

Mean forces, pressures and flow field velocities for circular cylindrical structures

Single cylinder with two-dimensional flow

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The preparation of this Data Item

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The Wind Engineering Panel, which took over the work on wind engineering previously monitored by the Fluid Mechanics Steering Group, first met in September 1979 and the Aerodynamics Committee, which first met in March 1942, have the co-operation of many engineers and scientists in industry, research establishments and universities from whom much assistance and information is being received.

MEAN FORCES, PRESSURES AND FLOW FIELD VELOCITIES FOR CIRCULAR CYLINDRICAL STRUCTURES: SINGLE CYLINDER WITH TWO-DIMENSIONAL FLOW

CONTENTS

	Page
1. NOTATION AND UNITS	1
2. PURPOSE, SCOPE AND USE OF THIS DATA ITEM	3
2.1 Purpose	3
2.2 Scope	4
2.3 Use of the Data Item	4
2.4 Applicability, Limitations and Uncertainties	5
3. DRAG COEFFICIENT OF ISOLATED CYLINDER NORMAL TO FLOW	5
3.1 The Roughness Factor λ_R	6
3.2 The Turbulence Factor λ_T	7
3.3 Calculation Procedure For Evaluating C_{D0}	7
3.4 Wind-tunnel Simulation of High Reynolds Number Flow	8
4. EFFECT OF CYLINDER INCLINATION TO FLOW	9
4.1 Subcritical Reynolds Numbers $Re_e < 3 \times 10^5$	9
4.2 Supercritical Reynolds Numbers ($Re_e > 3 \times 10^5$)	10
4.3 Curved Cylinders	11
5. CYLINDERS WITH SURFACE IRREGULARITIES	11
5.1 Stranded Cables	11
5.2 Perforated Cylinders and Shrouds	11
5.2.1 Derivation	12
5.3 Effect of Fins and Strakes	12
5.4 Effect of Other Spanwise Protrusions	13
5.4.1 Comments on the effects of protrusions	14
6. PROXIMITY EFFECT OF PLANE SURFACE PARALLEL TO CYLINDER AXIS	15
7. MEAN AND FLUCTUATING PRESSURE DISTRIBUTIONS, VELOCITY FLOW FIELD	17
7.1 Mean Pressure Distribution at Surface	17
7.1.1 Derivation of method for mean pressure distribution at surface	18
7.2 Fluctuating Pressure Distribution at Surface	18
7.3 Velocity Flow Field Away From Surface	20
7.3.1 Derivation of velocity flow field parameters	20

8.	EXAMPLES	20
8.1	Example 1	20
8.2	Example 2	21
8.3	Calculation Sheet for Examples (from Table 10.3)	23
8.4	Calculation Sheet for Example 1 (from Table 10.5)	24
9.	REFERENCES AND DERIVATION	25
9.1	References	25
9.2	Derivation	25
10.	TABLES	28
	FIGURES	33-49
	APPENDIX A GENERAL FEATURES OF THE FLOW AROUND A CIRCULAR CYLINDER	51
A1.	CYLINDER WITH TWO-DIMENSIONAL FLOW	51
A2.	INFLUENCE OF FREE-STREAM TURBULENCE	52
A3.	INFLUENCE OF SURFACE ROUGHNESS	53
	APPENDIX B SURFACE ROUGHNESS CHARACTERISTICS	55
B1.	EQUIVALENT SAND GRAIN ROUGHNESS	55
B2.	ADDITIONAL REFERENCES	56
B3.	TABLE OF EQUIVALENT ROUGHNESS VALUES FOR VARIOUS TYPES OF SURFACE FINISH	56
	APPENDIX C SUMMARY OF EQUATIONS FOR SOME OF THE GRAPHICAL DATA	57
C1.	EQUATION FITS TO THE DATA	57
	APPENDIX D RANGE OF EXPERIMENTAL DATA	60
D1.	SOURCES AND RANGE OF DATA USED	60

MEAN FORCES, PRESSURES AND FLOW FIELD VELOCITIES FOR CIRCULAR CYLINDRICAL STRUCTURES: Single Cylinder with Two-dimensional Flow

1. NOTATION AND UNITS

SI Unit

C_{D0}	mean drag coefficient of two-dimensional circular cylinder whose longitudinal axis is normal to flow; drag force per unit length/ $(\frac{1}{2}\rho V_\infty^2 D)$	
C_D, C_L C_N, C_T	set of force coefficients acting along and normal to flow direction and to cylinder axes respectively (see Sketch 4.1); force per unit length/ $(\frac{1}{2}\rho V_\infty^2 D)$	
C_p	mean pressure coefficient at cylinder surface, $(p - p_\infty)/(\frac{1}{2}\rho V_\infty^2)$	
$C_p(t)$	instantaneous value of the fluctuating pressure coefficient	
$\hat{C}_p(t)$	extreme value of $C_p(t)$	
C_{pb}	value of C_p on cylinder surface in wake region (see Sketch 7.1)	
C_{pm}	minimum value of C_p on cylinder surface (see Sketch 7.1)	
$C_p(r, \theta)$	mean pressure coefficient at distance r from cylinder centre and angle θ (see Sketch 7.2)	
D	cylinder diameter	m
d	protrusion height	m
f_ϕ	factor in Equation (4.7) accounting for effect of cylinder inclination on C_N	
g	crest or peak factor (see Equation (7.9))	
h	gap width between cylinder and plane surface (see Sketch 6.1)	m
I_u, I_v	intensities of turbulence; σ_u/V_∞ and σ_v/V_∞ (see Table 10.2 for typical values)	
L	length of cylinder	m
${}^rL_u$	lateral integral scale of u -component of turbulence (see Table 10.2 for typical values)	m
p	local static pressure	N/m ²
Re	Reynolds number, $V_\infty D \text{sec} \phi / \nu$	

