



ESDU 78043

Issued December 1978

With Amendment A

June 1979

Re-issued February 2013

Single disc springs – elastic deflections, stresses, stiffnesses and resiliences

ESDU Copyright material. For current status contact ESDU.

Institution of
**MECHANICAL
ENGINEERS**

Endorsed by
The Institution of Mechanical Engineers

AVA Documents
Document Sharing Hub

ESDU DATA ITEMS

Data Items provide validated information in engineering design and analysis for use by, or under the supervision of, professionally qualified engineers. The data are founded on an evaluation of all the relevant information, both published and unpublished, and are invariably supported by original work of ESDU staff engineers or consultants. The whole process is subject to independent review for which crucial support is provided by industrial companies, government research laboratories, universities and others from around the world through the participation of some of their leading experts on ESDU Technical Committees. This process ensures that the results of much valuable work (theoretical, experimental and operational), which may not be widely available or in a readily usable form, can be communicated concisely and accurately to the engineering community.

We are constantly striving to develop new work and review data already issued. Any comments arising out of your use of our data, or any suggestions for new topics or information that might lead to improvements, will help us to provide a better service.

THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item was monitored and guided by the Stress Analysis and Strength of Components Committee. This Committee first met in 1964 and now has the following membership:

Chairman	
Mr A.J. Batchelor	— Imperial Chemical Industries Ltd
Vice-Chairman	
Mr J.V.Vint	— C.A. Parsons and Company Ltd
Members	
Mr H.L. Cox	— Independent
Dr M.S.G. Cullimore	— University of Bristol
Mr C.E. Day	— Independent
Prof. J.P. Duncan*	— University of British Columbia
Mr D. Duval	— Foster Wheeler Power Products Ltd
Dr R.B. Heywood	— A. Macklow-Smith Ltd
Dr L.C. Laming	— Imperial College of Science and Technology
Dr R.M.G. Meek	— National Engineering Laboratory
Dr. J. Spence	— University of Strathclyde
Prof. C.E. Turner	— Imperial College of Science and Technology

* Corresponding Member

The construction and subsequent development of the Data Item was undertaken by

Mr R.P. Sharp — Senior Engineer.

The person with overall responsibility for the work in this subject area is Mr M.E. Grayley, Head of Strength Analysis Group.

SINGLE DISC SPRINGS – ELASTIC DEFLECTIONS, STRESSES, STIFFNESSES AND RESILIENCES

CONTENTS

	Page
1. NOTATION	1
2. NOTES	3
3. LOAD/DEFLECTION BEHAVIOUR	3
4. STRESS/DEFLECTION BEHAVIOUR	5
5. STIFFNESS	7
6. RESILIENCE	7
7. CORRECTION FOR LOADS ACTING NEAR AND BETWEEN THE PERIMETERS OF THE SPRING	8
8. DERIVATION AND REFERENCES	9
8.1 Derivation	9
8.2 References	9
9. EXAMPLES	9
9.1 Example 1	9
9.2 Example 2	11
FIGURES 1 to 13	12 to 25

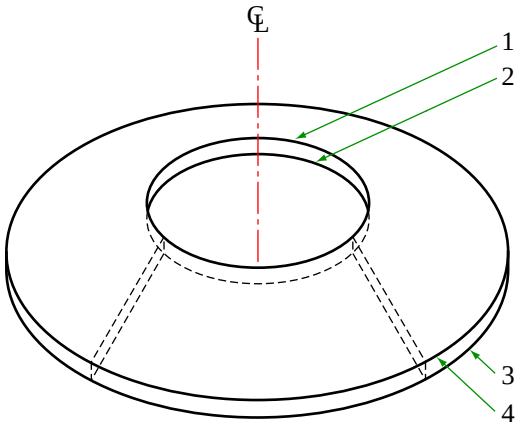
SINGLE DISC SPRINGS – ELASTIC DEFLECTIONS, STRESSES, STIFFNESSES AND RESILIENCES

