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Evaluation of the attenuation of broad-band sound by a non-uniform still atmosphere

Associated software: ESDUpac A7803
and B7803



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EVALUATION OF THE ATTENUATION OF BROAD-BAND SOUND BY A NON-UNIFORM STILL ATMOSPHERE

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EVALUATION OF THE ATTENUATION OF BROAD-BAND SOUND BY A NON-UNIFORM STILL ATMOSPHERE

1. NOTATION

A	accuracy parameter		
c	geometric band factor ($f_u = cf_l$)		
F	ratio of nominal to geometric band centre frequencies		
f	frequency	Hz	Hz
g_0	gravitational constant	m/s ²	ft/s ²
H	altitude	m	ft
I	intensity	W/m ²	lbf/s ft
K	normalized geometric band factor		
k	constant in $I = kf^{n-1}$		
m	total number of atmospheric layers or sub-layers		
n	spectrum slope	dB/ log ₁₀ Hz	dB/ log ₁₀ Hz
n'	smoothed spectrum slope	dB/ log ₁₀ Hz	dB/ log ₁₀ Hz
n''	average smoothed spectrum slope	dB/ log ₁₀ Hz	dB/ log ₁₀ Hz
δn	difference between two values of spectrum slope ($n_i - n_{i-1}$)	dB/ log ₁₀ Hz	dB/ log ₁₀ Hz
Δn	increase in spectrum slope due to atmospheric attenuation	dB/ log ₁₀ Hz	dB/ log ₁₀ Hz
Q	octave fraction		
R	gas constant for air	N m/kg K	lbf ft/slug K
S	band sound pressure level	dB*	dB*
S'	smoothed band sound pressure level	dB*	dB*
T	temperature	K	K
x	propagation distance	m	ft
α	discrete frequency attenuation rate	dB/100 m	dB/1000 ft

For footnote see end of Notation.

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β	discrete frequency attenuation due to vertical propagation across layer	dB	dB
γ	total attenuation along propagation path	dB	dB
δ	relative pressure		

Suffixes

B	bottom
b	band
d	discrete frequency
g	geometric centre frequency
i	i^{th} frequency band of spectrum
l	lower
M	middle
m	measured conditions
n	nominal centre frequency
p	layer number
s	source
T	top
u	upper

2. NOTES

This Item provides a means of estimating the attenuation of sound across a frequency band, referred to as band attenuation, by the atmosphere in which the temperature and relative humidity vary along the propagation path. The band attenuation rate is estimated by integrating the discrete frequency attenuation over the appropriate frequency range. Values of discrete frequency attenuation are found using ESDU 78002 which is based on the prediction method given in Derivation 1. Allowance is made for spectrum shape, and guidance is given on the use of a layering technique for the estimation of attenuation when atmospheric properties vary with altitude.

The atmospheric layering technique should only be used when the variation of the atmospheric parameters with altitude is known with sufficient confidence to enable the division of the atmosphere into layers within which temperature and relative humidity vary linearly with altitude. When considering horizontal propagation of sound where the temperature and relative humidity vary along the propagation path, the

suggested layering technique can be used by substituting linear distance for altitude. In this case any change in pressure between the layers due to variations in temperature is extremely small and should be neglected.

A computer program, known as ESDUpac A7803, to estimate band atmospheric attenuation assuming an ideal filter shape is described in Appendix B and accompanies this Item. An alternative graphical procedure for estimating one-third octave band atmospheric attenuation is described in Appendix A. The program can also be used to add or subtract the band attenuation from spectra input via the Common Data File system (see Appendices B and C).

2.1 Outline of the Numerical Method

The numerical method estimates the attenuation of a band of noise propagating through a non-uniform still atmosphere.

The band attenuation is found from the difference between the unattenuated and attenuated intensities integrated with respect to frequency between the upper and lower limiting frequencies of the band. As both the source spectrum shape and atmospheric attenuation are functions of frequency the integrations are performed numerically using Simpson's rule.