Re_{crit}	critical Reynolds number, <i>i.e.</i> Re at which $C_{D0} = 0.8$ in rapid drag fall (see Appendix A)	
Re_e	effective Reynolds number, $\lambda_T \lambda_R Re$ (see Section 3)	
Re_D	Re at which rapid fall in C_{D0} at transition begins (see Sketch 3.1)	
Re_x	$V_\infty x / \nu$	
r	distance between cylinder centre and point in flow field	m
$u(t), v(t)$	fluctuating components of free-stream velocity in direction of flow, and normal to flow direction and cylinder longitudinal axis, respectively	m/s
V_∞	mean free-stream velocity	m/s
V_s	local mean flow velocity at surface of cylinder	m/s
$V(r; \theta)$	local mean velocity in flow field at distance r from cylinder centre and angle θ	m/s
V_{eff}	effective mean free-stream velocity in boundary layer (see Equation (6.1))	m/s
V_z	local mean free-stream velocity in boundary-layer flow (see Sketch 6.1)	m/s
x	length of plane surface upstream of cylinder location	m
$\bar{\alpha}$	power-law index giving approximate variation of V_z with height from surface	
$\beta(r; \theta)$	angle between local flow direction and free-stream direction (see Sketch 7.2)	deg
ε	effective roughness height of surface (see Table 10.1 for typical values)	m
θ	angular location of point (see Sketches 5.2, 7.1)	deg
θ_b	value of θ where C_p first equals C_{pb}	deg
θ_m	value of θ where $C_p = C_{pm}$	deg
λ_R	parameter defining effect of surface roughness on Re_e (see Section 3.1)	
λ_T	parameter defining effect of flow turbulence on Re_e (see Section 3.2)	
ν	kinematic viscosity of free-stream	m ² /s
ρ	density of free-stream	kg/m ³
σ_{Cp}	standard deviation of fluctuating pressure coefficient	
σ_u, σ_v	standard deviations of $u(t)$ and $v(t)$	m/s

ϕ angle between free-stream direction and normal to longitudinal axis of cylinder (see Sketch 4.1) deg

ψ ratio of open area to total area

Subscripts

crit denotes value at critical Reynolds number

eff relates to effective value based on V_{eff}

h relates to value for gap width h

m relates to value where $C_p = C_{pm}$

max denotes maximum value

ϕ denotes value for cylinder inclined to flow at angle ϕ

0 denotes value for two-dimensional conditions (see Section 2.4)

∞ denotes free-stream value

2. PURPOSE, SCOPE AND USE OF THIS DATA ITEM

2.1 Purpose

The purpose of this Data Item is to provide data for estimating the mean forces induced by flow around a cylindrical structure of circular cross-section. Only single cylinders are considered in this Item; mean forces on cylinders in groups will be the subject of a separate Data Item. In the present context the data apply when the effect of the flow around the free end (or ends) can be ignored (*e.g.* for long cylinders). Reference 1 provides additional information in the form of correction factors which can be applied to these data to account for end effects and shear flow effects, such as those associated with cantilever structures in the non-uniform atmospheric wind.

The circular cylinder is one of the most commonly occurring shapes in engineering structures. Typical examples are chimney stacks, towers, storage tanks, silos, cables, pipe lines, space vehicles and missiles, and elements of structures such as lattice towers and off-shore structures. Other examples are pipes and struts inside ducts but in these cases the confinement effects due to the proximity of the duct walls must be taken into account (see ESDU 80024⁹). In many situations the overall response (mean and fluctuating) of the structure to the approaching flow, such as the atmospheric wind, is needed for design purposes. This Item provides data for estimating the mean components of loading. The fluctuating components, such as those arising from buffeting by a turbulent flow or from vortex shedding, must also be considered; ESDU 87035⁷ provides methods for estimating the maximum design loading due to buffeting by atmospheric turbulence (along the wind direction) and Reference 8 deals with the across-flow response due to vortex shedding.

2.2 Scope

The data presented (used in conjunction with Reference 1 to account for free-end effects and shear flow) form the basis for estimating mean force coefficients acting on circular cylindrical structures of many types in a variety of conditions. The following data can be obtained from this data Item.

<i>Data</i>	<i>Section No.</i>
Drag coefficients of plain cylinders	3
Force coefficients of inclined cylinders	4
Drag coefficients of stranded cables	5.1
Drag coefficients of perforated cylinders and perforated shrouds	5.2
Drag coefficients of cylinders with strakes	5.3
Drag and side force coefficients for cylinders with isolated spanwise protrusions such as ribs, joints, icing droplets, cables and pipes	5.4
Drag and side force coefficients of cylinder with its longitudinal axis parallel to nearby plane surface	6
Mean pressure distributions	7.1
Fluctuating pressure distributions	7.2
Velocity flow field around the cylinder	7.3

It must be emphasised that with circular cylindrical structures there are a number of parameters that can have a very significant effect on the flow-induced forces. The effects of the following parameters are particularly important and can be taken into account when estimating data from this Item.

<i>Parameter</i>	<i>Section No.</i>
Reynolds number (up to full-scale values)	3,
Roughness of cylinder surface (up to $\varepsilon/D = 0.12$)	3, 3.1
Turbulence characteristics of the approaching flow	3, 3.2
Cylinder inclination to flow direction (0 to 90°)	4

2.3 Use of the Data Item

To make the best use of this Data Item proceed as follows:

- (i) Select the appropriate calculation sheet, or sheets, from Tables 10.3 to 10.5. Read the general background notes referred to in the appropriate Sections of interest. (Section 8 contains two worked examples and Appendix A provides a general description of the features of the flow around a circular cylinder.)
- (ii) Take note of the general applicability and uncertainty of the data (see Section 2.4).
- (iii) If, for the structure under consideration, end effects are significant (*i.e.* length to diameter ratio less than about 25 as with most stacks and towers) and/or the approaching velocity profile is non-uniform, other correction factors must, in general, be applied to the values obtained from this Item; these correction factors are given in Reference 1.
- (iv) For repeated calculations many of the data in this Item can be programmed using the equations in the text and in Appendix C.

