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Free vibration of thin-walled, orthogonally stiffened, circular- cylindrical shells

Associated software: ESDUpac A8040
ESDUpac B8040



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THE PREPARATION OF THIS DATA ITEM

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FREE VIBRATION OF THIN-WALLED, ORTHOGONALLY STIFFENED, CIRCULAR-CYLINDRICAL SHELLS

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1. NOTATION

		Units	
		SI	British
A_r	ring cross-sectional area	m ²	in ²
A_s	stringer cross-sectional area	m ²	in ²
b_r	ring pitch	m	in
b_s	stringer pitch	m	in
C_1, C_2, C_3, C_4	non-dimensional amplitude		
D	flexural rigidity of shell given by $Et^3/[12(1-\nu^2)]$	N m	lbf in
E	Young's modulus of shell material	N/m ²	lbf/in ²
E_r	Young's modulus of ring material	N/m ²	lbf/in ²
E_s	Young's modulus of stringer material	N/m ²	lbf/in ²
e_r	ring eccentricity (distance from shell middle surface to ring centroid, positive for internal ring)	m	in
e_s	stringer eccentricity (distance from shell middle surface to stringer centroid, positive for internal stringer)	m	in
f	shell natural frequency	Hz	Hz
f_{min}, f_{max}	minimum and maximum frequencies of frequency range to be inspected	Hz	Hz
f_r	free ring natural frequency	Hz	Hz
G	shear modulus of shell material	N/m ²	lbf/in ²
G_r	shear modulus of ring material	N/m ²	lbf/in ²
G_s	shear modulus of stringer material	N/m ²	lbf/in ²
I_r	second moment of area of ring with respect to shell middle surface	m ⁴	in ⁴

I'_r	second moment of area of free ring with respect to principal axis normal to plane of ring (<i>i.e.</i> in-plane bending second moment of area)	m^4	in^4
I_s	second moment of area of stringer with respect to shell middle surface	m^4	in^4
J_r	torsional constant of ring	m^4	in^4
J_s	torsional constant of stringer	m^4	in^4
j	torsional mode number		
L	shell length	m	in
M	mass per unit area [see Equation (3.3)]	kg/m^2	$\text{lb s}^2/\text{in}^3$
m	axial mode number		
N_s	number of stringer stiffeners		
N_r	number of ring stiffeners (see Section 6.2)		
N_x	uniform axial force per unit length ($N_x = P/2\pi R$, tension positive)	N/m	lbf/in
n	circumferential mode number (number of circumferential waves)		
P	total initial axial force (tension positive)	N	lbf
p	pressure difference across shell (positive inwards)	N/m^2	lbf/in^2
R	shell mean radius	m	in
R_r	free ring mean radius	m	in
t	shell thickness	m	in
u	axial displacement/ R		
v	circumferential displacement/ R		
w	radial displacement/ R		
x^*, y^*, z^*	orthogonal curvilinear coordinate system (see Sketch 1.1)		
x, y	non-dimensional coordinates ($x^*/R, y^*/R$ respectively)		
Z	parameter defined in Equation (3.2)	m/s	in/s
δ_f	increment between successive test frequencies (step size)	Hz	Hz
ν	Poisson's ratio of shell material		

