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The compounding method of estimating stress intensity factors for cracks in complex configurations using solutions from simple configurations

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The Item was accepted for inclusion in the Stress and Strength Series by the Stress Analysis and Strength of Components Committee which first met in 1964 and now has the following membership:

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(Continued on inside back cover)

THE COMPOUNDING METHOD OF ESTIMATING STRESS INTENSITY FACTORS FOR CRACKS IN COMPLEX CONFIGURATIONS USING SOLUTIONS FROM SIMPLE CONFIGURATIONS

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THE COMPOUNDING METHOD OF ESTIMATING STRESS INTENSITY FACTORS FOR CRACKS IN COMPLEX CONFIGURATIONS USING SOLUTIONS FROM SIMPLE CONFIGURATIONS

1. INTRODUCTION

This Data Item presents a method of estimating values of stress intensity factors for cracks that have developed in complex configurations and are close to several boundaries and/or other cracks. The method, which is simple and easy to apply, entails compounding known stress intensity factor solutions for cracks in simple configurations. Many of these solutions may be found in the References in Section 4.

ESUD 80036 "Introduction to the use of linear elastic fracture mechanics in estimating fatigue crack growth rates and residual strength of components" describes how the crack behaviour of materials under both fatigue and static loading can be predicted using the concept of the stress intensity factor which characterises the highly stressed region around the tip of a crack. The resultant stress intensity factor obtained from the compounding method is only valid within the limitations of linear elastic fracture mechanics and therefore the geometric factors influencing the stress intensity factor will be in error if the net stresses reach a significant proportion of yield (say 70 per cent). Limited corrections for plastic zone size may help the accuracy of the resultant compounded stress intensity factor, obtained as suggested in ESDU 80036, provided the plastic zone size correction is small compared with the size of the ligament between the crack and the adjacent boundary.

2. NOTATION

A	cross-sectional area of stiffener	m^2	in^2
a, a_n	crack half length	m	in
a', a'_n	equivalent crack half length	m	in
b'	distance from centre of equivalent crack of half length a'_n to tip of equivalent crack of half length a , (see Sketch 3.12)	m	in
b_n	distance from centre of crack to n th boundary	m	in
b'_n	distance from centre of equivalent crack to n th boundary	m	in
c	distance from crack to centre of hole (see Sketch 3.3)	m	in
E_1, E_2	moduli of elasticity of sheet and stiffener materials, respectively	N/m^2	lbf/in^2
h	distance between centres of fasteners in stiffeners (see Sketch 3.6a)	m	in
K	stress intensity factor	$\text{N/m}^{3/2}$	$\text{lbf/in}^{3/2}$

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K_e	stress intensity factor correction due to interaction of boundaries	N/m ^{3/2}	lbf/in ^{3/2}
K_n	stress intensity factor for a crack in presence of n th boundary	N/m ^{3/2}	lbf/in ^{3/2}
K'_n	stress intensity factor due to effect of n th boundary acting on equivalent crack	N/m ^{3/2}	lbf/in ^{3/2}
K_0	stress intensity factor for a crack in absence of all boundaries	N/m ^{3/2}	lbf/in ^{3/2}
K'_0, K''_0	stress intensity factors for equivalent crack in absence of all boundaries	N/m ^{3/2}	lbf/in ^{3/2}
K_r	resultant compounded stress intensity factor	N/m ^{3/2}	lbf/in ^{3/2}
l	half length of plate (see Sketch 3.3)	m	in
N	total number of boundaries		
R	radius of hole	m	in
S	stress based upon gross cross-sectional area of component	N/m ²	lbf/in ²
$\bar{s}$	stiffness parameter defined by $\bar{s} = E_2 A / E_1 b t$ (see Section 3.2.1)		
t	thickness of sheet	m	in
W	width of plate (see Sketch 3.3)	m	in
α	geometrical factor in stress intensity factor		
λ	extensional stiffness parameter defined by $\lambda = 2atE_1/AE_2$ (see Section 3.2.3)		
λ'_n	extensional stiffness parameter due to effect of n th boundary acting on equivalent crack		
ρ	radius of curvature of boundary	m	in
<i>Suffixes</i>			
n	denotes n th boundary		

Both SI and British units are shown but any coherent system of units may be used.