2.4 Applicability, Limitations and Uncertainties

The data presented in this Item are principally derived from data associated with uniform, two-dimensional flow conditions. This means that either the cylinder is sufficiently long that end effects, which induce a three dimensional flow around the tip, are essentially localised to that region or that end effects are minimised by placing the cylinder between end plates. In this latter situation truly representative two-dimensional conditions will only prevail if the length to diameter ratio of the cylinder is greater than about 5 or 6; this then allows the three-dimensional cell-like structure of the wake associated with vortex shedding to develop naturally. Correction factors to account for finite-length cylinders in shear (or boundary-layer) flow, such as the atmospheric wind, are provided in Reference 1.

Other limitations on the use of the data are either implied by the comments in the text and the Figures or by the range of experimental data used to derive the various correlations summarised in the Table in Appendix D. Where possible the data have been interpolated and extrapolated to conditions other than those for which experimental data are available.

The uncertainties of the data presented are, in general, indicated on the appropriate Figures or discussed in the related text.

3. DRAG COEFFICIENT OF ISOLATED CYLINDER NORMAL TO FLOW

The calculation sheet in Table 10.3 provides a summary of the steps required to estimate a value of C_{D0} for a given set of input conditions. The following notes provide a background to the derivation of the data.

The flow pattern around a circular cylinder (and the resulting drag coefficient) are primarily determined by the position of the separation points at which the upstream boundary layer leaves the cylinder surface to form the wake region. The location of the separation points is primarily determined by the Reynolds number and turbulence characteristics of the approaching flow and by the roughness of the cylinder surface. The flow pattern development from very low to very high Reynolds numbers, and the general effects of turbulence and surface roughness, are described in Appendix A for conditions where compressibility effects can be ignored (Mach number less than 0.25)*. In practice, the drag coefficient of a two-dimensional circular cylinder can be correlated with flow and surface roughness conditions in the form

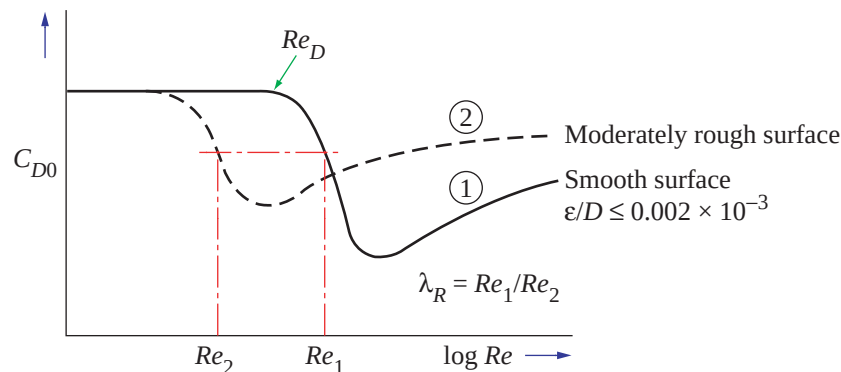
$$C_{D0} = f[Re_e, \epsilon/D]$$

as given by Figures 1a to 1c. The effective Reynolds number $Re_e (= \lambda_T \lambda_R Re)$ is a modified Reynolds number incorporating the factor λ_T , dependent on the turbulence characteristics of the approaching flow, and λ_R dependent on the surface roughness parameter ϵ/D .

The derivation of these factors is described in Sections 3.1 and 3.2 and their values for specified conditions are given by Figures 2 and 3. The procedure for evaluating C_{D0} is described in Section 3.3, or the calculation sheet in Table 10.3 can be used for this purpose. In addition Section 3.4 provides guidance on the use of these effects to provide simulation of high Reynolds number flow for wind-tunnel tests.

* The effects of compressible flow around circular cylinders such as axisymmetric bodies and stub cylinders normal or inclined to the flow will be considered in future Data Items to be included in the Aerodynamics Sub-series.

3.1 The Roughness Factor λ_R

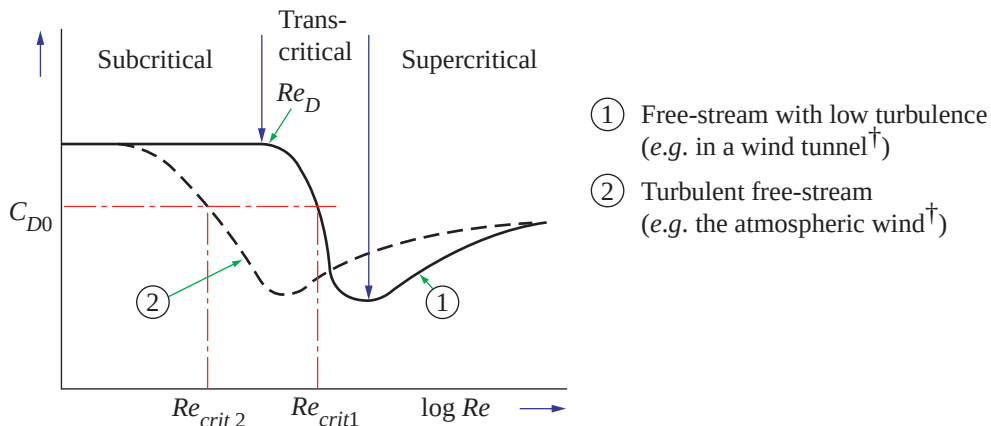


Sketch 3.1 Effect of surface roughness

For the reasons summarised in Appendix A, increasing surface roughness has the effect of decreasing the value of $Re_e (= Re_D)$ at which the rapid fall in C_{D0} begins (see Sketch 3.1) and λ_R is a parameter which characterises this effect. It may be defined as the ratio of Re for a smooth cylinder giving a specified value of C_{D0} to Re for a rough cylinder in the same free-stream giving the same C_{D0} , both measured in the transition region following Re_D ; in practice it is related to ϵ/D . This relationship is shown in Figure 2 which has been derived from an analysis of data for cylinders with uniformly distributed roughness. However, Figure 2 (and Figure 1c.) can be used for uniformly-distributed elements such as helical strakes taking ϵ as the element height (usually about $0.1 D$ for a helical strake).

