

STRESS CONCENTRATIONS AT GROOVES FOR RETAINING RINGS OR SEALS (WITH NOTES ON DESIGN AGAINST FATIGUE)

1. INTRODUCTION

This Item provides the theoretical elastic stress concentration factors for small circumferential grooves in circular cylinders. The data are provided for the design of such features as retaining ring or seal grooves that are sited in the stressed portions of shafts or tubes. *The data strictly apply to major loads carried by the shaft or tube; minor loads entering via the faces of the grooves must be negligibly small.* Notes on calculations for designing against fatigue are provided. These use the stress concentration factor coupled with fatigue limit data for notched specimens. Additional notes on the application of fracture mechanics methods are included.

2. NOTATION

a	crack depth (see Section 4.2.1)	m	in
D_g	diameter at bottom of groove	m	in
D_i, D_o	internal and external diameters of shaft, respectively	m	in
f_e	equivalent direct stress (see Section 3.3)	N/m ²	lbf/in ²
f_{max}	maximum direct stress in shaft	N/m ²	lbf/in ²
h	depth of groove, $h = (D_o - D_g)/2$ for external groove or $h = (D_g - D_i)/2$ for internal groove	m	in
K_{ab}	*stress concentration factor under axial loading and/or bending defined by $f_{max} = K_{ab}f_{ref}$ where, for external grooves, $f_{ref} = \frac{4P}{\pi(D_o^2 - D_i^2)} + \frac{32MD_o}{\pi(D_o^4 - D_i^4)}$ and, for internal grooves, $f_{ref} = \frac{4P}{\pi(D_o^2 - D_i^2)} + \frac{32MD_i}{\pi(D_o^4 - D_i^4)}$	N/m ²	lbf/in ²

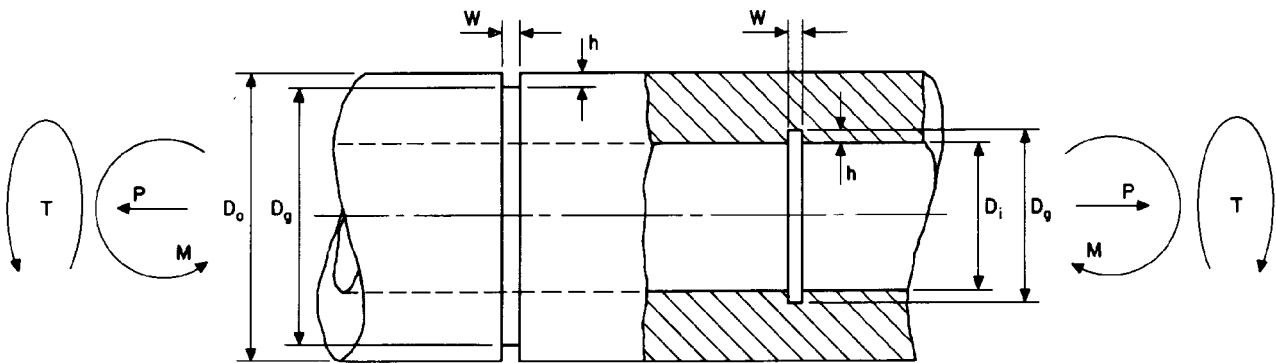
*For footnote see end of Notation Section

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K_{to}	* stress concentration factor under torsion defined by $q_{max} = K_{to}q_{ref}$ where, for external grooves, $q_{ref} = \frac{16TD_o}{\pi(D_o^4 - D_i^4)}$ and, for internal grooves , $q_{ref} = \frac{16TD_i}{\pi(D_o^4 - D_i^4)}$	N/m ²	lbf/in ²
$(\Delta K_I)_{TH}$	* threshold value of stress intensity factor range for shaft material, for mode I (see Section 4.2.1)	N/m ^{3/2}	lbf/in ^{3/2}
M	bending moment in shaft	N m	lbf in
P	axial load in shaft	N	lbf
q_{max}	maximum shear stress in shaft	N/m ²	lbf/in ²
R	S_{min}/S_{max} (see Section 4.2.1)		
r	radius at internal corners of groove	m	in
S_{max}, S_{min}	maximum and minimum values, respectively, of stress based on gross cross-sectional area of shaft	N/m ²	lbf/in ²
T	torque in shaft	N m	lbf in
W	width of groove	m	in
α	geometrical factor in stress intensity factor		

Both SI and British Units are quoted but any coherent system of units may be used.

* Note that the basic symbol K , with different prefixes and suffixes, is used for both stress concentration factors and stress intensity factors. These are distinct terms. Stress concentration factors are non-dimensional ratios of stresses whereas stress intensity factors are measures of intensity of stress in the vicinity of a crack tip and are dimensional.



Sketch 2.1

3. STRESS ANALYSIS

The theoretical stress concentration factors, based upon gross stress, for rectangular circumferential grooves in round shafts under direct and/or bending loading or in torsion are given in Figures 1 and 2, respectively. The data apply only to isotropic materials that obey Hooke's law. However, over the practical working range most engineering materials conform substantially in these respects and for them the data may be used without significant error. If the stress concentration factor is such that the maximum stress is above the elastic limit of the material, the stresses will be redistributed and will give rise to residual stresses on unloading.

