



ESDU 79005

Issued May 1979
With Amendment A
August 2010

Undamped natural vibration of shear buildings

**Associated software: ESDUpac A7905
VIEWpac 7905A**

Endorsed by
The Institution of Mechanical Engineers

AVA Documents
Document Sharing Hub

ESDU DATA ITEMS

Data Items provide validated information in engineering design and analysis for use by, or under the supervision of, professionally qualified engineers. The data are founded on an evaluation of all the relevant information, both published and unpublished, and are invariably supported by original work of ESDU staff engineers or consultants. The whole process is subject to independent review for which crucial support is provided by industrial companies, government research laboratories, universities and others from around the world through the participation of some of their leading experts on ESDU Technical Committees. This process ensures that the results of much valuable work (theoretical, experimental and operational), which may not be widely available or in a readily usable form, can be communicated concisely and accurately to the engineering community.

We are constantly striving to develop new work and review data already issued. Any comments arising out of your use of our data, or any suggestions for new topics or information that might lead to improvements, will help us to provide a better service.

THE PREPARATION OF THIS DATA ITEM

The work on this particular Item was monitored and guided by the following Working Party:

Prof. A.D.S. Barr	— University of Dundee
Mr R.W. Horsington	— University of Birmingham
Mr A.P. Jeary	— Building Research Station, Garston
Dr A.Y.T. Leung	— University of Bristol
Mr A.E. Saxon	— Ove Arup and Partners
Prof. W.H. Wittrick	— University of Birmingham

which was appointed by the Vibration and Structural Dynamics Steering Group. The Vibration and Structural Dynamics Steering Group, which also approved the final draft, has the following constitution:

Chairman	
Mr D.C.G. Eaton	— European Space Research and Technology Centre, Domeinweg, The Netherlands
Members	
Mr T.V. Lawson	— University of Bristol
Dr M.R. Smith	— University of Newcastle-upon-Tyne
Mr H.H.B.M. Thomas	— Royal Aircraft Establishment, Farnborough
Prof. W.H. Wittrick	— University of Birmingham

The Item was accepted for inclusion in the Wind Engineering Sub-series by the Fluid Mechanics Steering Group which has the following constitution:

Chairman	
Mr T.V. Lawson	— University of Bristol
Members	
Mr K.C. Anthony	— Ove Arup and Partners
Dr D.J. Cockrell	— Leicester University

(Continues on inside back cover)

UNDAMPED NATURAL VIBRATION OF SHEAR BUILDINGS

CONTENTS

	Page
1. NOTATION AND UNITS	1
2. INTRODUCTION	2
3. DATA PROVIDED	3
4. METHOD OF SOLUTION AND SHEAR BUILDING IDEALISATION	4
4.1 Method of Solution	4
4.2 Idealised Building	4
4.2.1 Mass and stiffness characteristics of the idealised building	4
4.3 Building Idealisation	5
4.3.1 Uniform stiffness type building idealisation	5
4.3.2 Uniformly varying stiffness type building idealisation	5
5. RANGE OF GRAPHICAL DATA PROVIDED	6
5.1 Natural Frequencies	6
5.2 Mode Shapes	6
5.3 Normal-mode Generalised Mass	7
5.4 Determination of Normal-mode Generalised Stiffness	7
6. ANALYTICAL EXPRESSIONS FOR UNIFORM STIFFNESS TYPE BUILDINGS	7
6.1 Natural frequencies	7
6.2 Mode Shapes	8
6.3 Normal-mode Generalised Mass	8
7. DERIVATION AND REFERENCES	9
7.1 Derivation	9
7.2 References	9
8. EXAMPLE	10
8.1 Building Idealisation	10
8.2 Natural Frequencies	11
8.3 Mode Shapes	12
8.4 Normal-mode Generalised Masses	12
8.5 Normal-mode Generalised Stiffnesses	13
FIGURES 1 to 12	14 to 25