When atmospheric layering is used the basic meteorological layers, within which temperature and relative humidity vary linearly with distance, are further subdivided, if necessary, such that the attenuation rate at a selected frequency varies approximately linearly across the layer. The atmospheric conditions at the mid-point of each layer or sub-layer are then used to estimate the discrete frequency attenuation rates at the band limiting and band centre frequencies for the layer. The total discrete frequency attenuations are calculated over the propagation path and these are integrated to give the required band attenuations.

In order to estimate the broad-band levels a simple power law relationship between intensity and frequency has been assumed to be valid over a restricted frequency range. The spectrum slopes of the measured broad-band spectrum are found from the measured spectrum after it has been smoothed. The source spectrum slope is then calculated from the smoothed values of the slope of the measured spectrum by accounting for the effects of atmospheric attenuation along the propagation path between the source and the measuring position. While smoothing spectra, the discrete frequency components can be identified enabling the attenuation of discrete frequencies to be considered independently of the band attenuation.

The numerical method takes no account of the spherical spreading of sound and the user should use the inverse square law to account for this effect.

3. DETERMINATION OF SPECTRUM SLOPE

3.1 Slope of the Measured Spectrum

The band intensity level is found by integrating the intensity-frequency function, with respect to frequency, between the upper and lower frequencies of the band. Acoustic intensity, I , may be expressed in terms of frequency, f , over a limited frequency range, by the relationship

$$I = kf^{n-1}, \quad (3.1)$$

where k is a constant and n the spectrum slope.

Assuming that the intensity/frequency relationship of Equation (3.1) is valid over two adjacent bands the spectrum slope can be simply determined from the difference between the adjacent band levels. Normalising

the band factor, c , with respect to a one-third octave so that

$$K = 10 \log_{10} \left[\frac{c}{10^{0.1}} \right],$$

the spectrum slope, n_i , is given by

$$n_i = \frac{S_i - S_{i-1}}{1 + K}, \quad (3.2)$$

where S is the band sound pressure level.

Values of K for various fractional octave bandwidths are given in Table 8.1

3.2 Slope of the Source Spectrum

Since atmospheric attenuation is a function of frequency for any sound measured at some distance from its source, the measured sound pressure level in any band is related to the source sound pressure level in the same band by

$$S_s = S_m + \gamma_b,$$

where γ_b is the band attenuation over the propagation path. The source spectrum slope is given by the measured spectrum slope plus an amount due to atmospheric attenuation. As the spectrum slope is considered to be constant over the band and assuming that the attenuation rate is proportional to the logarithm of frequency, the correction to spectrum slope, Δn , is

$$\Delta n_i = \frac{\gamma_d}{10 \log_{10} f_g}, \quad (3.3)$$

where γ_d is the discrete frequency attenuation at f_g , the band centre frequency. The discrete frequency attenuation, γ_d , is related to the attenuation rate, α , by

$$\gamma_d = \frac{\alpha x}{100},$$

where α is the attenuation rate in dB/100 m and x is the propagation distance in metres.

4. BAND ATMOSPHERIC ATTENUATION

The band attenuation is a function of propagation distance and is found from the difference between the source and measured band sound pressure levels. Using Equation (3.1) to represent the variation of source intensity with frequency, the variation of intensity with frequency for the measured sound is given by

$$I_m = k f^{n-1} 10^{\left(-\frac{\alpha x}{1000} \right)}.$$

Then the band attenuation over a distance x is found from

$$\gamma_b(x) = 10 \log_{10} \left[\int_{f_l}^{f_u} k f^{n_s-1} df \right] - 10 \log_{10} \left[\int_{f_l}^{f_u} k f^{n_s-1} 10^{\left(-\frac{\alpha x}{1000}\right)} df \right]. \quad (4.1)$$

As the discrete frequency attenuation rate is a function of frequency, Equation (4.1) can generally only be solved using numerical techniques. Integrating both parts with respect to log frequency using a single application of Simpson's rule, the band attenuation is given by

