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Inelastic shear stresses and strains in the adhesives bonding lap joints loaded in tension or shear

Associated software: ESDUpac A7916

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INELASTIC SHEAR STRESSES AND STRAINS IN THE ADHESIVES BONDING LAP JOINTS LOADED IN TENSION OR SHEAR

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INELASTIC SHEAR STRESSES AND STRAINS IN THE ADHESIVES BONDING LAP JOINTS LOADED IN TENSION OR SHEAR

1. INTRODUCTION, THEORY AND APPLICATION

1.1 INTRODUCTION

This Item, together with ESDUpac A7916, presents a computer program for the analysis of the distribution of the shear stresses and strains in the adhesives that bond lap joints of multi-step configuration loaded in either tension or shear*. The analysis can be applied to either single or double lap joints provided that the latter are symmetric. The program allows for the inelastic shear stress-strain properties of the adhesive and assumes that the behaviour of the adherend material is elastic. The program is applicable to joints made of orthotropic materials, such as composites, provided that the appropriate moduli are used. The method does not take into consideration the effect of bending in the joint and it ignores the local stress concentrations in the adherends.

The Data Item is provided in two main sections. Section 1 describes the theoretical solution and its application to practical bonded joints. The section includes figures that illustrate the influence of geometrical and material properties on the shear stresses and strains in the adhesive. Section 2 describes the use of the Fortran program, its input, output and messages. Example calculations are also included. The program is available in the form of an executable file accompanied by example input and output files.

1.2 NOTATION

Both SI and British units are quoted but any coherent system of units may be used.

A_1, A_2	constants in solution for shear stress (see Equation (1.8))	N/m ²	lbf/in ²
C_1, C_2	constants (see Equation (1.14))		
E_1, E_2	moduli of elasticity of materials of adherends 1 and 2 respectively	N/m ²	lbf/in ²
G_a	elastic shear modulus of adhesive material	N/m ²	lbf/in ²
G_1, G_2	elastic shear moduli of materials of adherends 1 and 2 respectively	N/m ²	lbf/in ²
L	length of joint overlap = $\sum_1^M l_i$ (see Sketch 1.1)	m	in
l_i	length of step i in joint (see Sketch 1.1)	m	in
M	number of steps in joint		
N	number of intervals per step for which output is required		
Q	shear load on joint per unit width, half load on double lap joint (see Sketch 1.1)	N/m	lbf/in

* This Data Item has been amended in order to incorporate a new Fortran program that produces more accurate results for long joints.