APPENDIX A COMPUTER PROGRAM FOR EVALUATION OF VIBRATIONAL PARAMETERS FOR UNCOUPLED MODES OF SHEAR BUILDINGS	27
A1. GENERAL	27
A2. DATA INPUT	27
A3. DATA OUTPUT	28
A3.1 Listing of the Input Data	28
A3.2 Organisation of the Data Output	28
A4. LISTING OF EXAMPLE OUTPUT	28
APPENDIX B COUPLED LATERAL AND TORSIONAL VIBRATIONS OF A CLASS OF SHEAR BUILDINGS	31
B1. ADDITIONAL NOTATION	31
B2. GENERAL	32
B3. BUILDING CLASSIFICATION	32
B4. METHOD OF CALCULATING VIBRATION PARAMETERS FOR COUPLED NORMAL MODES	33
B4.1 Natural Frequencies	33
B4.2 Mode Shapes	33
B4.3 Normal-mode Generalised Mass	35
B5. THE PARTICULAR CASE OF A UNIFORM SHEAR BUILDING	35
FIGURES B1 to B3	36

UNDAMPED NATURAL VIBRATION OF SHEAR BUILDINGS

1. NOTATION AND UNITS

Only the SI system of units is quoted but any coherent system of units may be used.

		SI UNIT	
		Lateral modes	Torsional modes
D	non-dimensional stepped-change in stiffness, $\Delta k/k_n$		
f_j	natural frequency for mode j	Hz	Hz
K_j	normal-mode generalised stiffness for mode j	N/m	N m/rad
k	constant storey stiffness of uniform stiffness type building	N/m	N m/rad
k_r	stiffness of r th storey of building	N/m	N m/rad
k_n	stiffness of n th storey of building	N/m	N m/rad
M_j	normal-mode generalised mass (or inertia) for mode j	kg	kg m ²
m	constant storey mass of idealised building	kg	kg m ²
m_r	lumped mass of r th storey	kg	kg m ²
m_n	lumped mass of n th storey	kg	kg m ²
n	number of storeys in building		
$q_{r,j}$	modal displacement of r th storey for mode j	m	rad
r	storey number		
Δk	constant stepped-change in stiffness per storey of an idealised building (defined to be positive when storey stiffness increases from top to bottom)	N/m	N m/rad
θ_j	generalised mass ratio for mode j , M_j/M_1		
$\mu_j(\xi_r)$	normalised displacement for mode j at r th floor, $q_{r,j}/q_{n,j}$		
v_j	natural frequency ratio for mode j , f_j/f_1		
ξ_r	the ratio r/n		
ρ_j	non-dimensional generalised mass for mode j , see Equation (5.7)		

Issued May 1979
With Amendment A, September 1998

ϕ_j frequency parameter for mode j , $f_j \sqrt{\frac{m}{k_n}}$

ψ_j parameter defined by Equation (6.3)

Subscripts

j relates to normal mode number

r relates to storey number (for ground level $r = 0$ and for top storey $r = n$).

Additional notation, required only for Appendix B, is given in the Appendix, Section B1.

2. INTRODUCTION

The construction techniques employed in modern high-rise buildings lead to structures of relatively high flexibility and low mass so that the effect of dynamic loads on the structure become increasingly important. A knowledge of the natural frequency parameters is necessary for the investigation of the response of a building to such loads. The natural frequency parameters, required for the calculation of the response of a building to fluctuating loads by the modal superposition method, are natural frequencies, mode shapes, and normal-mode generalised masses (or inertias) and stiffnesses. In considering lateral modes, generalised masses and lateral stiffnesses are used and for torsional modes, generalised rotational inertias and rotational stiffnesses are required. The way in which these data are used in the case of the calculation of the response of a building under the action of a fluctuating wind load is described in the Data Items in Sections 7, 8 and 9.