Some approximate values of the quantity ϵ for a number of different surfaces are given in Table 10.1. These values are equivalent sand-grain roughness heights, the derivation of which is discussed in Appendix B. Considerable variation in these values can apply as indicated in Table 10.1. It is also important to remember that the deterioration of a surface with time usually increases the surface roughness and, unless special maintenance procedures are employed, this should be taken into account when selecting an appropriate value of ϵ .

3.2 The Turbulence Factor λ_T



Sketch 3.2 Effect of turbulence in approaching flow

Turbulence in the approaching flow, such as the atmospheric wind, has a strong effect on the variation of C_{D0} with Re in the transition region as illustrated in Sketch 3.2. The physical reasons for this (and for the rapid variation of C_{D0} with Re in the transition region) are explained in Appendix A. An analysis of data from various sources (see Appendix D) shows that the effect of free-stream turbulence on C_{D0} becomes increasingly important for Re greater than about 10^4 although its effect decreases again for $Re > Re_D$ (the value of Re at which C_{D0} begins to fall in the transition region) and is negligible for $Re \geq 3 \times 10^6$. For a given surface roughness this effect is represented in Equation (3.1) by the factor λ_T . This parameter depends on Re and Re_{crit} , which in turn is determined by λ_{Tcrit} and λ_R , and is given by

$$Re_{crit} = \frac{4.5 \times 10^5}{\lambda_{Tcrit} \lambda_R}. \quad (3.1)$$

In general, Re_{crit} is the value of Re at which C_{D0} falls to a value of 0.8 in the transition region*. Figure 3a provides values of λ_{Tcrit} as a function of the turbulence parameter $I_u(D/\bar{r}L_u)^{1/5}$. The parameters I_u and $\bar{r}L_u$ represent the intensity and scale of the turbulence in the approaching flow (Appendix A explains the physical significance of these properties) and typical values are given in Table 10.2.

3.3 Calculation Procedure For Evaluating C_{D0}

- (1) Obtain a value of the surface roughness parameter, ϵ , from Table 10.1 and evaluate ϵ/D .
- (2) Determine λ_R from Figure 2.
- (3) Evaluate $Re = V_\infty D/\nu$. If $Re > 2 \times 10^6$ then the factor $\lambda_T \approx 1.0$ and steps 4 to 8 can be ignored.
- (4) Obtain typical values of the intensity (I_u) and scale of turbulence ($\bar{r}L_u$) from Table 10.2 and evaluate $I_u(D/\bar{r}L_u)^{1/5}$.

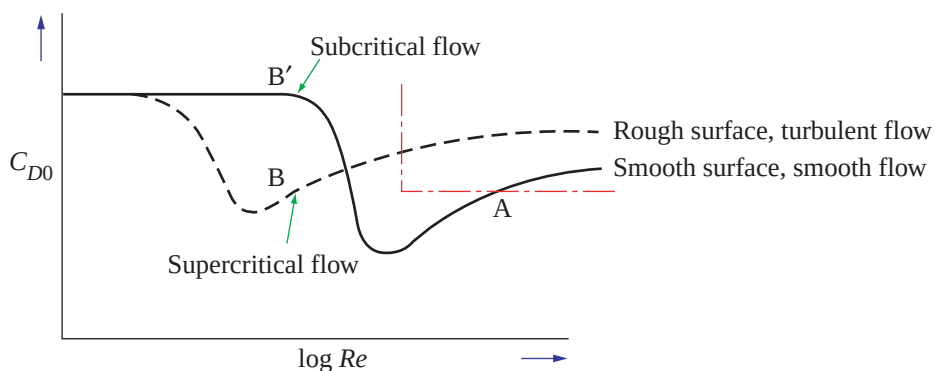
* Note that when ϵ/D is greater than about 8×10^{-3} then C_{D0} is always greater than 0.8 but the definition of Re_{crit} given by Equation (3.1) must be used because it is compatible with the way in which the method was derived.

[†] Typically, for a low-turbulence wing-tunnel flow, λ_{Tcrit} is about 1.4 whereas for the atmospheric wing λ_{Tcrit} is about 10.

- (5) Determine λ_{Tcrit} from Figure 3a.
- (6) Evaluate $Re_{crit} = 4.5 \times 10^5 / (\lambda_{Tcrit} \lambda_R)$.
- (7) Evaluate Re/Re_{crit}
- (8) Determine $(\lambda_T - 1)/(\lambda_{Tcrit} - 1)$ from Figure 3b and hence evaluate λ_T .
- (9) Evaluate $Re_e = \lambda_T \lambda_R Re$.
- (10) Determine C_{D0} from Figure 1a, 1b or 1c as appropriate.

3.4 Wind-tunnel Simulation of High Reynolds Number Flow

In wind-tunnel studies it is often difficult to simulate high Reynolds number flow conditions associated with many full-scale structures because of limitations on model size (and hence on Re). In practice, the data in Figures 1 to 3 show that the addition of moderate surface roughness (and sometimes an increase in turbulence in the approaching flow) can be used to promote supercritical flow conditions at Reynolds numbers when the flow would otherwise have been subcritical, as illustrated in Sketch 3.3. For example, referring to Sketch 3.3, the pressure distribution represented by conditions at A (relatively high Re , relatively smooth surface and low turbulence flow) is found to be more or less identical to the pressure distribution represented by conditions at B which relate to a rougher cylinder in turbulent flow but at a significantly lower Re .