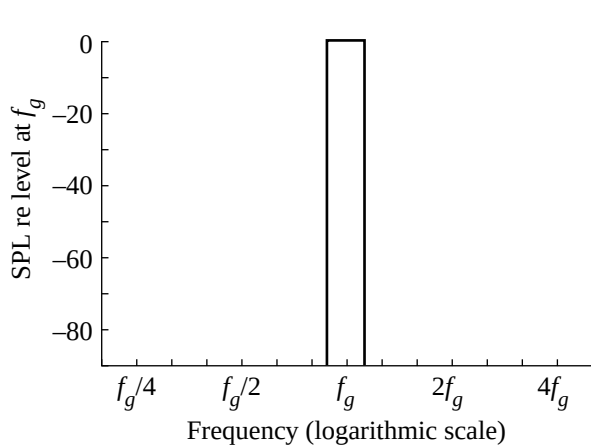
$$\gamma_b(x) = 10 \log_{10} \left[\frac{f_l^{n_s} + 4f_g^{n_s} + f_u^{n_s}}{f_l^{n_s} 10^{\left(-\frac{\alpha_l x}{1000}\right)} + 4f_g^{n_s} 10^{\left(-\frac{\alpha_g x}{1000}\right)} + f_u^{n_s} 10^{\left(-\frac{\alpha_u x}{1000}\right)}} \right], \quad (4.2)$$

where α is in dB/100 m and x is in metres.

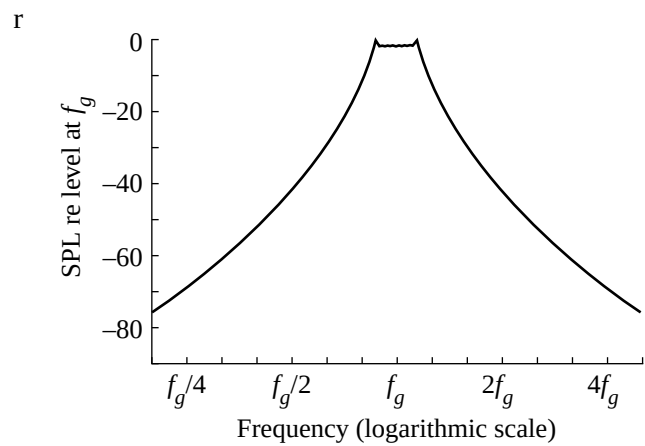
Values of the band nominal centre, band limiting and band geometrical frequencies for one-third octave bands are presented in Table 8.2.

4.1 The Effect of Filter Width

The analysis presented assumes that the spectrum has been analysed using ideal filters. An ideal filter only passes a signal from frequencies that lie between the lower and upper frequencies of the band being analysed. In practice the signal obtained from a real filter contains energy from frequencies outside the limiting frequencies of the frequency band being considered. However this additional energy is attenuated by the filter. In a real filter the attenuation due to the filter is dependent on the ratio of the frequency to the centre frequency of the filter passband. Typical characteristics for an ideal and a real one-third octave filter are illustrated in Sketches 4.1 and 4.2, respectively.



Sketch 4.1 One-third octave ideal filter



Sketch 4.2 One-third octave real filter

When compared with the band attenuation for an ideal filter the effect of the increased width of a real filter is to cause a decrease in the observed band attenuation when sound pressure level is decreasing with frequency and an increase when sound pressure level is increasing rapidly with frequency. These effects are significant only when the modulus of the slope of the measured spectrum is greater than the modulus of the filter characteristic slope.

Considering typical noise measurements, the effect of a non-ideal filter may become significant at the higher frequencies of the measured noise spectrum by which point the energy in the band has been attenuated so that its contribution to the overall sound pressure level is negligible. It is in this region that noise measurements should be treated with caution irrespective of the filter characteristic.

