

ELASTIC ADHESIVE STRESSES IN MULTISTEP LAP JOINTS LOADED IN TENSION

1. USE OF THE PROGRAM

1.1 Introduction

1.1.1 General notes

This Item presents a program for the analysis of the distribution of the elastic shear and direct (normal) peel stresses in the adhesive that bonds lap joints of multistep configuration loaded in tension. The program allows for dissimilar adherends of symmetric double lap or asymmetric single lap configuration and assumes that the properties of the adhesive and adherends are elastic. It takes account of the effect of bending in the joint and adjacent material but ignores local stress concentrations in the adherends. The program also provides for calculation of the stresses arising from temperature changes in joints where the adherends have different coefficients of thermal expansion.

Section 1 of the Item gives guidance on the application of the program, instructions for its use and examples of results to be expected, including details of comparative proving tests that have been run.

Section 2 gives supporting information on the theoretical derivation.

The program is available in two formats: (a) a Fortran source code with example input and output files, and (b) an executable version for PC's. Both versions can be found on the Compact disc and on disk in the Sub-series Software Volume.

Guidance on the use of this Data Item and other Data Items on bonded joints is provided in Data Item No. 81022*.

The basis of the Fortran program of ESDUpac A8039 is described in Section 1 of this Data Item. Every reasonable effort has been made to ensure that the program performs the intended calculation satisfactorily. However, if any difficulties are met in using the program, please contact ESDU and we will make every effort to resolve the problem. In common with other software suppliers, ESDU makes no representation as to the suitability or fitness of the program for any particular purpose and no liability for any loss occasioned by any persons as a direct or indirect result of use of the program whether arising from negligence or otherwise can be accepted. In no event shall ESDU or any individuals associated with the development of the program be liable for any damages, including loss of profit or consequential loss, arising out of or in connection with the program.

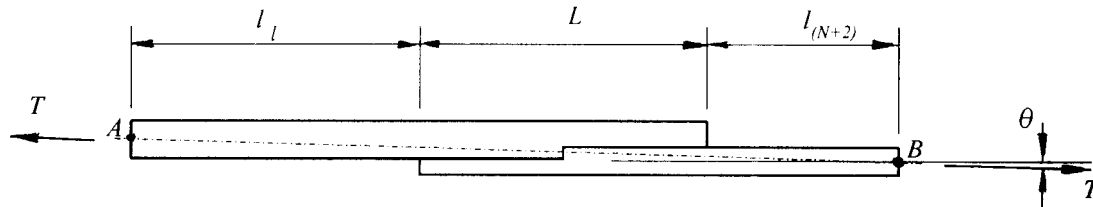
1.1.2 Scope and use of the program

The program is based on the elastic flexible joint analysis of Reference 3 (see Section 2.1.4) as described in Section 2. The analysis determines the distribution of shear stresses and direct (normal) peel stresses in the adhesive resulting from the transmission of in-plane tension loads from one adherend to the other through the adhesive assuming no externally applied bending moments (see Section 2.1).

For single lap joints the stresses in the adhesive are affected by the adherends beyond the bond length, L , that is, lengths l_1 and $l_{(N+2)}$, see Sketch 1.1. The ends of a single lap joint are taken to be the ends marked A and B on Sketch 1.1. The load is applied to the end faces which are free to rotate. The load line is taken to go through the mid-points of the end faces, that is, through points A and B, and is therefore not parallel

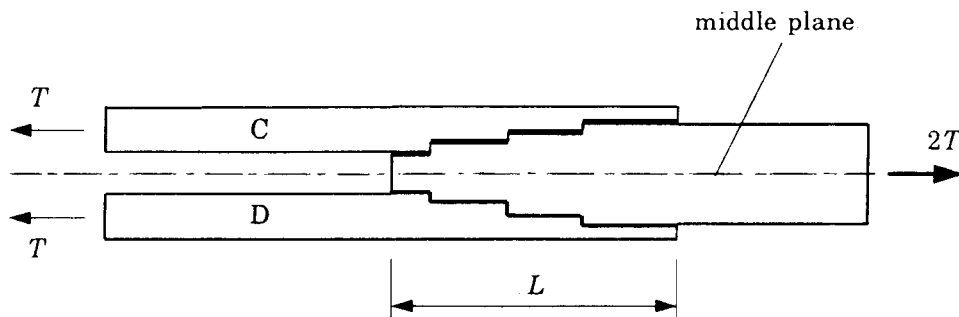
* Data Item No. 81022 "Guide to the use of Data Items in the design of bonded joints".