Sketch 3.3

This technique, however, has limitations. It cannot be used for groups of cylinders since wake-interference effects nullify the Reynolds number equivalence. It cannot be used when Re is less than about 3×10^4 since roughness then has no significant effect on the flow regime. It can only be used sensibly over a relatively small range of effective Reynolds numbers. However, the Data Item can be used to provide guidance in ascertaining the degree of additional roughness (and turbulence) that would be required to generate the appropriate supercritical flow conditions in a wind-tunnel test.

4. EFFECT OF CYLINDER INCLINATION TO FLOW

A number of structures, or their components, are naturally inclined to the approaching flow. Examples are leg members of lattice structures and off-shore structures, bridge cables and pipe lines. The data presented in this Section provide force coefficients giving either the drag force or the force normal to the cylinder axis.

The method of allowing for cylinder inclination depends on whether the flow is subcritical ($Re_e < 3 \times 10^5$) or supercritical ($Re_e > 3 \times 10^5$). For subcritical flow the simple cross-flow theory given in Section 4.1 applies. For supercritical flow the evidence is that this simple cross-flow theory *underestimates* $C_{D0\phi}$ or C_N for relatively smooth two-dimensional cylinders and should not be used; Section 4.2 provides guidance for this situation.

The critical flow velocity (corresponding to Re_{crit}) for an inclined cylinder is found to be lower than that for the same cylinder normal to the flow. In practice the critical Reynolds number is approximately independent of ϕ if expressed in terms of the streamwise components so that

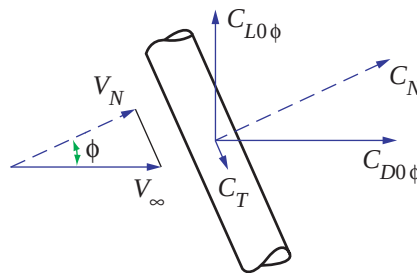
$$Re_{crit} = [V_{\infty} D \sec \phi / \nu]_{crit} = 4.5 \times 10^5 / (\lambda_{Tcrit} \lambda_R) \quad (4.1)$$

and more generally

$$Re_e = \lambda_T \lambda_R (V_{\infty} D \sec \phi / \nu) \quad (4.2)$$

The general calculation procedure for dealing with inclined cylinders is set down in the calculation sheet in Table 10.3 but Sketch 4.2 illustrates how the different methods for the subcritical and supercritical flow regions fit together.

4.1 Subcritical Reynolds Numbers $Re_e < 3 \times 10^5$



Sketch 4.1

For subcritical Reynolds numbers experimental data show that the force coefficients are dependent on the component of free-stream velocity normal to the cylinder axis, *i.e.* on $V_N = V_{\infty} \cos \phi$, and on the streamwise component of Reynolds number. Thus for inclined cylinders the normal force is given by

$$C_N = C_{D0} \cos^2 \phi \quad (4.3)$$

where C_{D0} is obtained from Figure 1 for $Re_e = \lambda_T \lambda_R (V_{\infty} D \sec \phi / \nu)$.

An analysis of data¹¹ shows that the axial force is small but that its variation with ϕ can be represented by (for $0 \leq \phi \leq 90^\circ$)

$$C_T / C_{D0} = k\phi \frac{\pi}{180} \left(2 - \phi \frac{\pi}{180} \right) \quad (4.4)$$

For long, smooth circular cylinders, $k = 0.033$ but for a rough or uneven cylinder, such as a stranded cable, $k = 0.045$ is a more reasonable estimate.

Drag and side forces parallel to, and normal to, the free-stream direction respectively can be obtained from a simple resolution of forces. Thus,

$$C_{D0\phi} / C_{D0} = \cos^3 \phi + C_T \sin \phi / C_{D0} , \quad (4.5)$$

$$C_{L0\phi} / C_{D0} = \cos^2 \phi \sin \phi - C_T \cos \phi / C_{D0} . \quad (4.6)$$

4.2 Supercritical Reynolds Numbers ($Re_e > 3 \times 10^5$)

For supercritical Reynolds numbers data¹⁵ indicate that the simple cross-flow theory described in Section 4.1 tends to underestimate C_N for relatively smooth, inclined circular cylindrical members. The reason for this is that while for $Re < Re_{crit}$ the laminar boundary layer (and the associated pressure distribution) tends to depend only on the cross-flow velocity, when transition to turbulent flow in the boundary layer has occurred ($Re > Re_{crit}$) the subsequent development and separation of the boundary layer are adversely affected by the three dimensional nature of the turbulent wake flow. This exerts a considerable influence on the pressure distribution and increases the flow-induced forces over those predicted using simple cross-flow theory.

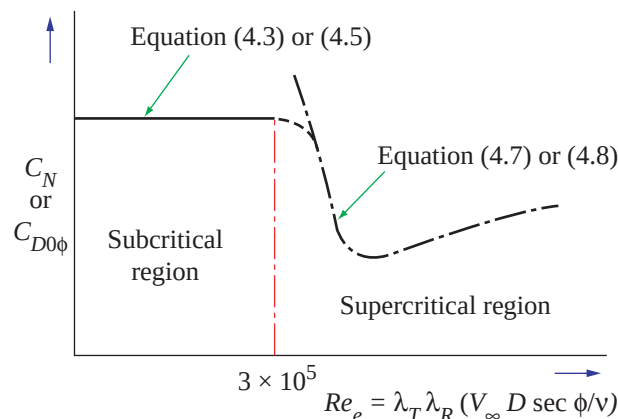
An analysis of the available data^{15, 44} indicates that in this flow regime the force coefficients may be estimated using

$$C_N = C_{D0} f_\phi , \quad (4.7)$$

$$C_{D0\phi} = C_N \cos \phi , \quad (4.8)$$

and $C_{L0\phi} = C_N \sin \phi \quad (4.9)$

where f_ϕ is given by Figure 4 and it is assumed that the axial force C_T can be ignored. Appendix C provides equations representing the data in Figure 4.