4.2 Accuracy of the Numerical Procedure

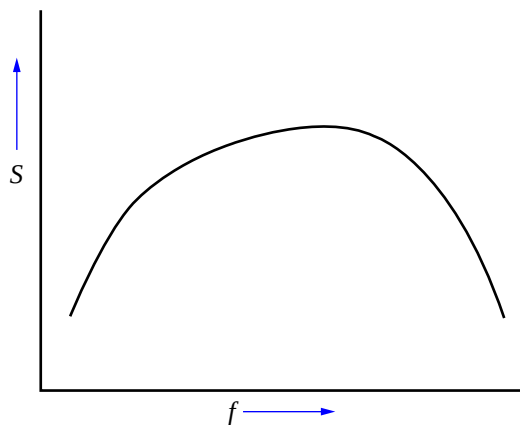
The accuracy of the numerical procedure has been assessed by considering the percentage difference between band attenuation rates calculated using one application of Simpson's rule to integrate over a whole one-third octave with the calculated values obtained by dividing the one-third octave band into many parts and using Simpson's rule to integrate over each part of the band separately. In fact using one application of Simpson's rule results are obtained within ± 0.2 per cent of those found applying Simpson's rule many times for typical values of attenuation and spectrum slope.

By way of example consider the atmospheric attenuation over a distance of 100 metres for one-third octave bands having typical values of spectrum slope and band attenuation less than 30 dB. The accuracy of the numerical integration procedure using the geometric centre frequency of the band is ± 0.2 per cent and using the band nominal centre frequency is ± 1 per cent. For values of the one-third octave attenuation greater than 100 dB over 100 metres, the accuracy becomes ± 1 per cent in both cases.

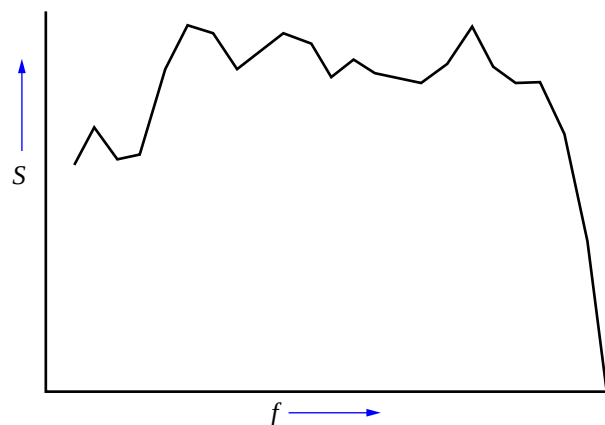
As the calculation procedures have assumed an ideal filter shape the accuracy of the calculated values of atmospheric attenuation is limited by the characteristic of the filters used in the determination of the measured sound pressure level. For real one-third octave filters the error in band atmospheric attenuation due to the filter width is in general less than one per cent for spectrum slopes with a modulus less than 10 and a band attenuation due to the atmosphere less than 30 dB. However, for a spectrum slope of -14 , for example, and atmospheric attenuation of 50 dB the error may be 10 per cent.

4.3 Determination of the Spectrum Slope for Irregular Spectra

The analysis presented in Section 3.1 assumes a smooth spectral shape as shown in Sketch 4.3. Measured spectra often show considerable variation from this ideal case and can be of the form shown in Sketch 4.4.



Sketch 4.3 Smooth spectrum



Sketch 4.4 Measured spectrum

It is necessary to smooth out the irregularities in a real spectrum to derive a spectrum with a smooth shape, similar to that in Sketch 4.3. For each frequency band the attenuation is estimated using the smoothed spectrum slope of the band considered.

For those bands, at the higher frequencies of the measured spectrum, where the spectrum slope is less than

-2 (i.e. $n < -2$) and the spectrum has been smoothed, consideration should be given to re-analysis using a narrower bandwidth in order to identify the spectral irregularity. Bands that, on re-analysis, are found to contain tones should be corrected using the approach outlined in Section 4.4. In the lower frequency range of the measured spectrum, differences between the smoothed and measured band sound pressure levels are generally not significant because at those frequencies there is little difference between band and tone attenuations.

A recommended procedure for estimating the spectrum slope for irregular spectra is given in Section 6.2.