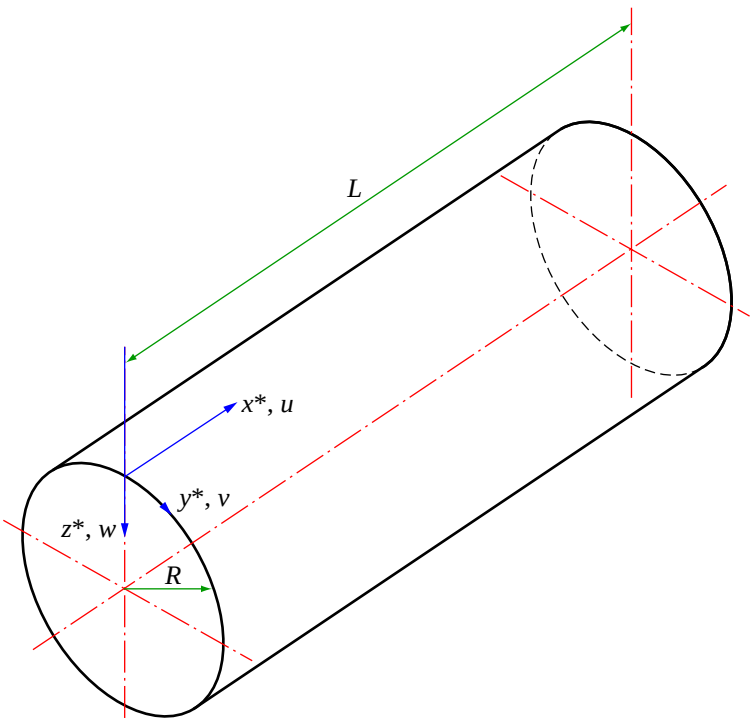
ρ	shell material density	kg/m^3	$\dagger$
ρ_r	ring material density	kg/m^3	$\dagger$
ρ_s	stringer material density	kg/m^3	$\dagger$

Shell end conditions

C	denotes clamped end
F	denotes free end
SD	denotes shear diaphragm end

Additional notation is given in Appendix A, Section A.1.

$\dagger$ A density value expressed in British units as pounds per cubic inch has to be divided by 386.4 before being used in this Item. (A force of 1 lbf acting on a mass of 1 lb produces an acceleration of 386.4 in/s².) For example, if the density is 0.097 lbf/in³, the value input into the computer program should be 2.51×10^{-4} lb s²/in⁴.



Sketch 1.1 Shell coordinate system

2. INTRODUCTION

Data on the free vibration of shells are an essential input in many analyses of forced vibration and flutter. The primary purpose of this Item is to provide modal data for circular-cylindrical, uniform shells, with either uniform stringer stiffeners or uniform ring stiffeners, or a combination of both types of stiffener, which can be modelled as homogeneous orthotropic shells. Such modelling is usually applicable when the stringers are uniformly and closely pitched around the shell circumference and the rings are uniformly and closely pitched along the shell longitudinal axis. The data provided should be used as guidelines to practical shell natural frequencies and mode shapes. When the orthotropic shell model is not applicable, for instance for stiffened shells that have changes in stiffness of either shell or stiffeners along their lengths or have irregularly or sparsely pitched stiffeners, finite element methods or other means, some of which are referred to in this Item, should be used for vibration analysis.

This Data Item should be used in conjunction with ESDU 78004 (Natural frequencies of thin-walled isotropic, circular-cylindrical shells). It is impossible to provide data for stiffened shells in a graphical form similar to that used for isotropic shells because of the larger number of parameters involved. Instead this Item provides computer programs for the estimation of the lower natural frequencies, and associated mode shapes, of the flexural modes of vibration of ring and stringer stiffened circular-cylindrical shells. A means of estimating the torsional natural frequencies and mode shapes for orthogonally stiffened shells is also given. The data provided are applicable to initially unstressed shells or to shells having initial stress due to static axial or pressure loading. All combinations of clamped, shear diaphragm and free shell end conditions are considered.

In Section 3 general information is given on the modes of free vibration of shells, and the effects on their vibrational characteristics of adding stiffeners and applying external static loading are discussed. The data given, and the methods of analysis used, are described in Section 4 together with information on the limitations of these analysis methods.

Details of two computer programs, known as ESDUpacs A8040 and B8040[†], are given in Section 5; and guidance on the use of these programs is given in Section 6. An example, with computer input and output data, which illustrates the application of the Item is given in Section 8.