to the interfaces of the unloaded joint but at the angle θ to them. In practical joints the angle θ is usually very small.

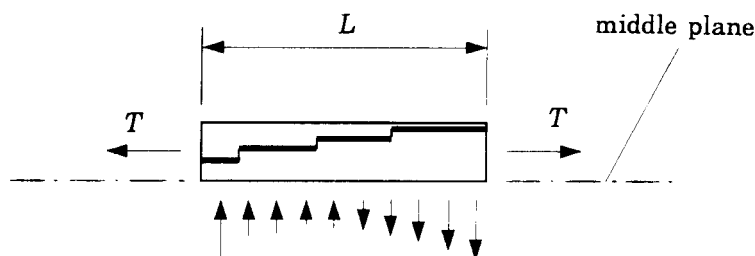


Sketch 1.1 Single lap, two step joint

For double lap joints that are symmetric about a middle plane, see Sketch 1.2, the load line is parallel to the unloaded joint interfaces and the middle plane of the joint is a line of symmetry. Because of this, the lengths of adherend beyond either end of the bond length, L , do not affect the shear or peel stresses calculated by the program. The program may be used to analyse such joints by representing them as one half of the joint subjected to half the total load and selecting the double lap option, see Section 1.3. This removes all exterior bending effects from the joint and constrains the middle plane to remain flat, compare Sketches 1.1 and 1.3. In such representations the force T gives a moment that is balanced by stresses on the middle plane as illustrated in Sketch 1.3.



Sketch 1.2 Symmetric double lap, four step joint



Sketch 1.3 Representation of the symmetric double lap joint of Sketch 1.2 as an asymmetric single lap joint subject to half the total load

Note that double lap joints in which the two adherends C and D, illustrated in Sketch 1.2, are joined to become one solid member to the left of L cannot be analysed accurately using the program. This is because there is continuity between members C and D across the end of the joint which locally constrains the peel stresses and also because, at a short distance beyond the length L , loads T shown in Sketch 1.2 become $2T$ on the middle plane.

The program is provided as an aid to the design of bonded joints. For a joint of given geometry and elastic properties the program will give the shear and direct normal stresses in the adhesive and their relation to the average shear stress in the joint. This allows an assessment of the relative importance of bending effects in the joint (peel effects). It can be particularly useful in helping to establish joint geometry.

The analysis disregards the shear distortion of the adherends in comparison with that of the adhesive and assumes the adhesive behaviour remains elastic. The first assumption may be invalid for fibre reinforced plastics and similar materials; also, in practice the adhesive is likely to behave in an inelastic or non-linear manner. In joints in which the load is such that the adhesive behaves in an inelastic non-linear manner the actual peak stresses in the adhesive are likely to be less than those given by the program. The influence of the inelastic shear properties of the adhesives on bonded joints is considered in Data Item Nos 78042 and 79016*. Many factors affect the cohesive strength of the adhesive and its adhesion to the adherends, including thermal and environmental effects, the method of bonding, curing and surface preparation. These factors should be taken into account in any assessment of the strength of a joint.

1.2 Notation

Both SI and lbf in units are quoted but any coherent system of units can be used.