4.4 Tone Influenced Bands

If on re-analysis a discrete frequency is found and its level and frequency are established then the atmospheric attenuation of each tone is estimated using ESDU 78002. The tone level is subtracted from the measured band level (using ESDU 66017) and the resulting band level corrected for atmospheric attenuation. The corrected measured band level is then found by combining the corrected tone and band levels using ESDU 66017.

5. APPLICATION TO REAL ATMOSPHERES

The prediction method for the atmospheric attenuation of discrete frequencies, ESDU 78002, is only applicable to a still homogeneous atmosphere. In order to account for the variation of temperature and humidity with altitude it is suggested that (Reference 3) the atmosphere is divided into horizontal layers inside which temperature and humidity can be assumed to vary linearly with altitude, and these are referred to as the basic meteorological layers. As the variation of attenuation rate with temperature and relative humidity is non-linear the basic meteorological layers should be divided into sub-layers such that the variation of attenuation rate with altitude across each sub-layer can be considered to be linear. The mean values of temperature and relative humidity are used to calculate the discrete frequency attenuation rates for the sub-layer. The band attenuation through a layer must be calculated using the discrete frequency attenuations given by the actual length of the propagation path through the layer; the propagation path is not necessarily the layer thickness.

5.1 Selection of Layer Thickness

The accuracy with which values of band attenuation may be calculated is dependent on the knowledge of the variations of temperature and relative humidity over the propagation path. For a reliable estimate of band attenuations, the atmospheric parameters must be measured at intervals along the propagation path such that the assumption of linear variation with distance for both temperature and relative humidity is valid between measuring stations.

When insufficient atmospheric data are available, such that large variations in temperature and relative humidity may be undetected, a cautious approach should be used in dividing the basic meteorological layers into sub-layers. When the assumption of constant variation with distance of either temperature or relative humidity is not valid it is recommended that no subdivision of atmospheric layers be made because the accuracy in estimated band attenuation is unlikely to be improved by layering.

5.1.1 Division of the atmosphere into layers

When the measured atmosphere is known to be stable and the assumption of constant lapse rates across the basic meteorological layers for both temperature and relative humidity can be made, consideration should be given to layering the atmosphere.

The thickness of the atmospheric layer required to approximate the condition of linear variation of attenuation rate with altitude on which the method is based is estimated using the attenuation rates calculated at a selected discrete frequency. Atmospheric attenuation increases with frequency so that the effect of a non-linear variation of attenuation rate with the atmospheric parameters will be most evident at the higher frequencies and it is recommended that the selected frequency should be the highest frequency within the range of interest.

To assess whether the variation of attenuation rate is sufficiently linear within a layer for the test case in hand the accuracy parameter, A , is estimated using Equation (5.1).

$$A = 100 \left(1 - \frac{\alpha_T + \alpha_B}{2\alpha_M} \right), \quad (5.1)$$

where α_B , α_M and α_T are the attenuation rates, calculated at the selected frequency, at the bottom, middle and top of the layer, respectively.

If A is greater than the maximum acceptable percentage error in band attenuation the layer is repeatedly subdivided until the value of A , for the sub-layer, is less than the acceptable percentage error in band attenuation. The error that is then achieved is not greater than A per cent.

Normally the error in estimated band attenuation would not be reduced to less than one per cent.

6. CALCULATION PROCEDURE

The procedure for estimating the band attenuation rate using this Data Item, and programmed for computer solution, is as follows.

6.1 Determination of the Layer Thickness

The following method can be used for an atmosphere that is stratified into either vertical layers (horizontal propagation of sound) or horizontal layers (non-horizontal propagation of sound). In the case of horizontal propagation, linear distance should be used instead of altitude.

- (i) Make a provisional estimate of the number of atmospheric layers required and their boundary altitudes. If there are insufficient data to use the method given in Section 5 then for a provisional estimate consider a layer boundary to be at each altitude at which atmospheric data were measured.
- (ii) For each layer calculate the temperature and relative humidity at the middle of the layer assuming a linear variation across the layer.
- (iii) For horizontal propagation, a constant value of relative pressure is used for all layers.