3. NOTES ON NATURAL MODES OF VIBRATION OF STIFFENED SHELLS

3.1 General

The information and data provided in this Item are specifically for orthogonally stiffened shells having either uniformly distributed longitudinal stiffeners (stringers) or circumferential stiffeners (rings), or a combination of both. Each type of stiffener is considered to be of uniform cross-section and continuous along the length, or circumference, of the shell. Unless specifically mentioned the shells are treated as being homogeneous and orthotropic with eccentricity effects included.

[†] The full ESDUpac number has an appended V followed by two digits. The digits indicate the ESDUpac version number. For example, the full ESDUpac number for version 2.0 of ESDUpac A8040 is A8040V20. The version appendage will not be used when referring to ESDUpacs in the text of this Item. See Introduction to ESDUpacs for a full description of the ESDUpac numbering system. The current version of each ESDUpac is given under the heading to the related Appendix B or C and on the appropriate example outputs in Sections B.4 and C.5

3.2 Boundary Conditions

The boundary conditions considered at the shell ends are all combinations of shear diaphragm (SD)[†], clamped (C) and free (F).

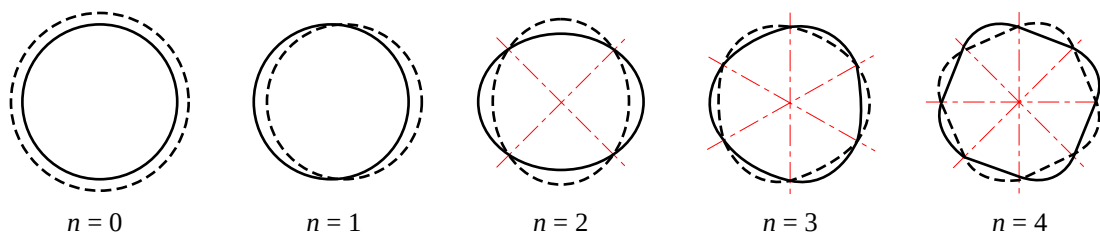
For the shear diaphragm end condition the shear diaphragm is assumed to be thin and stiff only within its own plane. Because the shear diaphragm is thin it has little stiffness in the shell axial direction and generates negligible bending moment about a tangent to the shell circumference at the shell-to-diaphragm junction; also negligible longitudinal membrane force is generated in the shell at the shell-to-diaphragm junction. The in-plane stiffness of the diaphragm ensures that there is no radial or tangential displacement of the shell surface at its ends.

The physical restraints which apply to the various end conditions considered in this Item are defined in Appendix A.

3.3 Mode Numbers

3.3.1 Circumferential mode number (n)

The circumferential mode number is defined for all shell end conditions as the number of circumferential waves around the shell circumference ($n = 0, 1, 2, \text{etc.}$). The circumferential mode shapes for the lower values of n are shown in Sketch 3.1. Such simple mode shapes will not in general apply to shells when the number of discrete stiffeners is small compared with the modal number.



Sketch 3.1 Circumferential mode shapes

For all sets of end conditions there are $2n$ longitudinal nodal lines on which radial displacements are zero. In general, shells do not have surface lines on which the radial, longitudinal and circumferential displacements are all zero.

3.3.2 Axial mode number (m)

The axial mode number, m , is defined as one plus the number of circumferential nodal circles ($m = 1, 2, 3, \text{etc.}$). In this definition the supported shell ends are not counted as nodes and the nodal circles are lines of zero radial displacement.

In the case of the SD–SD orthotropic shell the wave form is sinusoidal and the axial mode number is equal to the number of half waves in the length of the shell. This is not necessarily true for shells having discrete ring stiffeners, particularly if the stiffeners are sparsely distributed.

[†] In other technical literature the shear diaphragm condition may be referred to as “simply-supported” or “freely-supported”. However these terms are not strictly applicable to shells because they do not indicate that there is zero radial and tangential displacement of the shell surface at its ends.

3.3.3 Torsional mode number (j)

The torsional mode number, j ($j = 1, 2, 3, \text{etc.}$), defines the order in which torsional natural frequencies occur.