3.1 Stress Concentration under Direct and/or Bending Loading

To provide the data of Figure 1 for the practical range of retaining ring geometries, the plate data of Derivation 3 have been extrapolated, guided by the data of Derivation 2. As well as the inaccuracies arising from extrapolation there is the further discrepancy resulting from using plate data for a groove in a circular shaft. However, for practical retaining rings, for which $30 \leq D_o/h \leq 85$, the errors arising from this latter source are small. Thus, the values of K_{ab} given in Figure 1 are expected to be accurate to within ± 5 per cent.

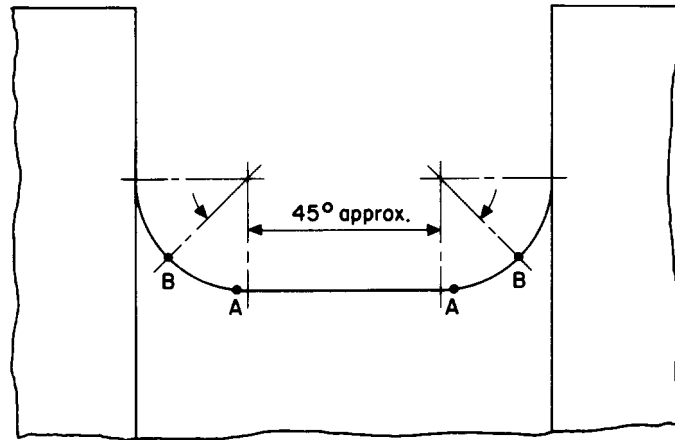
Because of the high values of D_o/h for retaining rings the data of Figure 1 are applicable to shafts subjected to either, or both, bending and axial loading. Similarly the data may be applied to hollow shafts, with internal or external grooves, provided that $(D_o - D_i)$ is greater than $10h$. In all cases the peak stress, f_{max} , occurs at points A shown in Sketch 3.1.

3.2 Stress Concentration under Torsion

The data of Figure 2 are based upon the transformation method described in Derivation 1. This allows calculation of the stress concentration factor for a hole in an infinite solid block loaded in shear parallel to the hole axis. The stress concentration factors are assumed to be representative of a shaft with a circumferential groove of depth equal to half the hole width. This ignores the effect of the curved free surface and assumes that the shaft diameter is very large in relation to the groove depth. (This assumption is met by the high D_o/h value for retaining ring grooves.) In addition the hole shape is described by a trigonometrical series and, because of this, precise geometrical representation of the rectangular hole with rounded corners is not possible. Values of K_{t0} are therefore reckoned to be accurate to within about ± 10 per cent.

The data may be applied to hollow shafts provided that $(D_o - D_i)$ is greater than $10h$. The maximum shear

stress, q_{max} , occurs at the points B shown in Sketch 3.1.



Sketch 3.1 Showing location of f_{max} (points A) and q_{max} (points B)

3.3 Combined Loadings

As Sketch 3.1 shows, the stress concentration factors resulting from torsion and direct and/or bending loading occur at two different locations around the radius so that simple combination of f_{max} and q_{max} is not strictly correct. However, locations A and B are about $0.7r$ apart and for practical retaining ring grooves this is usually less than 0.5 mm (0.020 in.). In consequence combination of f_{max} and q_{max} as if they occurred at the same point (and *simultaneously* for alternating loadings) will usually lead to only a slight overestimate of the resulting stress. The values of f_{max} and q_{max} may be combined to give f_e ; for example using the Mises-Hencky criterion gives

$$f_e = (f_{max}^2 + 3q_{max}^2)^{1/2}. \quad (3.1)$$

4. FATIGUE ANALYSIS

In general retaining rings and seals are employed in components in rotating machinery that are expected to experience many millions of cycles over their design life. Consequently design against the fatigue limit of the material is usually necessary. In order to ensure a proper fit for retaining rings, it is common practice to specify only a maximum value of r and not a minimum (see for example Reference 16*). Very small radii can lead to an excessively high stress under which fatigue cracks can be initiated. The cracks can grow to cause failure or may prove to be non-propagating. Because of this it becomes necessary to consider two approaches to the satisfaction of fatigue life criteria. Section 4.1 deals with cases where a minimum radius exists that is a major factor in controlling the fatigue strength. Section 4.2 deals with cases where r is either very small or undefined so that it becomes necessary to do a fracture mechanics calculation in which the groove is treated as an initial defect. In border-line cases both calculations should be carried out and the lower resulting fatigue limit accepted. It should be noted that in both cases the calculated fatigue limit will only apply if the radius can be maintained. Section 6 provides an example incorporating both approaches.

Attention is drawn to Item No. 71009 (in the Stress and Strength Sub-series) which provides notes on design principles and methods of reducing the stress concentrations at grooves.

* Complete ranges of retaining rings that accept higher values of r than those specified in Reference 16 are produced by some manufacturers.