3. METHOD OF COMPOUNDING

In this Item edges, surfaces, stiffeners and holes are termed boundaries and in general a structure or component will have several such boundaries. In addition cracks, other than the crack under consideration, are also termed boundaries. The stress distribution and hence the value of the stress intensity factor at the tip of a crack in a structure or component will be influenced by the boundaries. The compounding method may be used to estimate the value of the stress intensity factor for a crack in a structure containing several boundaries, by separating a complex configuration into a number of simpler ancillary configurations each containing the crack and a single boundary and having known solutions. These known solutions are then compounded to obtain the desired solution. The known solutions are published in the literature, for example References 10 to 14, and many of these solutions define the stress intensity factor*, K , as

$$K = \alpha S \sqrt{\pi a}. \quad (3.1)$$

There are two general compounding equations that are dependent on whether or not the crack crosses a boundary and these are presented in Sections 3.1 and 3.2. Both of these equations are expressed as sums of the simple cases.

3.1 A Crack that Does Not Cross a Boundary

The general compounding equation for this condition consists of the stress intensity factor in the absence of all boundaries, K_0 , the stress intensity factors for the ancillary configurations, K_n , and also a term, K_e , to account for the effect of any interaction between the boundaries. Thus, the resultant compounded stress intensity factor is given by

$$K_r = K_0 + \left[\sum_{n=1}^{n=N} (K_n - K_0) \right] + K_e, \quad (3.2)$$

where* $\frac{K_r}{K_0} = S \sqrt{\pi a}. \quad (3.3)$

Because the solutions for the ancillary configurations are often presented in terms of K_n/K_0 , see for example Reference 14, Equation (3.2) is rewritten as

$$\frac{K_r}{K_0} = 1 + \left[\sum_{n=1}^{n=N} \left(\frac{K_n}{K_0} - 1 \right) \right] + \frac{K_e}{K_0}. \quad (3.4)$$

If the term K_e/K_0 can be estimated or shown to be negligible, Equation (3.4) can be used to build up solutions to complex configurations from known simpler ones. In general, if the boundaries are not too close to each other relative to the size of the crack, the boundary interaction term K_e/K_0 can be neglected. Often in components containing cracks at circular holes and where the ratio of the hole diameter to the width of sheet is large, the boundary interaction term cannot be ignored and a method of obtaining a solution for K_e/K_0 given in Derivation 4 (see also Section 3.3). For stiffened sheet panels the boundary interaction term has been shown to be very much less important than the effect of the crack crossing a stiffener or

* Equations (3.1) and (3.3) may not necessarily be expressed in this form. For example, where a load, P , is applied at a point in the close vicinity of a crack it is usually found convenient to express K_0 as $K_0 = P/(\pi a)^{1/2}$ (where P is the force per unit thickness).

approaching a stiffener (see Section 3.2). It has generally been found that good approximations to the resultant stress intensity factors for complex configurations may be obtained (ignoring the boundary interaction term) from the simplified equation

$$K_r = K_0 \left[1 + \sum_{n=1}^{n=N} \left(\frac{K_n}{K_0} - 1 \right) \right], \quad (3.5)$$

which is the stress intensity factor in the absence of all boundaries multiplied by a correction term. Equation (3.5) may also be expressed as,

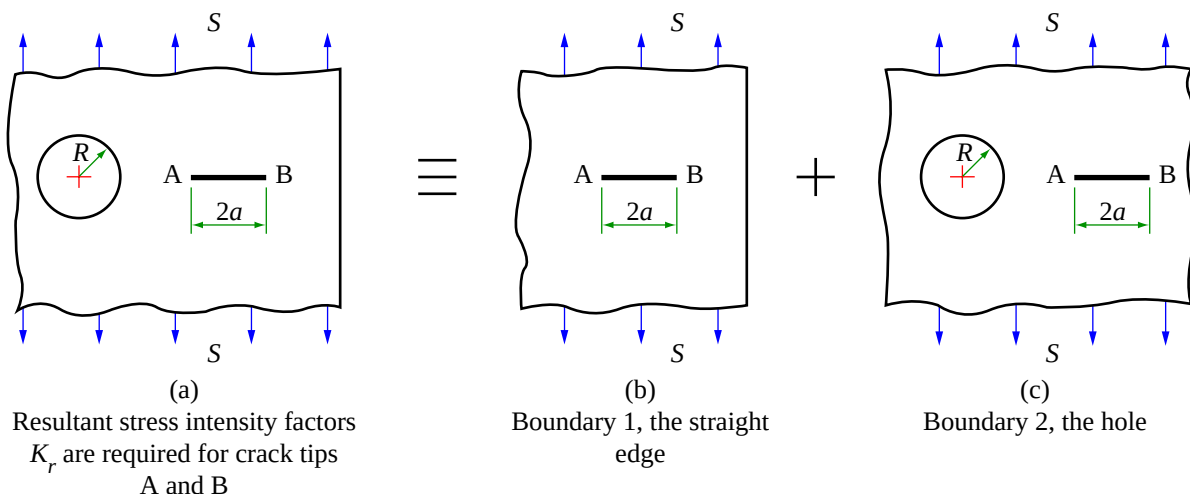
$$K_r = \left[\sum_{n=1}^{n=N} K_n \right] - K_0(N-1) \quad (3.6)$$