B, C, d	coefficients of Equations (2.7) and (2.8) (see Section 2.1)		
D	flexural rigidity of adherend	N m	lbf in
E_a	effective modulus of elasticity of adhesive material (see Section 2.1.1)	N/m ²	lbf/in ²
E_z	modulus of elasticity of adherend material in z-direction	N/m ²	lbf/in ²
E_1, E_2	moduli of elasticity of materials of adherends 1 and 2, respectively, in x-direction	N/m ²	lbf/in ²
f_a	direct normal peel stress in adhesive material in z-direction	N/m ²	lbf/in ²
f_1, f_2	peak direct stresses in adhesive material at ends of adherends 1 and 2, respectively, in z-direction	N/m ²	lbf/in ²
G_a	shear modulus of adhesive material	N/m ²	lbf/in ²
G_{xz}	shear modulus of adherends material in z-direction	N/m ²	lbf/in ²
g	effective modulus of an adherend in cylindrical bending ($= E_1 / (1 - \nu_{1x}\nu_{1y})$) or $E_2 / (1 - \nu_{2x}\nu_{2y})$)	N/m ²	lbf/in ²
i	zone number		
l_i	length of i th zone (i varies from 1 to $N + 2$, see Sketch 1.4)	m	in

* Data Item No. 78042 "Shear stresses in the adhesives in bonded joints. Single step double lap joints loaded in tension".
Date Item No. 79016 "Inelastic shear stresses and strains in the adhesives bonding lap joints loaded in tension or shear".

L	total bonded length (zones 2 to $N + 1$, see Sketch 1.4)	m	in
M	bending moment	N m	lbf in
m, n	coefficients of Equation (2.8) (see Section 2.1)		
N	number of bonded (overlapping) steps in joint, $N \leq 6$		
Q	shear force in z-direction	N/m	lbf/in
q_a	shear stress in adhesive in xz plane (see Sketch 1.4)	N/m ²	lbf/in ²
q_{av}	average shear stress in adhesive ($= T/L$)	N/m ²	lbf/in ²
q_1, q_2	peak shear stresses in adhesive material at ends of adherends 1 and 2, respectively	N/m ²	lbf/in ²
T	tensile load on joint per unit width, half load on double lap joint (see Sketches 1.4 and 1.5)	N/m	lbf/in
t	thickness of adherend in balanced single step joint (see Section 2.1)	m	in
t_a	thickness of adhesive layer, constant (see Sketch 1.4)	m	in
t_{1i}, t_{2i}	thickness at zone i of adherends 1 and 2, respectively, where i varies from 1 to $N + 2$ (see Sketch 1.4)	m	in
u, w	displacements in x- and z-directions, respectively	m	in
x	in-plane coordinate along joint length (see Sketch 1.4)		
y	in-plane coordinate across joint width (see Sketch 1.4)		
z	out-of-plane coordinate, normal to plane of joint (see Sketch 1.4)		
$\bar{z}$	position of joint neutral axis measured from upper surface of adherend 1 in z-direction (see Section 2.1)	m	in
α_1, α_2	coefficients of thermal expansion of adherends 1 and 2, respectively, in x-direction	°C ⁻¹	°C ⁻¹
$\Delta\phi$	joint working temperature minus stress free temperature	°C	°C
θ	angle between unloaded joint interface(s) and line of applied load (see Sketch 1.1 and Equation (2.2))	rad	rad
ν_{1x}, ν_{1y} ν_{2x}, ν_{2y}	Poisson's ratios for materials of adherends 1 and 2 in x- and y-directions, respectively		