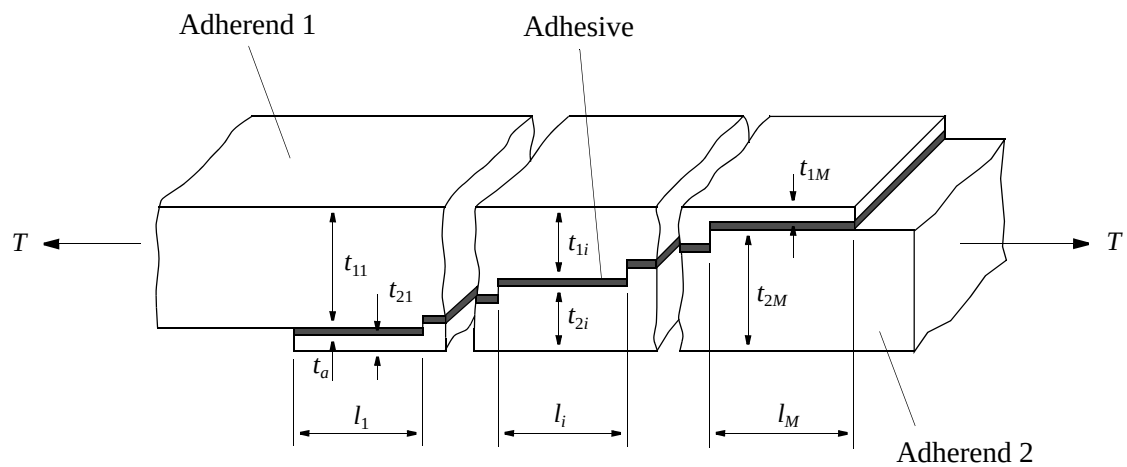
T	tensile load on joint per unit width, half load on double lap joint (see Sketch 1.1)	N/m	lbf/in
T_F	tensile load per unit width that causes failure in joint	N/m	lbf/in
T_1	tensile load per unit width on adherend 1	N/m	lbf/in
T_2	tensile load per unit width on adherend 2	N/m	lbf/in
t_a	thickness of adhesive layers, assumed to be equal for all steps (see Sketch 1.1)	m	in
t_{1i}	thickness of adherend 1 at step i (see Sketch 1.1)	m	in
t_{2i}	thickness of adherend 2 at step i (see Sketch 1.1)	m	in
u_1, v_1	displacements of adherend 1 in x and y directions respectively	m	in
u_2, v_2	displacements of adherend 2 in x and y directions respectively	m	in
x, y	in-plane co-ordinates (see Sketch 1.1)	m	in
α	parameter for adhesive stress-strain curve (see Equation (1.18) and Sketch 1.3)	N/m ²	lbf/in ²
β	parameter for adhesive stress-strain curve (see Equation (1.19) and Sketch 1.3)	N/m ²	lbf/in ²
ϕ	constant (see Equation (1.30))	N ² /m ⁴	lbf ² /in ⁴
γ	shear strain in adhesive		
γ_{all}	maximum allowable shear strain in adhesive		
γ_E	shear strain in adhesive at elastic limit		
γ_F	shear strain at which cohesive failure occurred in adhesive under test		
γ_{max}	maximum value of shear strain in adhesive for particular joint		
ϵ_1, ϵ_2	tensile strains in adherends 1 and 2 respectively		
λ	expression used in derivation (see Equation (1.7))	m ⁻¹	in ⁻¹
σ_1, σ_2	tensile stresses in adherends 1 and 2 respectively	N/m ²	lbf/in ²
τ	shear stress in adhesive material	N/m ²	lbf/in ²
τ_A	shear stress to which theoretical stress-strain curve of adhesive is asymptotic (see Sketch 1.3)	N/m ²	lbf/in ²
τ_E	shear stress in adhesive at elastic limit (see Sketch 1.3)	N/m ²	lbf/in ²

τ_F	shear stress at which cohesive failure occurred in adhesive under test	N/m ²	lbf/in ²
τ_{max}	maximum value of shear stress in adhesive for a particular joint	N/m ²	lbf/in ²
ψ	constant (see Equation (1.29))	N/m ²	lbf/in ²

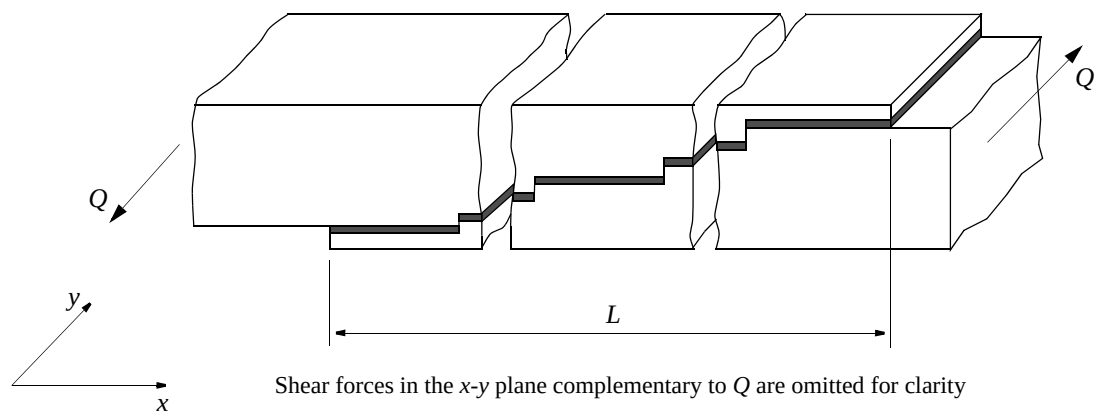
1.2.1 Suffixes

Suffix i indicates a joint step number ranging from 1 to M .