Sketch 4.2 Variation of force coefficients for inclined cylinder

The data for f_ϕ in Figure 4 for smooth or relatively smooth cylinders can be taken to apply up to say $\varepsilon/D < 0.1 \times 10^{-3}$. For very rough cylinders the adverse effect of the roughness elements causing earlier separation of the boundary layer from the cylinder is unlikely to be made significantly worse by the three-dimensional wake effects induced by cylinder inclination. For this reason it is probable that the simple cross-flow theory described in Section 4.1 can be applied to rough cylinders say ($\varepsilon/D > 0.1 \times 10^{-3}$). Between $\varepsilon/D = 0.1 \times 10^{-3}$ and 10×10^{-3} values of f_ϕ between the two extreme recommendations are likely to apply as indicated in Figure 4 and by the Equations in Appendix C. No data have been found to verify these tentative recommendations and this is clearly an important area needing further research.

4.3 Curved Cylinders

Data²⁴ suggest that, for $Re_e < 3 \times 10^5$, Equations (4.3), (4.5) and (4.6) may be applied locally to long, not too highly curved cylinders by taking ϕ as the local inclination of the curved cylinder to the free-stream and then summing the local forces along the cylinder length. In the absence of any other information a similar procedure may be used for $Re_e > 3 \times 10^5$ using the appropriate data in Section 4.2.

5. CYLINDERS WITH SURFACE IRREGULARITIES

5.1 Stranded Cables

Figure 5 gives drag coefficient data for stranded wire cables as a function of Reynolds number and the ratio of strand to cable outside diameter (d/D). The correlation follows the trend of a collection of data^{22, 47} which show a scatter of about 0.1 in C_{D0} about the smoothed mean correlation curves. The experimental data indicate that for smooth flow conditions ($I_u < 0.01$) the rapid fall in C_{D0} with Re occurs at a Reynolds number of between 2×10^4 and 3×10^4 . It is not expected that turbulence will have a very significant effect on C_{D0} but data⁴⁷ indicate that for $I_u = 0.11$ the value of Re_D , at which the rapid fall in C_{D0} with increasing Re occurs (see Sketch 3.1), is decreased to about 1×10^4 .

Insufficient data are available for $d/D < 0.07$ to indicate how C_{D0} varies for $Re > 3 \times 10^4$. Extrapolation of Figure 5 to lower values of d/D is therefore not recommended.

For stranded cables that are inclined to the approaching flow it may be assumed that the simple cross-flow theory described in Section 4.1 applies. In this case Re_{crit} (based on streamwise components) can be taken as the value of Re in Figure 5 at which the rapid fall in C_{D0} begins.

5.2 Perforated Cylinders and Shrouds



(a) Perforated cylinder

(b) Cylinder enclosed by perforated shroud

Sketch 5.1

Perforated cylinders, either in the form of (a) in Sketch 5.1 or in the form of (b) where a perforated shroud is fitted over a circular cylindrical structure, can be used to inhibit flow-induced oscillations. For example, dampers consisting essentially of a perforated cylinder of 60 per cent open-area ratio ($\psi = 0.6$) and $L/D = 4$ when attached to overhead cables are often used in reducing the amplitude of wind-induced galloping motions⁵¹. The use of shrouds ($\psi = 0.20$ to 0.36) fitted to the top 25 per cent of circular cylindrical structures has also been shown to be effective in reducing oscillatory motion due to vortex shedding. The perforations in the cylinder act to reduce the tendency to shed strong vortices.

Drag coefficient data for a uniformly perforated cylinder are presented in Figure 6 as a function of the open-area ratio (ψ). Similar data for a cylinder enclosed by a perforated shroud are given in Figure 7. In this case data are provided giving C_{D0} for the cylinder-shroud combination and the component of this total C_{D0} acting on the shroud.

Particular features to note in Figure 6 and 7 are that (i) the drag coefficient of a perforated cylinder can be greater than that for an equivalent solid cylinder and (ii) that with shrouded cylinders the drag coefficient of the inner cylinder can be negative.

Since the data on which Figures 6 and 7 are based are limited to specific configurations they have been extrapolated to other open-area ratios and cylinder-to-shroud diameters using methods outlined in Section 5.2.1.

5.2.1 Derivation

Data are available⁵¹ for a perforated cylinder with an open-area ratio of 0.6 and values of L/D from 2.7 to 8. An approximate estimate of the drag of a perforated cylinder is given by considering two perforated plates in series using data presented in Item Nos 70015 and 81027. By comparing these estimates with the measured data, a correction factor can be obtained which is found to be only slightly dependent on L/D . Correction factors for the extreme cases where $\psi = 0$ and 1.0 can also be determined which, when correlated with the values for $\psi = 0.6$, then provide a basis for a general method of estimating C_D for a perforated cylinder from $\psi = 0$ to 1.0. This forms the basis for Figure 6.