This Item provides a means of estimating the natural frequencies, mode shapes and normal-mode generalised masses* for both lateral and torsional undamped modes of vibration of shear buildings. A relationship is given from which the generalised stiffnesses for lateral and torsional modes might be obtained using other data provided. This Item is primarily intended for use with uncoupled modes of vibration although coupling between lateral and torsional modes is considered in Appendix B where expressions for the estimation of coupled mode parameters are given for a special class of shear building.

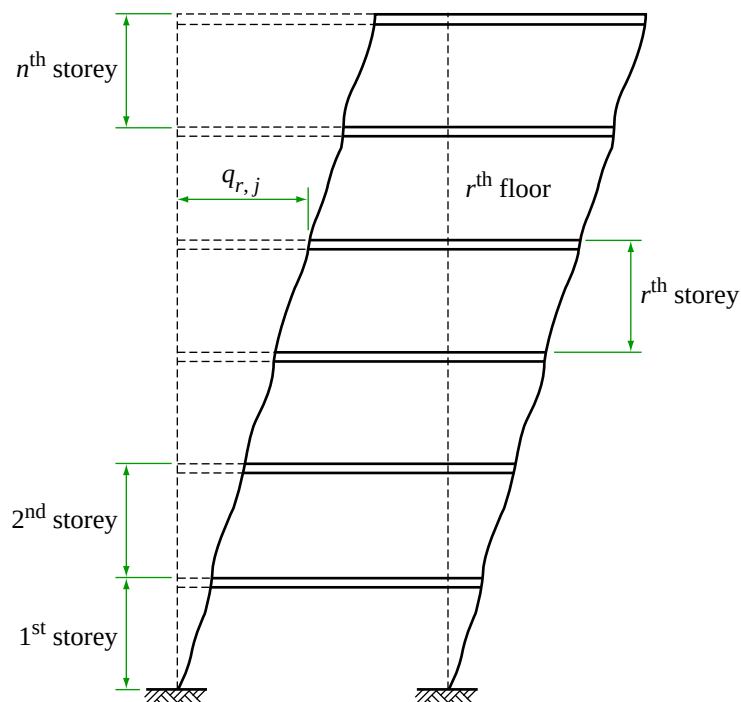
The data provided are restricted to use for those buildings that may be modelled as shear type buildings. A shear building is defined as a building that has inextensible columns and rigid floors which prevent joint rotations. In this Item it is assumed that the reductions in column stiffness due to the axial loads applied by the supported structure are negligible. This stiffness effect causes a reduction in frequency and may modify the mode shapes but is generally small. Further, it is assumed that the buildings have a rigid base. Thus, by implication, the displacement at ground level is assumed to be zero and it is assumed that no soil/structure interaction takes place.

The mathematical model of the shear building, illustrated in Sketch 3.1, is further idealised (as described in Section 4) to provide a graphical solution for the natural mode of vibration parameters. A computer program is provided in the **Wind Engineering Series via the website** that allows the solution of the shear building model without the necessary further idealisation imposed by the graphical solution. A number of modes of vibration may significantly contribute to the dynamic loads on a building although it is generally the lowest frequency mode which has the greatest influence on the dynamic response. Graphical data are provided for the first four natural modes of vibration for both lateral and torsional oscillations.

* The terms mass, stiffness and displacement are used in the generic sense to include rotational inertia, rotational stiffness and rotational displacement, respectively, for the torsional vibration case.

3. DATA PROVIDED

The natural frequencies, mode shapes and normal-mode generalised masses (or inertias) of the first four uncoupled lateral or torsional modes of two types of idealised shear buildings with 4 to 60 storeys are provided in a graphical form. The terms mass, stiffness and displacement are used in the generic sense to indicate mass, lateral stiffness and lateral displacement, respectively, in the calculation of lateral mode parameters, and rotational inertia, rotational stiffness and rotational displacement, respectively, in the calculation of torsional mode parameters. The method by which a real building is represented by one of two idealisations is presented in Section 4.3. Buildings that cannot be represented by either of the two idealised types should be analysed using the computer program described in Appendix A. A method for computing natural frequencies and mode shapes for coupled lateral and torsional vibrations of a class of shear buildings is provided in Appendix B.