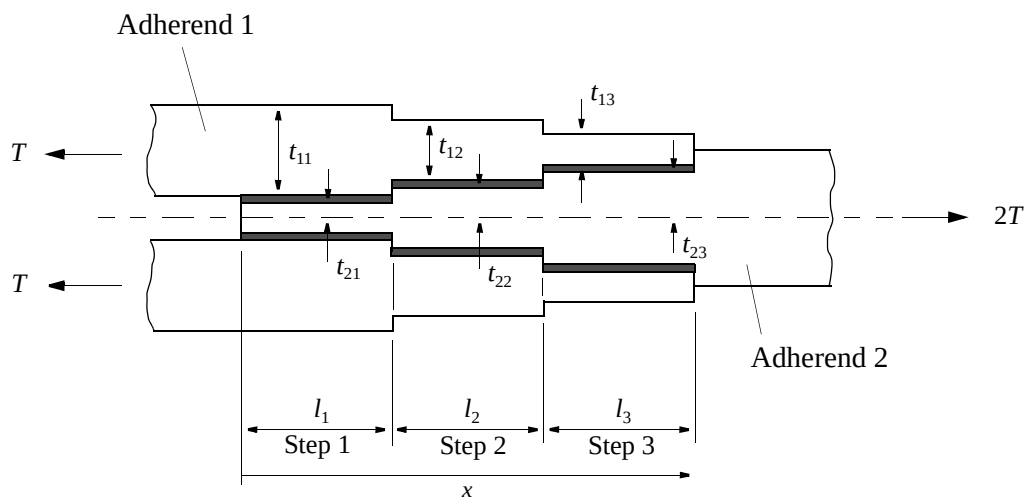
Suffixes L and R indicate values at the left- and right-hand ends respectively of a particular step.



(a) Joint of M steps loaded in tension



(b) Joint loaded in shear



(c) Three step ($M = 3$) double lap joint loaded in tension

Sketch 1.1 Multi-step joint geometry

1.3 ANALYSIS OF MULTI-STEP BONDED LAP JOINTS

The analysis is based on the method detailed in Derivation 1 of Section 1.5.1, a summary of which is given in Sections 1.3.1 and 1.3.2. The analysis determines the distribution of shear stress in the adhesive that results from the transmission of the in-plane tensile or shear loads from one adherend to the other via the adhesive. The analysis takes account of the inelastic shear stress-strain characteristics of the adhesive. It assumes that the adherends behave elastically and are flexurally rigid. It does not therefore consider the effect on the adhesive or the adherends of through-the-thickness (peel) stresses or of bending effects that may result either from the transmission of the load from one adherend to the other or from external bending moments. The effects of these factors are discussed in ESDU 80039* and ESDU 81022†.

The analysis is concerned with the internal cohesion of the adhesive. It does not consider the adhesion of the adhesive to the adherend which may vary with adherend material, surface condition, *et cetera*. For the purposes of the analysis the stresses in the adherends are assumed to be uniform through their thickness.

Owing to the fact that the analysis does not consider through-the-thickness (peel) and bending effects within the joint, it is most relevant to joints where these effects are at a minimum, for example balanced double lap joints with rigid adherends. In joints where the through-the-thickness and bending effects are significant, for example single lap joints, the analysis may underestimate the resultant stresses at a given load and therefore overestimate the load a joint can carry. In such cases it is suggested that this analysis is used in conjunction with an elastic analysis which, in the inelastic range of the adhesive, will overestimate the stresses and therefore underestimate the strength of the joint. Examples of elastic analyses can be found in References 2 and 5 and in ESDU 80039*.

