both in terms of the directly related pressure change, resulting from  $dp_a/dz$ , and in terms of the associated state of disequilibrium which is known to affect the remaining components.

In addition to this group of three Items, two other Items are available that give guidance on more general aspects of two-phase flow. ESDU FMI2<sup>3</sup> is a guide on the flow relationships and statistical methods used in ESDU 76018<sup>1</sup> and 77016<sup>2</sup> (dealing with the components  $dp_f/dz$  and  $dp_g/dz$  respectively) and ESDU 78018<sup>4</sup> gives guidance on the calculation procedures to adopt for solving typical problems in two-phase flow.

In this Item, Section 3 introduces the causes of momentum change and its effects on pressure change and Section 6 describes two major causes of disequilibrium.

## 3. INTRODUCTION TO THE MOMENTUM-CHANGE COMPONENT OF PRESSURE CHANGE

The component of pressure gradient,  $dp_g/dz$ , that results from a change in momentum flux of a two-phase mixture arises from, for example, evaporation or condensation induced by heat transfer and flashing. However, it can also result simply from a change in density associated with the changing pressure of the flow just as in single-phase flow. In either case, the pressure change due to momentum change is always independent of the flow path. Thus when calculating a finite overall pressure difference between two points, it is not necessary to integrate  $dp_g/dz$  along the flow path because, provided that the momentum flux of the flow is known at both points, the pressure loss due to momentum change can be equated to the gain of momentum flux between the two points. However, unlike single-phase flow, the evaluation of the momentum flux is difficult because it is strongly dependent on the two-phase flow pattern which cannot be predicted reliably. This Item recommends methods for the evaluation of momentum flux that are considered the most appropriate for the problem. It is not possible to recommend a single method as the most appropriate for all situations but, instead, guidance is given in Section 5 on the more suitable choice of two alternative methods. These, depending on the circumstances, will underestimate or overestimate the true change in momentum flux of the flow and can be selected accordingly.

The momentum flux of a fluid is the integral of the product of mass flow rate and velocity divided by the area of cross section through which it flows. However, in a two-phase mixture, both the local elemental mass flow rates, *i.e.*  $d\dot{m}$  and the local velocities,  $u$ , are generally not known in detail because the two phases can be distributed within the pipe in a variety of ways\* and can flow with greatly differing velocity distributions. The two theories presented in this Item represent extremes of flow structure where the phases are either perfectly mixed (homogeneous theory) or fully independent (separated flow theory). It is possible to devise other theories from more sophisticated models of the flow structure, but these have not been included because they offer no measurable benefit to compensate for their additional complexities.

Descriptions and calculation procedures may be found for the homogeneous theory in Appendix P and for the separated flow theory in Appendix Q. Appendices A to G appear in ESDU 76018<sup>1</sup> and detail methods for calculating  $dp_f/dz$ . Appendices H to O appear in ESDU 77016<sup>2</sup> and detail methods for calculating  $dp_g/dz$ .

\* ESDU FMI 2<sup>3</sup> discusses the several flow patterns that can arise.