1.2.1 Suffixes

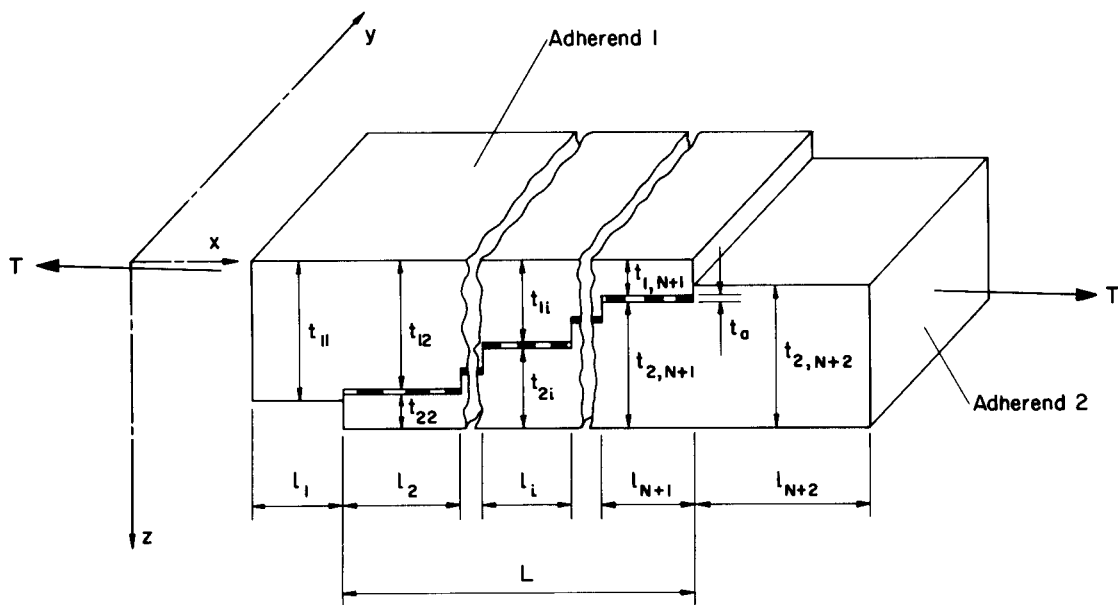
i	i th zone
$1, 2, i$	refers to adherend 1, right hand end of the i th zone
$1, 1, i$	refers to adherend 1, left hand end of the i th zone
$2, 1$ or $2, i$ or $2, i + 1$	refers to adherend 2, first or i th or $i + 1$ zone

1.2.2 Superscripts

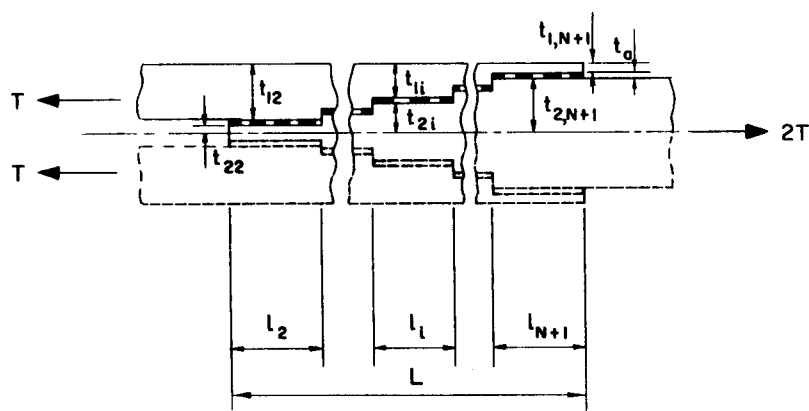
'(prime)	refers to the end of a zone of bonded length
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1.2.3 Direct stress convention

tensile stresses are positive
compressive stresses are negative



Sketch 1.4 Geometry of asymmetric single lap stepped joint



Sketch 1.5 Geometry of symmetric double lap stepped joint

1.3 Data Input Information

Many parameters are required to define a multistep joint and these are needed in the program input. It is therefore recommended that the program is run in the user friendly ESDUview environment. The ESDUview files may be saved and can be quickly and easily accessed later for modification to provide data for analysis of a similar joint. Table 1.1 is given as a guide to the input for the Fortran program outside ESDUview and an example input for this is given in Section 1.5.

Prior to the input of material values 3 lines are provided for run identification details. These lines may be left empty or can carry up to 75 alphanumeric characters each. The contents of the lines appear in the output.

Also prior to the input of materials values, 2 lines are provided for the declaration of the units in which all the input data are to be supplied. These lines may be left empty or can carry up to 75 alphanumeric characters each. These lines also appear in the output. (Note that this is NOT an option to select units but simply the invitation to state the units used in the input data and to echo this statement in the output.)