The two following sections present a summary of the derivation of the formulae on which the analyses carried out by the Fortran program are based. Only the derivation of the basic formulae is presented here; for a fuller explanation of the theory and application of the analyses of multi-step bonded lap joints see Derivation 1.

Section 1.3.1 deals with the analysis of a joint in which the adhesive behaves elastically, that is τ_{max} is less than τ_E . Section 1.3.2 then summarises the derivation of the formulae used to analyse joints in which some or all of the adhesive is stressed above its elastic limit, that is τ_{max} is greater than τ_E .

The derivations in Sections 1.3.1 and 1.3.2 are based on a lap joint loaded in tension. As stated in Derivation 1 and Reference 5, the analysis of a joint loaded in shear is directly analogous to that for a joint loaded in tension, the quantities T , E_1 , E_2 , u_1 and u_2 being replaced by their respective counterparts Q , G_1 , G_2 , v_1 and v_2 .

* ESDU 80039 and ESDUpac A8039, "Elastic adhesive stresses in multistep lap joints loaded in tension".

† ESDU 81022, "Guide to the use of Data Items in the design of bonded joints".

1.3.1 Derivation of Basic Formulae for Joints where the Adhesive Behaves Elastically ($\tau_{max} < \tau_E$)

In a particular step in a joint the rates of change of the tensions in the adherends are equated to the shear stresses in the adhesive. The tension in adherend 1 decreases with increasing x and is given by

$$\frac{dT_1}{dx} = -\tau \quad (1.1a)$$

and similarly the tension in adherend 2 is given by

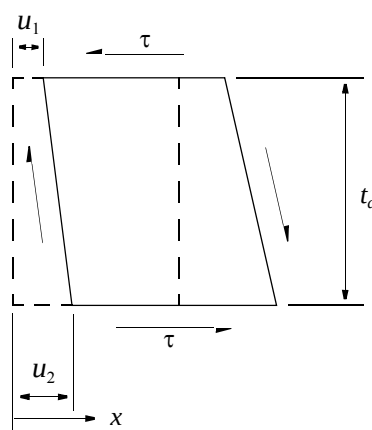
$$\frac{dT_2}{dx} = \tau. \quad (1.1b)$$

The shear strain in the adhesive can be stated in terms of the displacements as

$$\gamma = \frac{u_2 - u_1}{t_a}. \quad (1.2)$$

In an elastic analysis this gives the following shear stress in the adhesive

$$\tau = \frac{G_a}{t_a} (u_2 - u_1). \quad (1.3)$$



Sketch 1.2 Element of adhesive showing shear stresses and displacements

The direct strains in adherends 1 and 2 respectively are

$$\varepsilon_1 = \frac{du_1}{dx} = \frac{\sigma_1}{E_1} = \frac{T_1}{t_1 E_1} \quad (1.4a)$$

and

$$\varepsilon_2 = \frac{du_2}{dx} = \frac{\sigma_2}{E_2} = \frac{T_2}{t_2 E_2}. \quad (1.4b)$$

Differentiating τ twice with respect to x gives, from Equation (1.3),

$$\frac{d^2\tau}{dx^2} = \frac{G_a}{t_a} \left(\frac{1}{t_2 E_2} \frac{dT_2}{dx} - \frac{1}{t_1 E_1} \frac{dT_1}{dx} \right) = \frac{G_a}{t_a} \left(\frac{1}{t_2 E_2} + \frac{1}{t_1 E_1} \right) \tau \quad (1.5)$$

which can be written as

$$\frac{d^2\tau}{dx^2} = \lambda^2 \tau, \quad (1.6)$$

where

$$\lambda^2 = \frac{G_a}{t_a} \left(\frac{1}{t_2 E_2} + \frac{1}{t_1 E_1} \right). \quad (1.7)$$

The general solution to Equation (1.6) is

$$\tau = A_1 e^{\lambda x} + A_2 e^{-\lambda x}. \quad (1.8)$$

To avoid numerical problems in the program when dealing with long steps Equation (1.8) is re-expressed as follows

$$\tau = A_1 e^{-\lambda(x_R - x)} + A_2 e^{-\lambda(x - x_L)}, \quad (1.9)$$

where x_L is the value of x at the left-hand end of the step, x_R is the value of x at the right-hand end of the step and $x_L \leq x \leq x_R$.