1. NOTATION

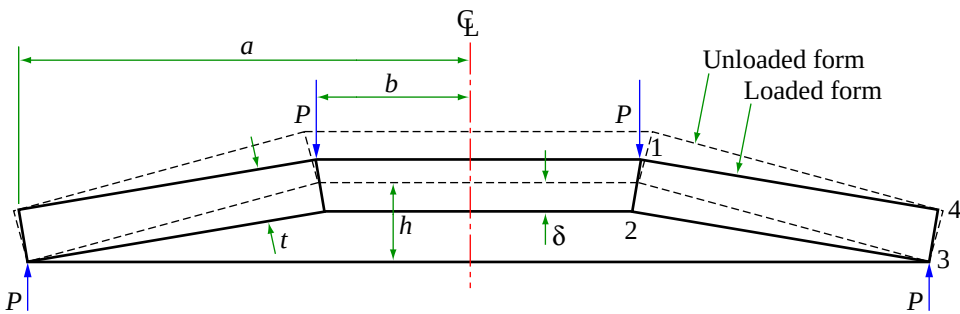
a	external radius of spring (see Sketch 1.2a)	m	in
b	internal radius of spring (see Sketch 1.2a)	m	in
$C_{1,2,3,4,5}$	geometric factors (see equations on Figures 1, 2, 8, 9 and 11, respectively)		
E	modulus of elasticity of spring material	N/m ²	lbf/in ²
$f_{1,2,3,4}$	stresses at perimeters 1 to 4, respectively, of coned disc spring (see Sketch 1.1)	N/m ²	lbf/in ²
h	internal unloaded height of spring (see Sketch 1.2a)	m	in
P	total load applied uniformly to perimeters 1 and 3 of coned disc spring (see Sketch 1.2a)	N	lbf
P'	total load applied uniformly near and between perimeters of spring (see Sketch 7.1)	N	lbf
P_f	total load applied uniformly to perimeters 1 and 3 of a flat disc spring (see Sketch 1.2b)	N	lbf
P_0	value of P at which $\delta = h$	N	lbf
R, R_f	resiliences of coned and flat disc springs, respectively	N/m ²	lbf/in ²
S, S_f	stiffnesses of coned and flat disc springs, respectively	N/m	lbf/in
t	thickness of disc (see Sketch 1.2a)	m	in
x	distance between loading points if load applied near and between perimeters of spring (see Sketch 7.1)	m	in
α	ratio of external to internal radii of spring = a/b		
δ	deflection of spring under load P (see Sketch 1.2a)	m	in
δ'	deflection between loading points if load applied near and between perimeters of spring (see Sketch 7.1)	m	in
ν	Poisson's ratio of spring material		

Tensile stresses are considered positive. Both SI and British units are quoted but any coherent system of units may be used.

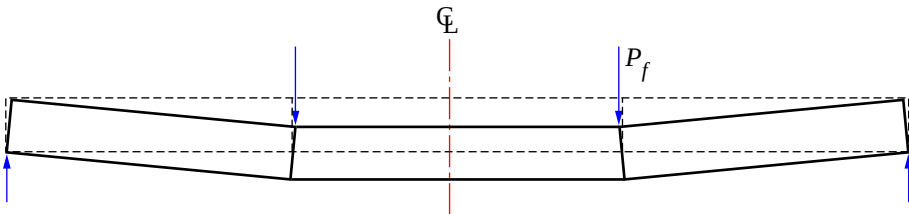
Issued December 1978
With Amendment A, June 1979
Re-issued February 2013



Sketch 1.1 Initially coned disc spring showing perimeter reference numbers



Sketch 1.2a Section through initially coned disc spring showing basic notation



Sketch 1.2b Section through initially flat disc spring (notation as in Sketch 1.2a)

2. NOTES

This Item presents, in both graphical and algebraic form, the elastic deflections, stresses, stiffnesses and resiliences of initially coned disc springs (Belleville washers) and initially flat disc springs. The Item deals with springs of rectangular section mounted either on shoulders or on flat plates and loaded uniformly around the upper and lower edges of the section or around a radius inside the edges of the section. ESDU 79010* presents a method of calculating the load/deflection behaviour of stacks of disc springs and also gives guidance on calculating the stiffnesses of spring stacks.