For the purposes of the input the program treats the geometry in the form of a single lap joint with the adhesive joining adherends 1 and 2 as shown in Sketch 1.4. For a double lap joint the values entered should therefore be the values of the top half of the joint viewed about the axis of symmetry as shown in Sketch 1.5. Note that the lengths l_1 and l_{N+2} do not influence the double lap joint and are therefore not required in the input for such joints. The load in this case will be half the total load on the double lap joint. The total length of the structure, including for single lap joints the unbonded (free) lengths of the adherends beyond the ends of the joint, is partitioned into zones of constant geometry numbered 1 to $N + 2$ from the left as shown in Sketches 1.4 and 1.5. The positions of adherends 1 and 2 relative to the zone number are not interchangeable; zone 1 begins at the thicker end of adherend 1. It should be noted that t_{21} and $t_{1(N+2)}$ will always be zero.

When using the program the limitations on its use set out in Section 1.1.2 should be carefully considered. When considering temperature effects it should be noted that the program will not compute with a tensile load, T , of zero and the introduction of a small nominal load is therefore necessary to obtain a solution.

A consistent set of units must be used throughout (for example, if the thicknesses and lengths are in mm, E should be in N/mm^2 and consequently the stress will be in N/mm^2).

1.4 Output Data Interpretation

In a successful run the output header is followed by a list of all the entered data including;

- (1) type of joint and number of steps,
- (2) materials properties of adherend 1, adherend 2 and the adhesive,
- (3) joint direct and thermal loading,
- (4) joint geometry.

The shear stress and direct stress distributions are then given in two forms. First the actual values of the shear stresses and direct stresses are given at the ends of 20 sub-divisions of each step in turn. The stress distributions are then repeated for the same number of sub-divisions as ratios of the average shear stress, T/L , in the following columns.

Examples of the computer output are given in Section 1.5. Examples of the results to be expected for various configurations are given in Section 1.6.

During the course of reading the input data the program checks each value. With the exception of the temperature, $\Delta\phi$, all negative values are rejected. In these cases a note saying which value is negative is output and the program is terminated. The control parameters indicating the joint type and the number of steps are also checked. If they are not valid an appropriate note is output and the program is terminated. Similarly, if the applied load is input as zero then a note to this effect is output and the program is terminated.

TABLE 1.1 PROGRAM INPUT DATA (see Section 1.3)

Entry order	Enter the numerical values for the quantities indicated in free format. Each entry must be in the order given and made on a separate line with no blank lines in between (except Entries 1 and 2). The values given for each entry must have a space or comma between them.
(1)	Enter run identification data , input file title or notes. Three lines 75 characters wide allowed. Lines may be left blank.
(2)	Enter description of units of input (see Section 1.3). Two lines 75 characters wide allowed. Lines may be left blank.
(3)	The number of steps. Enter: N Note that $1 \leq N \leq 6$.
(4)	Selection of joint type. Enter: 1 for an asymmetric single lap joint. 2 for a symmetric double lap joint.
(5)	First adherend material properties. Enter: $E_1, \nu_{1x}, \nu_{1y}, \alpha_1$
(6)	Second adherend material properties. Enter: $E_2, \nu_{2x}, \nu_{2y}, \alpha_2$
(7)	Adhesive properties, thickness and temperature difference. Enter: $E_a, G_a, t_a, \Delta\phi$
(8)	Joint geometry. For single lap joints: For each step enter: l_i, t_{1i}, t_{2i} , where i varies from 1 to $N + 2$. Note that $t_{21} = 0$ and $t_{1(N+2)} = 0$ in all cases. For double lap joints: For each step enter: l_i, t_{1i}, t_{2i} where i varies from 2 to $N + 1$.
(9)	Applied load. Enter: T Note that T is the full load on single lap joints and 1/2 the applied load on double lap joints. Also T must be positive and cannot be zero.