Substituting Equation (1.9) into Equations (1.1a) and (1.1b) and integrating gives the loads in adherends 1 and 2 as

$$T_1 = \frac{1}{\lambda} \left(A_1 e^{-\lambda(x_R - x)} - A_2 e^{-\lambda(x - x_L)} \right) + t_1 E_1 C_1 \quad (1.10a)$$

and

$$T_2 = \frac{1}{\lambda} \left(-A_1 e^{-\lambda(x_R - x)} + A_2 e^{-\lambda(x - x_L)} \right) + t_2 E_2 C_2, \quad (1.10b)$$

where C_1 and C_2 are the constants of integration. To determine their values, Equation (1.3) may be differentiated with respect to x and Equations (1.4a) and (1.4b) inserted to give

$$\frac{d\tau}{dx} = \frac{G_a}{t_a} \left(\frac{du_2}{dx} - \frac{du_1}{dx} \right) = \frac{G_a}{t_a} \left(\frac{T_2}{t_2 E_2} - \frac{T_1}{t_1 E_1} \right). \quad (1.11)$$

Equation (1.9) is differentiated to obtain an alternative expression for $d\tau/dx$. That expression is substituted into Equation (1.11) together with values for T_1 and T_2 from Equations (1.10a) and (1.10b), and λ from Equation (1.7), and Equation (1.11) then reduces to

$$C_1 = C_2. \quad (1.12)$$

In addition to this, constants C_1 and C_2 must satisfy the condition

$$T_1 + T_2 = T \quad (1.13)$$

over the entire length of the joint. Substituting again for T_1 and T_2 from Equations (1.10a) and (1.10b), and with $C_1 = C_2$ from Equation (1.12), Equation (1.13) gives

$$C_1 = C_2 = \frac{T}{t_1 E_1 + t_2 E_2}. \quad (1.14)$$

The values of the constants A_1 and A_2 for each step can be obtained by equating the tensions in adherend 1 and the shear stresses in the adhesive at the ends common to adjacent steps, and by using the following boundary conditions which are the same for both the elastic and inelastic cases.

At the left-hand end of the joint, that is at $x = 0$,

$$T_1 = T \quad (1.15)$$

and at the right-hand end, that is at $x = L$,

$$T_1 = 0. \quad (1.16)$$

Hence, T_1 and T_2 may be evaluated using Equations (1.10a), (1.10b) and (1.14) enabling τ and γ to be evaluated from Equations (1.2), (1.3) and (1.9).

1.3.2 Derivation of Basic Formulae for Joints where the Adhesive Behaves Inelastically ($\tau_{max} > \tau_E$)

The five Equations (1.1a), (1.1b), (1.2), (1.4a) and (1.4b) apply to both the elastic and the inelastic regions of the joint. In an elastic region the assumption that τ equals $G_a \gamma$ leads to the differential equation, Equation (1.6) and hence Equation (1.9). In regions where the adhesive behaves inelastically, that is τ is greater than τ_E , the assumption that τ is represented by Equation (1.17) is made. Equation (1.17) is a simple explicit stress-strain relationship for the adhesive that has been chosen for convenience of calculation, the results calculated being relatively insensitive to the precise form of the curve. A graphical illustration of the terms used in Equations (1.17) to (1.19) is given in Sketch 1.3.