The advantage of the coned disc spring is that, in addition to axial compactness in the direction of loading, the geometry of the spring can be chosen to give a wide variety of load/deflection characteristics. Either positive, zero or negative stiffness can be achieved by simply varying the unloaded cone height, spring thickness and the working range of the deflection, see Sketch 3.1. At low height/thickness ratios the stiffness is always positive and approximately linear so that coned disc springs can be used as compact simple springs. At higher values of the ratio h/t the spring rate becomes non-linear and may change from positive to negative as the load rises so that, if the spring is mounted on shoulders, it can be made to snap through the flat position, where the deflection is equal to the unloaded height, to a stable position where the deflection is greater than the unloaded height. On removal of the load the spring will snap back through; hence the spring can be used in a safety valve mechanism. Alternatively, springs with large height/thickness ratios can be made such that once a certain load has been applied the spring will snap through and will remain in that position upon removal of load. Such springs can be used in one-way release valves.

The theory upon which the data in the Item are based is given in Derivation 1. The theory assumes that radial stresses are negligible and that a radial section of the disc does not distort but rather rotates about a neutral point. This method has been found to agree with test data for values of α , see Sketch 1.2a, of less than 5.0 and for discs having cone angles less than 15 degrees.

The load/deflection behaviour is discussed in Section 3 and given in Figures 3, 4 and 5, the stress characteristics in Section 4 and Figures 6 and 7, the stiffness in Section 5 and Figures 8, 9 and 10 and the resilience in Section 6 and Figures 11, 12 and 13. Constants dependent on the outside to inside radius ratio, α , are given in Figures 1 and 2. For cases where loads are applied inside the edges of the spring the relevant correction factor can be found in Section 7. Examples on the use of the data are provided in Section 9.

3. LOAD/DEFLECTION BEHAVIOUR

Under an applied load, distributed around perimeter 1 and seated uniformly at perimeter 3, see Sketch 1.1, the load/deflection curves will be as illustrated in Sketch 3.1. As the load is increased the spring will deflect to the flat position, $\delta = h$, and, if the spring is mounted on shoulders, through the flat position to deflections greater than $\delta = h$ and at higher load to deflections greater than $\delta = 2h$. The characteristics of the load/deflection behaviour, that is the stiffness, are discussed in Section 5. The major factor controlling the load/deflection behaviour is the ratio of unloaded internal cone height to spring thickness, h/t , and if the value of this parameter is altered the load/deflection behaviour changes as shown in Sketch 3.1.

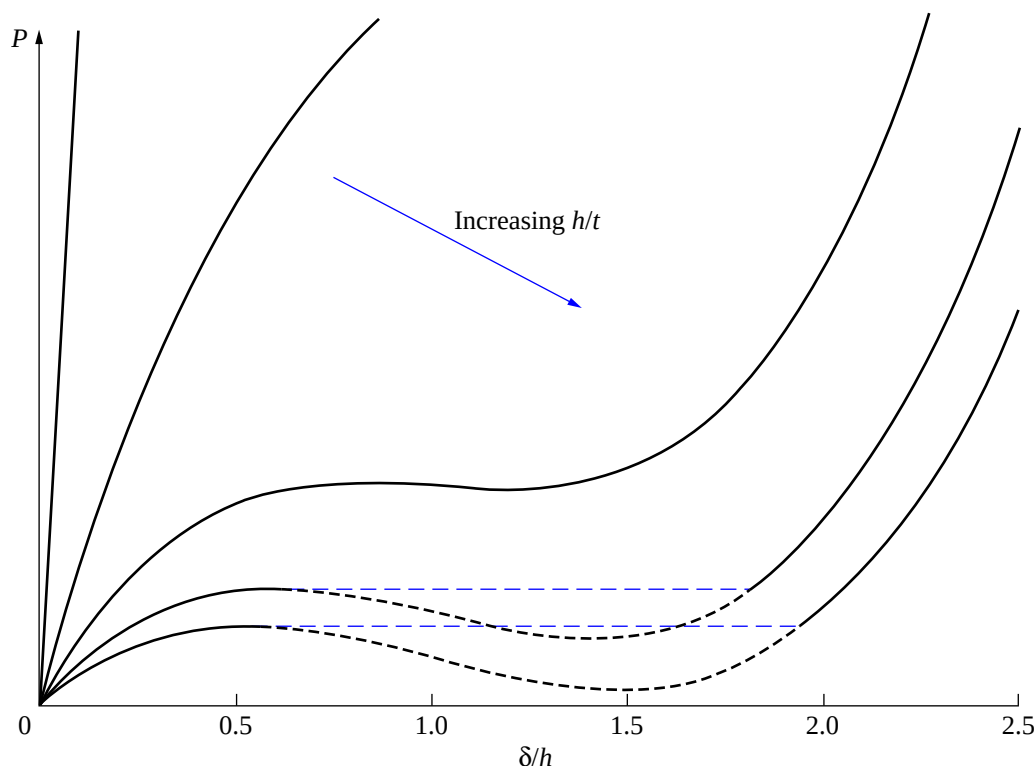
The load/deflection relationship for any geometry is given in Equation (3.1)