Sketch 3.1 n -storey shear building model

4. METHOD OF SOLUTION AND SHEAR BUILDING IDEALISATION

4.1 Method of Solution

The lumped parameter method (References 4 and 8) has been used to calculate the vibrational properties of shear buildings. The method assumes that the building may be represented by lumped masses, or in the case of torsional modes lumped inertias, concentrated at floor levels and connected by massless elastic columns. The stiffness of a storey is defined as the lateral force, or in the case of torsional mode the torque, at the floor required to produce a unit displacement of the storey floor relative to the floor below (see Sketch 3.1). For a shear-frame building the effects of all the individual columns of a storey are summed to obtain the storey stiffness. It should be noted that the lumped mass at each floor level must include the whole of the fixed mass and a proportion of the movable mass associated with the floor loading. The fixed mass includes the mass of the floor and the mass of the columns and walls adjacent to it together with finishes, partitions and fixed equipment. The proportion of the movable mass (*e.g.* people) included in the lumped floor mass should be that defined in the code of practice to which the building is to apply or some appropriate proportion. Typically the proportion of the movable mass included in the lumped floor mass lies between 25% and 100% of the total movable mass. Guidance on the contribution of the movable mass to the lumped mass may be found in References 6, 7, 9 and 10.

The equations of motion of a lumped shear building with non-uniform distribution of either the lumped mass or the stiffness of the storeys have been solved numerically. However, for the uniform stiffness type building described in Section 4.2 an analytical solution of the equations has also been obtained and is presented in Section 6.

4.2 Idealised Building

The idealised shear building for which graphical data are provided has a constant lumped storey mass and either a constant stiffness for all the storeys (a uniform stiffness type building) or a storey-wise stiffness variation with a constant stepped-change in stiffness per storey (a uniformly varying stiffness type building). The procedure for this idealisation is given in Section 4.3.

4.2.1 Mass and stiffness characteristics of the idealised building

The idealised building can be a uniform stiffness type or a uniformly varying stiffness type as defined in Section 4.2.

For the uniform stiffness type building the lumped storey mass and the storey stiffness characteristics are described by Equations (4.1) and (4.2) to be

$$m_r = m \quad (r = 1, 2, 3, \dots, n), \quad (4.1)$$

$$k_r = k \quad (r = 1, 2, 3, \dots, n). \quad (4.2)$$

The uniformly varying stiffness type building has similar lumped mass characteristics but the storey stiffness is characterised by Equation (4.3) as follows,

$$k_r = k_n + (n - r)\Delta k. \quad (4.3)$$

The term Δk , the constant stepped-change in stiffness per storey, is defined to be positive when the storey

stiffness increases from top to bottom and is expressed in the non-dimensional form, D , as

$$D = \Delta k / k_n. \quad (4.4)$$

4.3 Building Idealisation

Since a real building rarely falls exactly into either of the ideal categories considered in this Item, to use the graphical data provided it is necessary to represent the building by one of the idealisations as discussed in Section 4.3.1 and 4.3.2. It must be emphasised that the accuracy of the data obtained will depend on how closely the idealisation represents the real shear building. Results obtained for buildings for which the stiffness or mass variation does not conform closely to the idealised cases should be treated with caution and it is recommended that in such cases the buildings should be analysed with the aid of the computer program described in Appendix A.

4.3.1 Uniform stiffness type building idealisation

Where a real building can be approximated by the uniform stiffness idealisation, the constant lumped mass and storey stiffness for the idealised building are

$$m = \frac{1}{n} \sum_{r=1}^n m_r \quad (4.5)$$

and
$$k = \frac{1}{n} \sum_{r=1}^n k_r \quad (4.6)$$

where the m_r are the lumped storey masses and the k_r are the storey stiffnesses of the real building.