$$\tau = \tau_E + \beta \left(\frac{\alpha}{\alpha + \beta} \right) = \tau_E + \frac{\alpha \beta}{\alpha + \beta}, \quad (1.17)$$

where

$$\alpha = G_a \gamma - \tau_E \quad (1.18)$$

and

$$\beta = \tau_A - \tau_E. \quad (1.19)$$

Differentiating Equation (1.2) with respect to x and substituting for du_1/dx and du_2/dx from Equations (1.4a) and (1.4b) gives

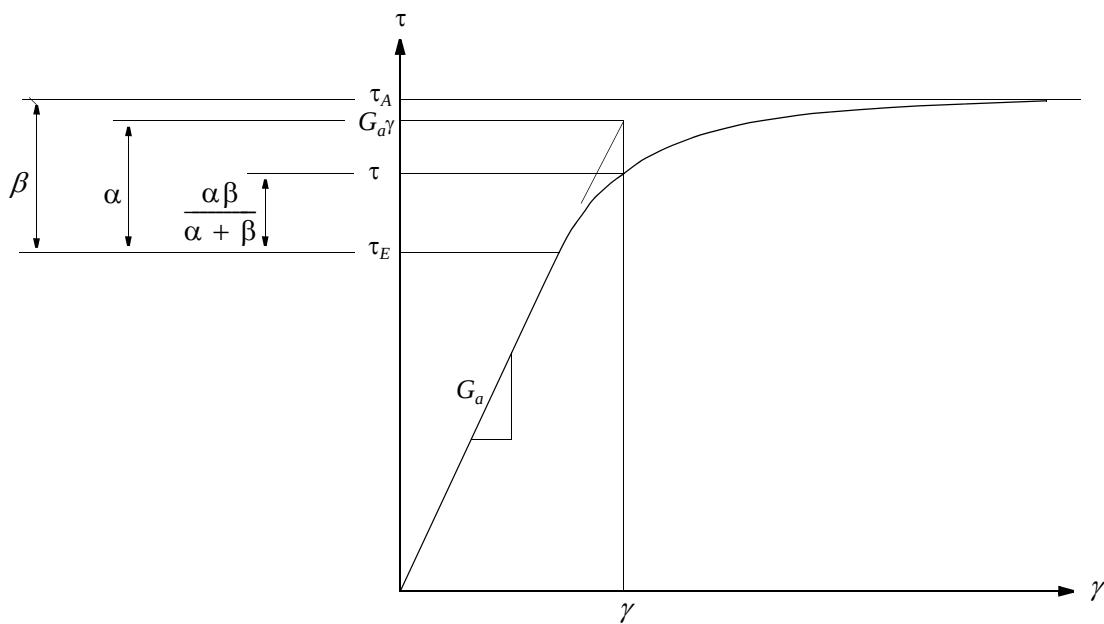
$$\frac{d\gamma}{dx} = \frac{1}{t_a} \left(\frac{T_2}{t_2 E_2} - \frac{T_1}{t_1 E_1} \right). \quad (1.20)$$

From Equation (1.18)

$$\frac{d\alpha}{dx} = G_a \frac{d\gamma}{dx}. \quad (1.21)$$

Substituting Equation (1.20) into Equation (1.21) gives

$$\frac{d\alpha}{dx} = \frac{G_a}{t_a} \left(\frac{T_2}{t_2 E_2} - \frac{T_1}{t_1 E_1} \right). \quad (1.22)$$



Sketch 1.3 Theoretical adhesive stress-strain curve

Differentiating again with respect to x and substituting for dT_1/dx and dT_2/dx from Equations (1.1a) and (1.1b) gives

$$\frac{d^2\alpha}{dx^2} = \frac{G_a}{t_a} \left(\frac{\tau}{t_2 E_2} + \frac{\tau}{t_1 E_1} \right) = \lambda^2 \tau, \quad (1.23)$$

where the coefficient λ is the same as for the elastic analysis in Section 1.3.1.

By substituting for τ from Equation (1.17) the above equation becomes

$$\frac{d^2\alpha}{dx^2} = \lambda^2 \left(\tau_E + \frac{\alpha\beta}{\alpha + \beta} \right). \quad (1.24)$$