$$P = \frac{\delta E}{(1 - \nu^2)a^2 C_1} \left[t(h - \delta) \left(h - \frac{\delta}{2} \right) + t^3 \right], \quad (3.1)$$

* ESDU 79010 "Characteristics of disc springs in combination (to be used in conjunction with ESDU 78043)".

where C_1 is a parameter dependent upon the ratio of the radii of perimeters 1 and 3, see Sketch 1.1, and is given by Equation (3.2) and Figure 1.

$$C_1 = \frac{1}{\frac{\pi}{6} \left(\frac{\alpha}{\alpha - 1} \right)^2 \log_e \alpha} \quad (3.2)$$



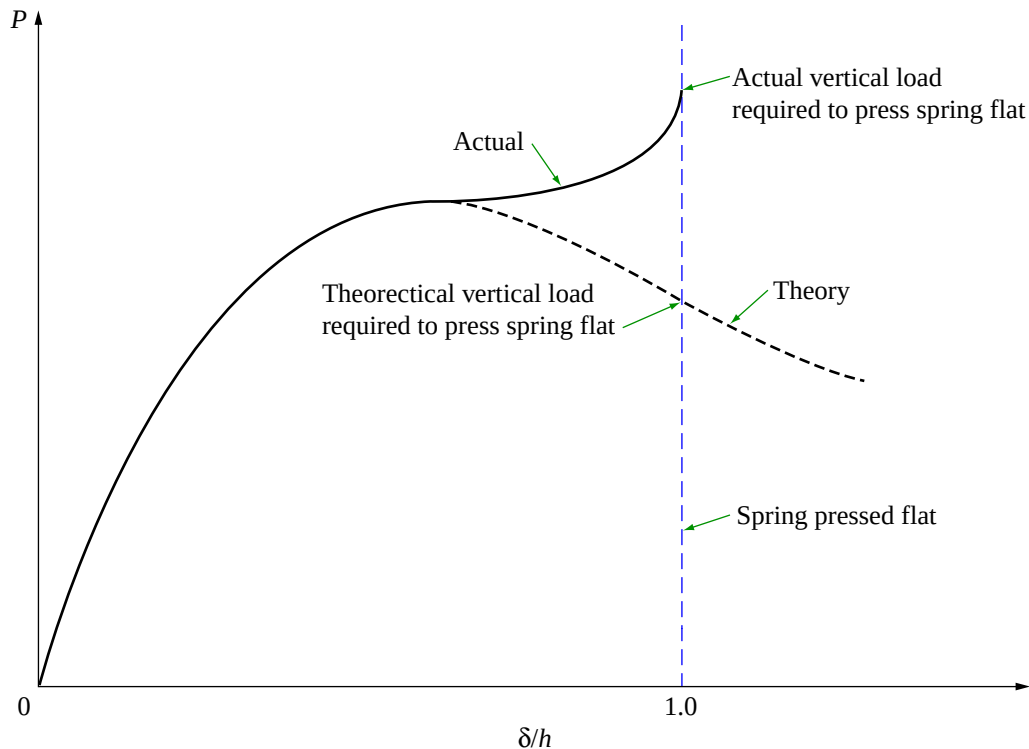
Sketch 3.1 Load/deflection curves for different h/t values for a typical coned disc spring