1.5 Example

It is required to determine the adhesive stresses in a two step single lap bonded joint subject to a tensile load of 510 N/mm. The joint is both cured and worked at room temperature so that no temperature effects are present. The joint has the following geometry and properties.

Joint geometry (see Sketch 1.4)

Number of steps: $N = 2$.

Unbonded length of adherend 1 (zone 1)

$$l_1 = 144 \text{ mm}, t_{11} = 5.1 \text{ mm}, t_{21} = 0.$$

First step (zone 2)

$$l_2 = 36 \text{ mm}, t_{12} = 5.1 \text{ mm}, t_{22} = 2.55 \text{ mm}.$$

Second step (zone 3)

$$l_3 = 36 \text{ mm}, t_{13} = 2.55 \text{ mm}, t_{23} = 5.1 \text{ mm}.$$

Unbonded length of adherend 2 (zone 4)

$$l_4 = 144 \text{ mm}, t_{14} = 0, t_{24} = 5.1 \text{ mm}$$

Adherend Properties

$$E_1 = E_2 = 210\,000 \text{ N/m}^2.$$

$$\nu_{1x} = \nu_{1y} = \nu_{2x} = \nu_{2y} = 0.3.$$

$$\alpha_1 = \alpha_2 = 2.3 \times 10^{-5} \text{ }^\circ\text{C}^{-1}.$$

Adhesive Properties

$$E_a = 2130 \text{ N/mm}^2.$$

$$G_a = 642 \text{ N/mm}^2.$$

$$t_a = 0.1 \text{ mm}.$$

From the given data the applied load is 510 N/mm and $\Delta\phi = 0$. Section 1.5.1 gives the input file; note that the units are newtons and millimetres throughout. Section 1.5.2 gives the results of running the program with the input file of Section 1.5.1.

1.5.1 Input for Example

Example 1

Data Item No. 80039.

August 1995

Force: Newtons

Length: mm

2

1

210000.0	0.3	0.3	2.3e-5
210000.0	0.3	0.3	2.3e-5
2130.0	642.0	0.1	0.0
144.0	5.1	0.0	
36.0	5.1	2.55	
36.0	2.55	5.1	
144.0	0.0	5.1	
510.0			

1.5.2 Results of running ESDUpac A8039 with the input file of Section 1.5.1

ESDU International plc

Program A8039

ESDUpac Number: A8039V10

ESDUpac Title: ELASTIC ADHESIVE STRESSES IN MULTISTEP LAP JOINTS
LOADED IN TENSION.

Data Item Number: 80039

Data Item Title: ELASTIC ADHESIVE STRESSES IN MULTISTEP LAP JOINTS
LOADED IN TENSION.

ESDUpac Version: 1.0 Issued November 1995

See Data Item for full input/output specification and interpretation.

INPUT DATA

Example 1

Data Item No. 80039.

August 1995

Force: Newtons

Length: mm

No errors detected in the input data.

Asymmetric single sided stepped lap joint with 2 steps.

FIRST ADHEREND

Thickness T1 = .5100E+01

Modulus E1 = .210E+06

Poissons ratio x-direction nu1x = .300E+00

Poissons ratio y-direction nu1y = .300E+00

Coefficient of expansion alpha1 = .230E-04

SECOND ADHEREND

Thickness T(N+2) = .5100E+01

Modulus E2 = .210E+06

Poissons ratio x-direction nu2x = .300E+00

Poissons ratio y-direction nu2y = .300E+00

Coefficient of expansion alpha2 = .230E-04

ADHESIVE

Thickness Ta = .1000E+00

Modulus Ea = .213E+04

Shear modulus Ga = .642E+03

TEMPERATURE DIFFERENCE Working

minus stress free temp. Delta0 = .000E+00

JOINT GEOMETRY

Position	Length	Thickness adherend 1	Thickness adherend 2
End	.144E+03	.510E+01	.000E+00
Step 1	.360E+02	.510E+01	.255E+01
Step 2	.360E+02	.255E+01	.510E+01
End	.144E+03	.000E+00	.510E+01