For non-horizontal propagation, estimate the value of relative pressure for the conditions at every layer boundary and the middle of each layer using:

- (a) for layers in which there is a finite temperature difference across the layer

$$\delta_T = \delta_B \left(\frac{T_T}{T_B} \right)^{\left[\frac{g_0 (H_T - H_B)}{R (T_T - T_B)} \right]}, \quad (6.1)$$

- or (b) for layers in which the temperature is constant with altitude

$$\delta_T = \delta_B \exp \left[-\frac{g_0 (H_T - H_B)}{R T_B} \right]. \quad (6.2)$$

The derivation of Equations (6.1) and (6.2) can be found in ESDU 77022.

Values for g_0 and R are given in the following table.

	<i>SI</i>	<i>British</i>
g_0	9.806 65 m/s ²	32.174 0 ft/s ²
R	287.052 87 N m/kg K	3 089.811 4 ft lbf/slug K

- (iv) Using the highest frequency within the range of interest calculate the discrete frequency attenuation rate using ESDU 78002 for the atmospheric conditions at the boundaries and mid-point of each layer.
- (v) For each layer, the accuracy parameter, A , is evaluated using Equation (5.1) and compared with the permissible error. If the calculated value of A is greater than the permissible error the layer is subdivided and Steps (ii) to (v) are repeated.

6.2 Determination of the Spectrum Slope

In estimating the smoothed spectrum slope of the last two frequency bands of interest (*i.e.* for which band attenuations are required) the spectrum slopes of the following two higher frequency bands are needed (see Step xii). If spectral data for these two frequency bands are unavailable, the spectrum slopes of these additional bands are set equal to the slope of the highest band for which band attenuation is required. However if measured sound pressure levels are available for the frequency bands higher than the highest band for which attenuations are required, these can be used to calculate the actual spectrum slopes. As the last two frequency bands, used in the spectrum slope calculations, are outside the true frequency range of interest they are neglected in the subsequent estimation of band attenuation.

- (vi) Starting with the second frequency band and for each subsequent band calculate the spectrum slope using Equation (3.2) and the measured band levels.
- (vii) Starting with the third frequency band and for each subsequent band calculate the magnitude of the rate of change of slope from

$$\delta n = |n_i - n_{i-1}|.$$

(viii) For those bands having $\delta n > 5$ spectrum smoothing is applied according to the following criteria.

(a) For $n_i > 0$ and $n_i > n_{i-1}$

a smoothed value of sound pressure level is given by

$$S'_i = 0.5 [S_{i+1} + S_{i-1}]$$

and $S'_{i-1} = S_{i-1}$.

(b) For $n_i \leq 0$ and $n_{i-1} > 0$

a smoothed value of sound pressure level is given by

$$S'_{i-1} = 0.5 [S_i + S_{i-2}]$$

and $S'_i = S_i$.

(c) For $n_i > 0$ and $n_i < n_{i-1}$

and also $n_i < 0$ and $n_{i-1} < 0$

no smoothing is required so that

$$S'_{i-1} = S_{i-1}$$

and $S'_i = S_i$.

(d) When $\delta n_i > 5$ in the last band (highest frequency), for that band the smoothed value is given by

$$S'_i = S_{i-1} + (1 + K)n_{i-1}.$$

(ix) For those bands having $\delta n_i \leq 5$ no spectrum smoothing is required and $S'_i = S_i$.

(x) Starting with the third frequency band and for each subsequent band calculate a new set of spectrum slopes, n' , using Equation (3.2) and the smoothed values of sound pressure level, S' .

(xi) For the first two bands set the value of n'_i equal to the slope of the second band, n_2 .

(xii) To enable an average value of spectrum slope, n''_i , to be calculated, two hypothetical bands beyond the last band are created. The slopes of the hypothetical (last +1) and (last +2) bands are set equal to the smoothed slope of the last band (n'_i last).

(xiii) For each band calculate the arithmetic average of the three adjacent slopes from

$$n''_i = \frac{(n'_i + n'_{i+1} + n'_{i+2})}{3}.$$