Load/deflection behaviour is shown in non-dimensional form in Figure 3 in which δ/h is plotted against the applied load divided by P_0 , the load required to press the spring into the flat position ($\delta = h$), for a range of values of h/t . The load P_0 can be calculated using Equation (3.3):

$$P_0 = \frac{Eht^3}{(1 - \nu^2)a^2 C_1} \quad (3.3)$$

For low values of the ratio h/t (less than about 0.5) the load/deflection curve is approximately linear, a constant stiffness condition, see Sketch 3.1. At higher ratios of h/t the deflection per load increment increases until, at values of h/t of about $\sqrt{2}$, the spring exhibits a region of large deflection for small increases in load. This is sometimes termed a constant load condition. This is followed by a return to an approximately linear load/deflection curve. Above $h/t = \sqrt{2}$ the spring, if mounted between shoulders can, once a certain load has been reached, exhibit negative stiffness. This causes the spring to snap through the flat position into a stable position in the region where δ/h is greater than 1.0, see Sketch 3.1. Increase in load beyond this level will give further deflection. On reduction of load below a certain value the spring will snap back through the flat position or alternatively, if $h/t > 2\sqrt{2}$, once the spring has 'snapped through' the spring will remain in this position. Application of load in the opposite direction can bring the spring back to its original position.

When the spring is mounted on a flat surface rather than between shoulders, the load/deflection behaviour alters as the spring approaches the flat position, see Sketch 3.2; a higher load than would be given by using the data in this Item is then required due to metal to metal contact arising from meridional strain caused by Poisson's ratio effects. This region can be avoided if desired by using Figure 4 and ensuring that the chosen spring lies in a region of positive spring rate and by keeping the deflection below a value of $\delta/h = 0.75$.



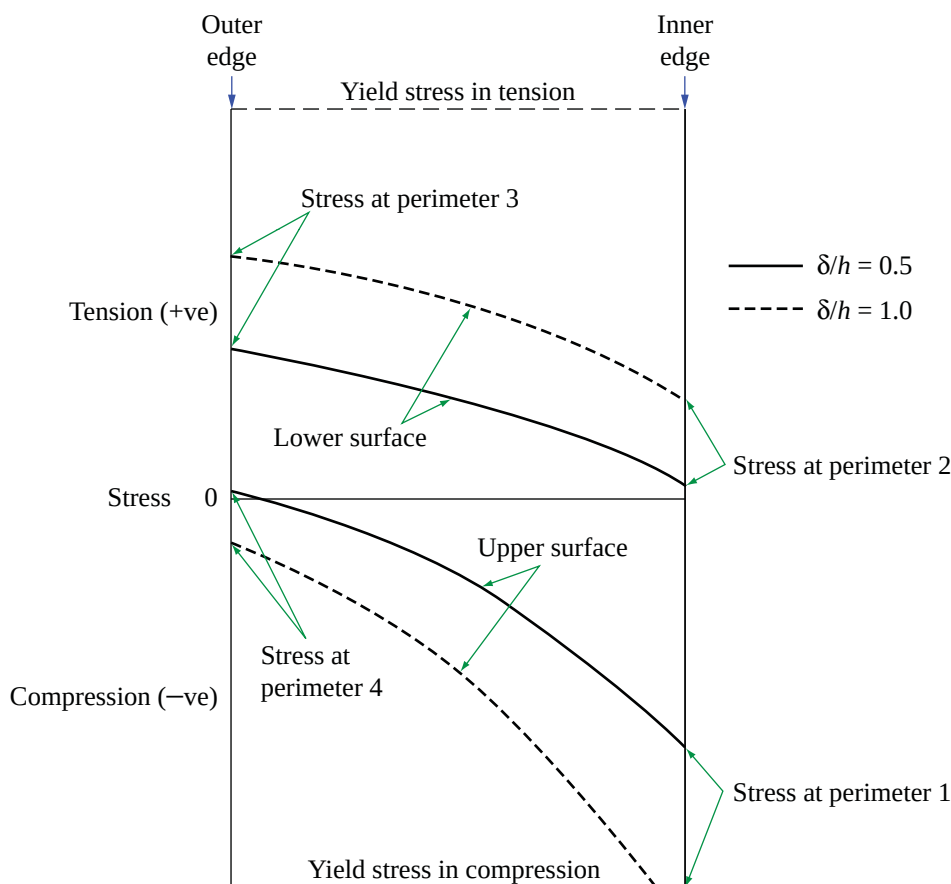
Sketch 3.2 Deviation from theory for coned disc spring pressed between flat surfaces