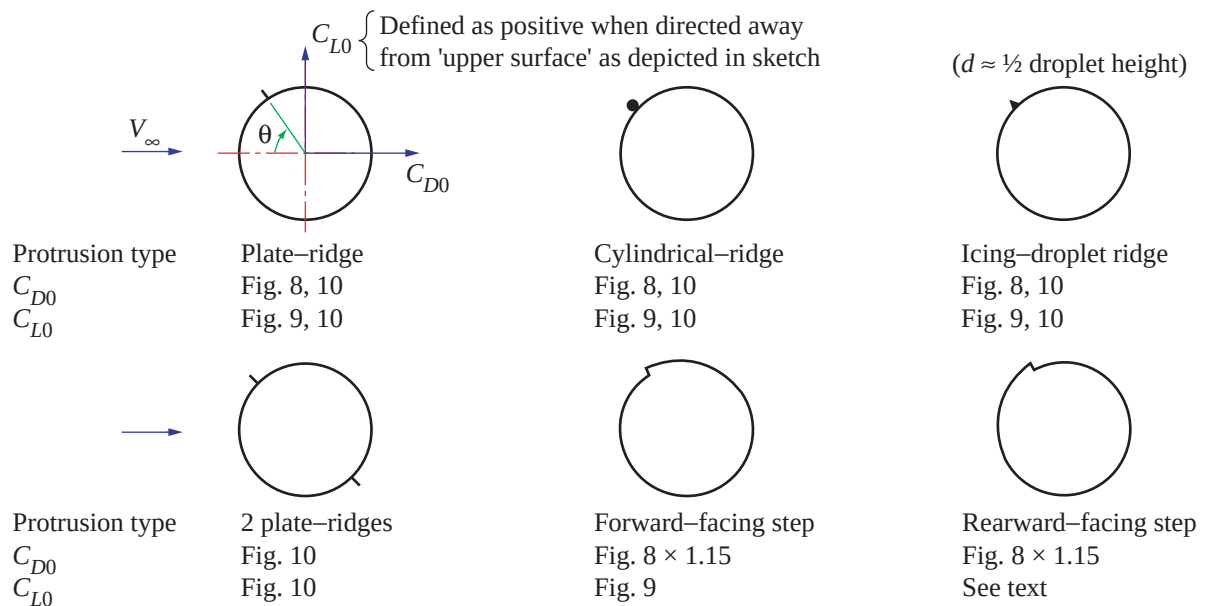
For a cylinder enclosed in a perforated shroud, data for open-area ratios of 20 per cent and 36 per cent are available²⁸ for a range of cylinder-to-shroud diameters from 0.73 to 0.85. Values of C_{D0} for both the shroud-cylinder combination and the contributions acting on the shroud were measured. These data have been extrapolated through to values of the cylinder-to-shroud diameter ratio equal to zero (given by Figure 6) and unity, when C_{D0} (at subcritical Re) will be about 1.2 for the cylinder-shroud combination. Of this total approximately $(1 - \psi)C_{D0}$ will be the contribution acting on the shroud when the cylinder and shroud diameters are equal.

5.3 Effect of Fins and Strakes

Helical strakes of the type fitted to stacks and towers to inhibit vortex-induced oscillations cause a significant increase in the drag coefficient. An analysis of data¹⁸ for cylinders fitted with helical strakes over part of their length shows that a good estimate of C_D for the straked portion can be obtained by equating the strake protrusion height (d) to ε and using Figures 1 to 3 with the corresponding value of ε/D . Typically, for $Re > 3 \times 10^5$, $C_{D0} = 1.15(1 + 2d/D)$.

A similar procedure can be used to estimate C_{D0} for cylinders with fins or ribs spanning the length of the cylinder at regularly spaced intervals around the circumference. Section 5.4 provides guidance on the effect of isolated spanwise protrusions.

5.4 Effect of Other Spanwise Protrusions



Sketch 5.2 Cylinder with various types of protrusions along its length

For practical engineering structures it is rare that the outside surface will be free from surface irregularities. Circular cylindrical structures are no exception and are often encumbered with spanwise protrusions such as pipes, ladders, brackets, overlap joints and ice droplets many of which can be idealised by the simple protrusion shapes illustrated in Sketch 5.2.

The effect of these spanwise protrusions is twofold. First, the maximum C_{D0} of the cylinder with a protrusion is usually significantly greater than that of the plain cylinder. Secondly, the addition of a spanwise protrusion causes the configuration to become asymmetrical with the result that a side force (represented by C_{L0}) is induced. The physical explanation for the origin, direction and variation of these forces with Re is summarised in Section 5.4.1.

Figures 8 and 9 provide data giving the variation of the maximum values of C_{D0} and C_{L0} with Re_e ; Figure 10 provides guidance on typical variations of C_{D0} and C_{L0} with the angular location of the protrusion for the subcritical and supercritical regimes. These data apply for two-dimensional conditions; the correction factors for finite aspect ratio and shear flow effects from Reference 1 can be used where necessary. The calculation sheets on Tables 10.4 and 10.3 summarise the steps necessary to obtain C_{D0} and C_{L0} .

Although there are few data sources dealing with the protrusion problem it is clear that to a large extent both the maximum C_{D0} and C_{L0} tend to be independent of the protrusion shape. Data presented in Figures 8 and 9 provide typical values of C_{D0} and C_{L0} based on data for plate-type^{20, 38}, step-type³⁸ and cylindrical-type³⁸ protrusions and closely spaced ice droplets²⁵. It may be assumed, in the absence of other information, that the maximum values apply for similarly shaped protrusions although the qualifications noted later for 'steps' should be taken into account.

In some situations a gap may exist between the protrusion and the cylinder surface (e.g. as with a pipe running along the cylinder length). In this case, providing the gap width (h) is not large (less than about $0.25 D$) the data in Figures 8 to 10 can be assumed to apply approximately by taking the protrusion height as $d + h$.

The original data were obtained in low turbulence ($I_u < 0.005$) flow but the application of Figures 8 and 9 has been tentatively extended to cover other conditions using the effective Reynolds number ($Re_e = \lambda_T \lambda_R Re$) to correlate the data. In this way the turbulence factor λ_T (Section 3.2) can be used to account for the effect of turbulence in the approaching flow. In addition the factor λ_R (Section 3.1) can be used to account approximately for the effect of slightly rough surfaces (up to $\epsilon/D \approx 0.05 \times 10^{-3}$) but for rougher surfaces it is likely that the addition of a protrusion invalidates the use of λ_R as given by Figure 2.