4.3.2 Uniformly varying stiffness type building idealisation

Where a real building can be approximated by a uniformly varying stiffness idealisation the constant lumped mass, the constant stepped-change in stiffness per storey, and the non-dimensional stepped-change in stiffness for the idealised building are

$$m = \frac{1}{n} \sum_{r=1}^n m_r, \quad (4.7)$$

$$\Delta k = \frac{2}{n(n-1)} \sum_{r=1}^n (k_r - k_n), \quad (4.8)$$

and
$$D = \frac{\Delta k}{k_n} \quad (4.9)$$

where the m_r are the lumped storey masses and the k_r are the storey stiffnesses of the real building.

5. RANGE OF GRAPHICAL DATA PROVIDED

For the two idealised types of shear building graphical data are provided for natural frequencies, mode shapes (normalised with respect to the top storey displacements) and the normal-mode generalised masses for buildings from 4 to 60 storeys for $D = 0$ to 1; the value of $D = 0$ applies to uniform stiffness type buildings.

5.1 Natural Frequencies

For the fundamental frequency, the frequency parameter, ϕ_1 , is plotted against the number of storeys for a range of values of D in Figure 1. The value of the fundamental frequency is obtainable from the expression

$$f_1 = \phi_1 \sqrt{\frac{k_n}{m}} . \quad (5.1)$$

The frequencies of the higher modes are given by

$$f_j = v_j f_1 . \quad (5.2)$$

Values of v_2 , v_3 and v_4 are given in Figures 2, 3 and 4 respectively for a range of values of D and n .

5.2 Mode Shapes

The normalised mode shapes, $\mu_j(\xi_r)$, for the first four natural modes are presented in Figures 5 to 8 where, for mode j ,

$$\mu_j(\xi_r) = q_{r,j}/q_{n,j} \quad (5.3)$$

$$\text{and} \quad \xi_r = r/n . \quad (5.4)$$

The terms $q_{r,j}$ and $q_{n,j}$ are the modal displacements of the r th and n th storeys respectively.

The idealised building is assumed to have a rigid base. Hence,

$$\mu_j(\xi_0) = 0 \quad (5.5)$$

For each mode, the lumped parameter representation gives n discrete values of modal displacement for an n storey building. To cover the range of storeys considered in this Item, mode shapes are presented as continuous curves of normalised displacements plotted against r/n . These curves are calculated for a 40-storey building but provide a reasonable approximation to the mode shapes at other values of n . The errors in mode shape will increase as n is reduced and for the lower values of n (say $n = 12$) the computer program should be used if accurate mode shapes are required.

It is sometimes necessary, for response calculations, to normalize the mode shapes with respect to the maximum floor displacement. If this normalization is required the values of $\mu_j(\xi_r)$ obtained from Figures 5 to 8 should be divided by the maximum value of $\mu_j(\xi_r)$ for the mode under consideration.

5.3 Normal-mode Generalised Mass

For a lumped mass building, M_j the normal-mode generalised mass for mode j , is defined as

$$M_j = \sum_{r=1}^n m_r \mu_j^2(\xi_r) \quad (5.6)$$

or in terms of the non-dimensional generalised mass, ρ_j ,

$$M_j = \rho_j \sum_{r=1}^n m_r. \quad (5.7)$$

For the idealised building the values of ρ_1 are presented in Figure 9 for a range of values of D and n .

The value of the first-mode generalised mass of the idealised building is determined from Equation (5.7) and generalised mass values for the higher modes are obtained from

$$M_j = \theta_j M_1. \quad (5.8)$$

Values of θ_j are plotted against n for modes 2, 3 and 4 in Figures 10, 11 and 12, respectively, for a range of values of D .

If the mode shapes are reorganized relative to the maximum floor displacement, to obtain the compatible value of M_j the value obtained using the corresponding θ_j (obtained from Figures 10 to 12 for $n > 12$) must be divided by the square of the maximum value of $\mu_j(\xi_r)$ for the mode under consideration.