3.4 Natural Modes of Vibration

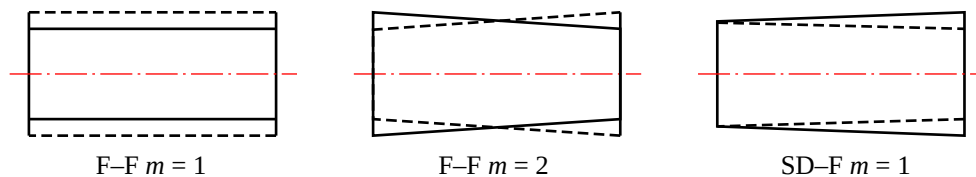
3.4.1 Coupled modes

For each combination of m and n there are three natural frequencies of vibration. The modes associated with each of these frequencies correspond to motion which is primarily radial, axial or circumferential. The lower natural frequencies are usually associated with a motion which is primarily radial.

3.4.2 Low frequency modes for free-free and free-shear diaphragm shells

For unstiffened shells, inextensional modes exist in which the middle surface of the shell deforms without stretching ($n \geq 2$). For this condition to apply to circular-cylindrical shells the shell generating line must remain straight during vibration.

For the case of a F-F shell there are two sets of such modes of vibration for which approximate natural frequencies may be derived. These modes for the unstiffened shell are referred to as the Rayleigh modes ($m = 1$) and the Love modes ($m = 2$). The $m = 1$ modes of the F-SD shell may also be derived from inextensional theory. The mode shapes for the three inextensional theory frequencies are shown in Sketch 3.2. For stiffened shells, similar modes exist but are not necessarily inextensional.



Sketch 3.2 Inextensional mode shapes

3.4.3 Torsional modes of vibration

In the torsional modes of vibration of circular-cylindrical shells there are no radial or longitudinal deflections so that cross-sections remain circular. There are three types of boundary conditions to be considered for shell torsional vibration modes. Using the smeared stiffener theory (see Section 4.2) the natural frequencies and mode shapes for these boundary conditions may be estimated using the following relationships (Derivation 8).

- (i) For C-C and SD-SD boundary conditions the torsional natural frequencies are given by

$$f = \frac{jZ}{2L}, \quad (3.1)$$

where

$$Z = \left[\frac{1}{M} \left\{ Gt \left(1 + \frac{t^2}{4R^2} \right) + \frac{G_s J_s}{R^2 b_s} + N_x \right\} \right]^{1/2} \quad (3.2)$$

and M is the mass per unit area of the stiffened shell given by

$$M = \rho t + \frac{\rho_s A_s}{b_s} + \frac{\rho_r A_r}{b_r}. \quad (3.3)$$

For thin-walled shells

$$\left(1 + \frac{t^2}{4R^2}\right) \approx 1;$$

also the effect of N_x on torsional natural frequencies is small, so that for practical shells the equation for Z may be written as

$$Z = \left[\frac{1}{M} \left\{ Gt + \frac{G_s J_s}{R^2 b_s} \right\} \right]^{1/2}. \quad (3.4)$$

The mode shape in terms of v , the non-dimensional[†] tangential displacement of the shell median surface is of the form

$$v = C_1 \sin \frac{j\pi R x}{L}, \quad (3.5)$$

where x is the non-dimensional distance along the shell and C_1 is an amplitude ratio.

- (ii) For F–F boundary conditions the torsional natural frequencies are the same as those for the C–C and SD–SD boundary conditions, *i.e.*

$$f = \frac{jZ}{2L}.$$

The mode shape is of the form

$$v = C_2 \cos \frac{j\pi R x}{L}, \quad (3.6)$$

where C_2 is an amplitude ratio.

- (iii) For C–F and SD–F boundary conditions, when the free end is at $x = L$, the torsional natural frequencies are given by

$$f = \frac{Z}{4L}(2j-1), \quad (3.7)$$

[†] In this Item linear dimensions are non-dimensionalised by dividing by R .