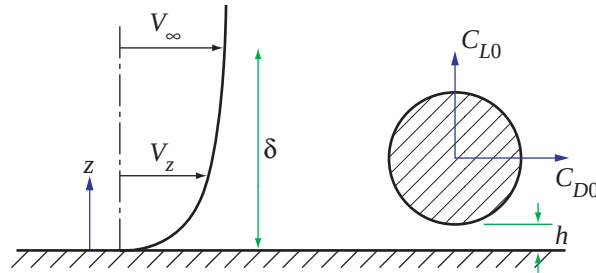
For forward-facing or rearward-facing steps (typical of overlap joints generated when a metal sheet is rolled into a cylinder) the data³⁸ indicate that C_{D0} is approximately 15 per cent higher than the values given in Figure 8 for subcritical Reynolds numbers ($Re_e < 4.5 \times 10^5$). The C_{L0} data are less well defined and are based only on data for plate protrusions²⁰ (supercritical Re) and forward-facing steps³⁸ (subcritical Re). For a rearward facing step, and subcritical Re , C_{Lmax} appears to be up to about 50 per cent greater than the values given in Figure 9 for values of d/D greater than 0.03. At supercritical Re ($Re_e > 4.5 \times 10^5$) it may be assumed that there is no dependence on protrusion shape within the limited applicability of Figure 9.

5.4.1 Comments on the effects of protrusions

The variation of C_{D0} with Re for a cylinder with a protrusion is similar to that for a plain cylinder but with some important differences which are also associated with the occurrence of significant values of C_{L0} . As with the plain cylinder C_{D0} is a maximum at lower values of Re but its variation with Re goes through two transition stages. A premature boundary layer transition to turbulent flow on the upper surface (with reference to Sketch 5.2) is caused by turbulence produced by the protrusion element and results in delayed flow separation, larger suction pressures, higher base pressure and an initial decrease in C_{D0} with increasing Re . This is followed at a higher Re (corresponding approximately to Re_{crit} for a smooth cylinder) by a further drop in C_{D0} as the lower surface boundary layer becomes turbulent at separation.

At subcritical Re the larger suction pressures on the upper surface (due to the delayed boundary layer separation) cause the lift force to be positive (*i.e.* directed upwards). At supercritical Re the protrusion causes a larger separated flow region to occur on the upper surface, thus eliminating its region of high suction pressures, but increases the suction pressures on the lower surface (typically by about 30%); this results in a negative lift force (directed downwards). The maximum C_{L0} occurs at just supercritical Re (giving negative C_{L0}) when there is a maximum differential between the flow separation points on the upper and lower surfaces. In general the maximum values of C_{D0} and C_{L0} do not occur for the same circumferential position of the protrusion; except at high Re , C_{Lmax} tends to occur for a value of θ at which C_{D0} is a minimum, and vice versa.

6. PROXIMITY EFFECT OF PLANE SURFACE PARALLEL TO CYLINDER AXIS



Sketch 6.1 Cylinder near plane surface

Interference effects associated with the proximity of a plane surface parallel to the longitudinal axis of a circular cylinder occur in various practical situations such as with pipe lines on the sea bed or with stacks located near a boiler house wall. The proximity of the plane surface not only causes large variations in C_{D0} but also induces significant side forces (C_{L0}) associated with the asymmetry of the flow which acts to repel the cylinder from the surface.

The calculation sheet in Table 10.4 provides a summary of the steps to obtain C_{D0} and C_{L0} given in Figures 11 and 12. These data provide information for an idealised situation; in practice conditions may be significantly different from this and in such circumstances the data should only be used as a guide. Note in particular that V_∞ is the local free-stream velocity which in the presence of other nearby surfaces or obstructions may be significantly different from the undisturbed free-stream velocity.

The information presented in Figures 11 ($h=0$) and 12 ($h>0$) is based on limited sources^{33, 42, 45, 49} giving the mean value of C_{D0} and C_{L0} for a range of values of h/D and Re . The data apply to long cylinders where end effects can be ignored.

Figure 11 deals with the case when the cylinder is resting on the surface. The forces induced are dependent on the thickness of the boundary layer on the surface just upstream of the cylinder location. These boundary layer effects can be approximately accounted for if C_{D0} and C_{L0} are defined as the mean drag or lift force per unit length/ $(\frac{1}{2}\rho V_{eff}^2 D)$ where V_{eff} is a mean effective velocity given by

$$\frac{V_{eff}^2}{V_\infty^2} = \frac{1}{D} \int_h^{D+h} V_z^2 / V_\infty^2 dz \quad (6.1)$$

which, assuming that $V_z/V_\delta = V_z/V_\infty = (z/\delta)^{\bar{\alpha}}$, gives (for $\delta/h \geq 1$)

$$\frac{V_{eff}^2}{V_\infty^2} = \frac{\delta}{D} \frac{1}{2\bar{\alpha} + 1} \left[1 - \left(\frac{h/D}{\delta/D} \right)^{2\bar{\alpha} + 1} \right] + 1 + \frac{h}{D} - \frac{\delta}{D}. \quad (6.2)$$

Values of δ for a turbulent boundary layer on a smooth flat plate can be obtained from Item No. 68020² or from the correlating equation representing these data:

$$\frac{\delta}{D} = \frac{x}{D} \frac{\delta}{x} = \frac{0.078(\log_{10} Re_x)^{-1.06}}{1 - 3.03(\log_{10} Re_x)^{-1.06}} \frac{x}{D}. \quad (6.3)$$