5.4 Determination of Normal-mode Generalised Stiffness

The normal-mode general stiffness K_j , for mode j of the idealised building, is determined from the corresponding normal-mode generalised mass value using Equation (5.9):

$$K_j = (2\pi f_j)^2 M_j. \quad (5.9)$$

6. ANALYTICAL EXPRESSIONS FOR UNIFORM STIFFNESS TYPE BUILDINGS

For a uniform stiffness type building the equations of motion have been solved using the theory of difference equations (References 5 and 8). The resultant expressions for the required vibrational parameters are given by Equations (6.1) to (6.8).

6.1 Natural frequencies

The natural frequency parameter, ϕ_j , and frequency ratio, v_j , for mode j are

$$\phi_j = \frac{\sin \psi_j}{\pi} \quad (6.1)$$

$$\text{and} \quad v_j = \frac{\sin \psi_j}{\sin \psi_1}, \quad (6.2)$$

$$\text{where} \quad \psi_j = \frac{\pi(2j-1)}{2(2n+1)}. \quad (6.3)$$

6.2 Mode Shapes

The modal displacement of the r th storey, and the normalised mode shape for mode j are

$$q_{r,j} = C \sin(2r\psi_j) \quad (6.4)$$

$$\text{and} \quad \mu_j(\xi_r) = \frac{\sin(2r\psi_j)}{\sin(2n\psi_j)}, \quad (6.5)$$

respectively, where C is a constant and ψ_j is defined by Equation (6.3).

6.3 Normal-mode Generalised Mass

The normal-mode generalised mass, M_j , the non-dimensional generalised mass, ρ_j , and the generalised mass ratio, θ_j , are

$$M_j = \frac{m(2n+1)}{4 \sin^2(2n\psi_j)}, \quad (6.6)$$

$$\rho_j = \frac{(2n+1)}{4n \sin^2(2n\psi_j)}, \quad (6.7)$$

$$\text{and} \quad \theta_j = \frac{\sin^2 2n\psi_1}{\sin^2(2n\psi_j)}. \quad (6.8)$$

7. DERIVATION AND REFERENCES

7.1 Derivation

The derivation lists selected sources that have assisted in the preparation of this item.

1. JOSEPH, M.G. Natural pattern of vibration of building frames. *J. Sound Vib.*, Vol. 46, RADHAKRISHNAN, R. No. 1, pp.15-32, May 1976.
2. WITTRICK, W.H. On the coupled torsional and sway vibrations of a class of shear buildings. To be published in *Earthquake Engineering and Structural Dynamics*.
HORSINGTON, R.W.
3. – Private communications with Mr R.W. Horsington and Professor W.H. Wittrick, Department of Civil Engineering, the University of Birmingham.

7.2 References

The references given are recommended sources of information supplementary to that in this Item.

4. HURTY, W.C. *Dynamics of structures*. Prentice-Hall Inc., 1964.
RUBINSTEIN, M.F.
5. CONTE, S.D. *Elementary numerical analysis*. McGraw-Hill Book Company, 1965.
6. MITCHELL, G.R. Floor loading in office building – the results of a survey. BRE CP 3/71, WOODGATE, R.W. Building Research Estab., Garston, England, 1971.
7. MITCHELL, G.R. Floor loading in retail premises – the results of a survey. BRE CP 25/71, WOODGATE, R.W. Building Research Estab., Garston, England, 1971.
8. McCALLION, H. *Vibration of linear mechanical systems*. Longman Group Limited, London, 1973.
9. – First order reliability concept for design codes – Appendix B – Basic notes on actions. CEB Bull. No. 112, Comité Européen de Béton, Munich, July 1976.
10. MITCHELL, G.R. Floor loading in domestic buildings – the results of a survey. BRE CP WOODGATE, R.W. 2/77, Building Research Estab., Garston, England, 1977.