Since local yielding may significantly affect the load/deflection behaviour of the spring it may be necessary to check that stresses remain within the elastic range of the spring material, see Section 4. Local yielding at the perimeters of the spring may be acceptable in springs in which the volume of material that has yielded is small compared with the total volume of the spring.

The load/deflection behaviour of flat disc springs, see Sketch 1.2b, is given in Figure 5 or can be determined from Equation (3.1) by setting $h = 0$. Only positive stiffnesses are possible with this type of spring.

4. STRESS/DEFLECTION BEHAVIOUR

The maximum stress induced by the load for a wide range of geometries and deflections is a tangential compressive stress along perimeter 1. The magnitude of this stress is determined by the deflection and exhibits a maximum for a particular deflection, see Figures 6a to 6f. At high deflections, for example $\delta/h > 2.0$, the maximum absolute stress can occur at the lower inner edge of the section, that is at perimeter 2, see Sketch 1.1 and Figures 6a to 6f, and will be tensile. The distribution of stress across the section of a typical spring is illustrated in Sketch 4.1.



Sketch 4.1 Distribution of stress across a typical disc spring

If the stress at any point on the spring exceeds the yield stress of the spring material, either in tension or compression, plasticity will occur at the relevant perimeter of the spring. This will cause local plastic flow and deformation of the spring that will alter the unloaded cone height and load/deflection behaviour. The stress at any perimeter can be determined using the following equations.

$$f_1 = - \frac{\delta E}{(1 - \nu^2)a^2} \left(\frac{\alpha}{\alpha - 1} \right)^2 \left[\left(\frac{\alpha - 1}{\log_e \alpha} - 1 \right) \left(h - \frac{\delta}{2} \right) + \left(\frac{\alpha - 1}{2} \right) t \right], \quad (4.1)$$

$$f_2 = - \frac{\delta E}{(1 - \nu^2)a^2} \left(\frac{\alpha}{\alpha - 1} \right)^2 \left[\left(\frac{\alpha - 1}{\log_e \alpha} - 1 \right) \left(h - \frac{\delta}{2} \right) - \left(\frac{\alpha - 1}{2} \right) t \right], \quad (4.2)$$

$$f_3 = - \frac{\delta E}{(1 - \nu^2)a^2 \alpha} \left(\frac{\alpha}{\alpha - 1} \right)^2 \left[\left(\frac{\alpha - 1}{\log_e \alpha} - \alpha \right) \left(h - \frac{\delta}{2} \right) - \left(\frac{\alpha - 1}{2} \right) t \right] \quad (4.3)$$

and

$$f_4 = - \frac{\delta E}{(1 - \nu^2)a^2 \alpha} \left(\frac{\alpha}{\alpha - 1} \right)^2 \left[\left(\frac{\alpha - 1}{\log_e \alpha} - \alpha \right) \left(h - \frac{\delta}{2} \right) + \left(\frac{\alpha - 1}{2} \right) t \right] \quad (4.4)$$

or, from Figures 6a to 6f for varying values of α , see Sketch 1.2a, and for a range of geometries and deflections.