3.1.1 A simple example

A step-by-step illustration of how the compounding equation for determining values of K_r can be obtained is shown in Sketches 3.1. The values of K_r are required for crack tips A and B for the configuration shown in Sketch 3.1a. The crack configuration has two boundaries, 1 the straight edge and 2 the circular hole. The ancillary crack configurations for which the stress intensity factor solutions are known (see Figures 15, 109 and 110 of Reference 14) are shown in Sketches 3.1b and 3.1c.

The resultant stress intensity factors K_r for the crack tips A and B are calculated separately, using Equation (3.6) with $N = 2$ so that,

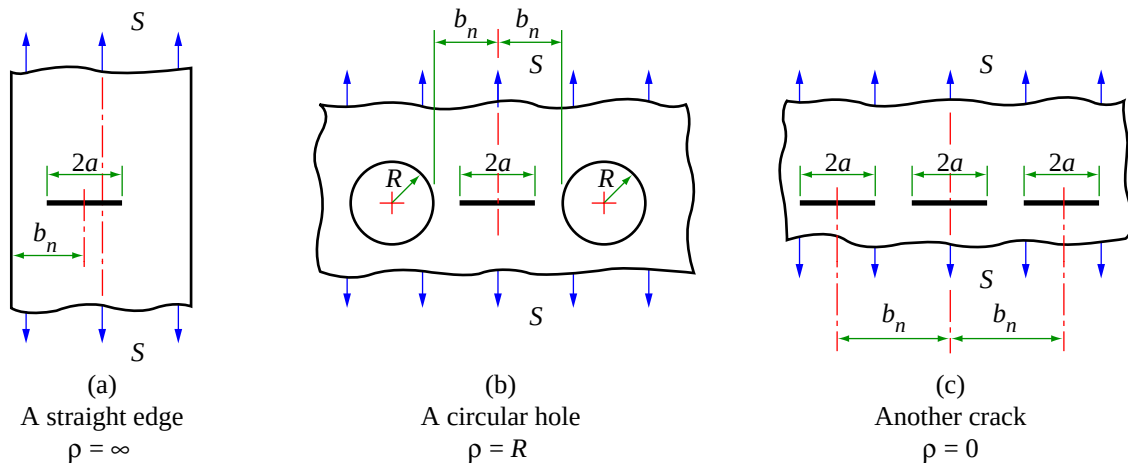
$$K_r = K_1 + K_2 - K_0. \quad (3.7)$$



Sketch 3.1

3.1.2 Errors when applying the method to plane sheets

Equation (3.5) has been used to compound solutions of stress intensity factors for three types of plane sheet specimens each containing a crack adjacent to boundaries. Each boundary is characterised by a different radius of curvature, ρ , as shown in Sketch 3.2.



Sketch 3.2

The evaluated results for these boundaries have been compared with known solutions obtained from Reference 14. The approximate percentage errors between the values of the stress intensity factors obtained using the compounding method and those obtained from known analytical solutions, for configurations similar to those illustrated in Sketch 3.2, are given in Table 3.1. For these cases the compounding method underestimates the resultant stress intensity factor.

TABLE 3.1 APPROXIMATE PERCENTAGE UNDERESTIMATION OF K_I

$\frac{a}{b_n}$	Straight boundaries $\rho = \infty$	Circular boundaries $\rho = R$	Crack boundaries $\rho = 0$
0.2	2	2	1
0.4	4	2	2
0.6	9	3	—
0.8	25	6	—

The results indicate the following trends.

- (a) At a fixed ratio of crack half length to boundary distance (a/b_n) errors tend to decrease with decreasing radius of curvature of the boundary.
- (b) Errors increase as the ratio of crack half length to boundary distance (a/b_n) increases.
- (c) Straight boundaries have the largest errors and they are significant at relatively large crack lengths (for example about 14 per cent at $a/b_n = 0.7$).

A worked example of a case where a crack does not cross a boundary follows.

3.1.3 Worked example

A steel bulkhead made from BS 4360 Grade 50B material, of yield strength 370 MN/m^2 , is made up of two 13 mm thick plates welded together by means of a butt weld transverse to the direction of loading. The weld has been dressed and stress relieved so that the weld is considered to be as strong as the plate material.