

STRESS CONCENTRATION FACTORS AXIALLY LOADED LUGS WITH CLEARANCE-FIT PINS

1. NOTATION

a	distance from centre of hole to lug end (see Sketches 1.1a and 1.1b)	m	in
d	diameter of hole	m	in
e	pin-to-hole clearance as a percentage of d		
E_l, E_p	moduli of elasticity of lug and pin materials, respectively	N/m ²	lbf/in ²
f_{max}	maximum tensile stress in lug around hole perimeter	N/m ²	lbf/in ²
K'_e	stress concentration factor at e per cent clearance for lugs with t/d less than 0.5, defined by	N/m ²	lbf/in ²
$f_{max} = K'_e f_{ref}$ where $f_{ref} = \frac{P}{(W - d)t}$ (Thus $K'_{0.5}$ relates to 0.5 per cent clearance between pin and hole and K'_{100} is used for the limiting case of point loading. Note that for round-ended lugs f_{ref} remains as defined here even when $a < W/2$.)			
K''_e	value of K'_e when t/d is greater than 0.5 (see Section 2.5)		
K_s	stress concentration factor for waisted lug with flat and slot (related to K_{wf} , see Section 2.4)		
K_w	stress concentration factor for waisted lug (related to particular value of K'_e , see Section 2.4)		
K_{wf}	stress concentration factor for waisted lug with additional flat (related to a particular value of K'_e , see Section 2.4)		
L	distance between lug hole and slot in waisted lug (see Section 2.4)	m	in
P	tensile load on lug applied via pin	N	lbf
t	thickness of lug (see Sketch 1.1c)	m	in

W width of lug (see Sketches 1.1a and 1.1b) m in

η_e correction factor for clearance e based on $K'_{0.2}$ and defined by

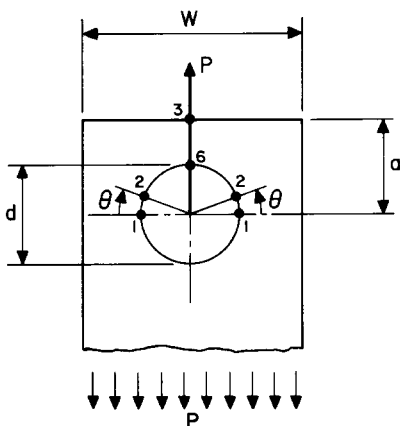
$$\eta_e = \frac{K'_e - K'_{0.2}}{K'_{100} - K'_{0.2}}$$

so that

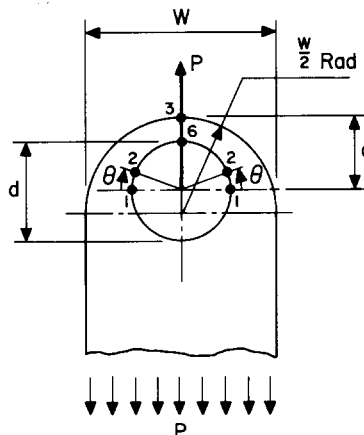
$$K'_e = K'_{0.2} + \eta_e (K'_{100} - K'_{0.2})$$

θ angle indicating position of f_{max} (see Sketches 1.1a and 1.1b) degree degree

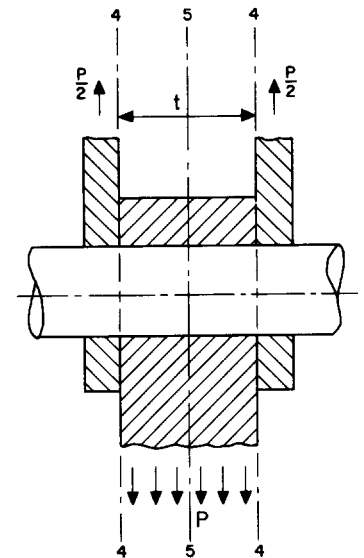
Both SI and British units are quoted but any coherent system of units may be used.



Sketch 1.1a Square-ended lugs



Sketch 1.1b Round-ended lugs



Sketch 1.1c Section through lug assembly centre line

2. GENERAL NOTES

This Item provides the maximum elastic stress concentration factors at the perimeter of the hole in lugs loaded axially by single central clearance-fit pins. In Figures 1 to 3 data are provided for thin square- and round-ended plain lugs to which the following notes up to Section 2.3 apply. In Figures 5, 6, 7 and 8 data are given on particular examples of thin waisted round-ended lugs as guidance on the stress concentrations to be expected for such geometries. The notes in Section 2.4 qualify the use of the waisted lug data and their relationship to the data for plain lugs. In Figure 4 data on the effect of lug thickness are given, notes on the use of this figure being given in Section 2.5.

In addition to lug geometry the pin/hole clearance significantly affects the stress concentration. For both square- and round-ended lugs having perfect-fit pins the theoretical position of f_{max} is at Points 1 shown in Sketches 1.1a and 1.1b. As clearance between the pin and hole is introduced the value of f_{max} increases and its position moves. Thus for most practical good quality clearance-fits f_{max} occurs at Points 2 on the sketch for which θ is typically between 10 and 35 degrees. Section 2.3 provides data that may be used to account for pin-to-hole clearances when estimating K'_e . Note that if $(a - d/2)$ is small then significant local bending of the thin section $(a - d/2)$ wide will occur so transferring the maximum tensile stress in the lug to Point 3 on the outside of the lug. This will only occur outside the range of geometries covered by this Item.

Figures 1 and 2 provide curves for the limiting condition of $a/W = \infty$. In practice this limit applies at values of a/W in excess of about 1.5. For lugs with $t/d < 0.5$, variation in the ratio E_p/E_l between 1.0 and 3.0 has no significant effect on K'_e . See Section 2.5 for lugs with $t/d > 0.5$.

2.1 Square Ended Lugs with $t/d < 0.5$

Figure 1 provides curves of K'_e plotted against d/W for three values of a/W . Two sets of curves are provided and each set relates to a different quality pin-fit. The dashed curves relate to very small clearances between pin and hole, such as would normally only be obtained in laboratory specimens, that is by careful manufacture and individual pairing of lug-hole and pin. Typically $e < 0.1$ per cent for such cases which therefore represent a lower practical limit to the stress concentration factor. The solid curves relate to assemblies with pin clearances of about 0.2 per cent of d . These are therefore $K'_{0.2}$ values. An estimate of the stress concentration factor for the upper limit case of point loading, that is K'_{100} , may be obtained from Figure 2, see Section 2.2. (Note that point loading is strictly a line loading applied uniformly across the thickness of the lug at location 6, indicated on Sketches 1.1a and 1.1b.)

2.2 Round-Ended Lugs with $t/d < 0.5$

Figure 2 provides curves of K'_e plotted against d/W for three values of a/W . The chain dotted curves relate to point loading and are therefore K'_{100} values. The solid lines relate to pin-fits having about 0.2 per cent clearance and are therefore $K'_{0.2}$ values. (The solid lines strictly relate to a sinusoidal type loading applied to the hole. Comparison of these data with Figure 1 and with experimental results shows that they approximate to 0.2 per cent clearance.)

2.3 Effect of Pin-to-Hole Clearance, $t/d < 0.5$

Figure 3 provides values of η_e plotted against e . The data are based on a datum value of $K'_{0.2}$, that is, the solid curves of Figures 1 and 2, and strictly relate to round-ended lugs with $d/W = 0.5$ or 0.7 and with $a/W = 0.5$. The curve of Figure 3 may be used to provide an indication of the effect of e on the stress concentration factor for other geometries and lug forms. However, within set tolerances the effect of non-circularity coupled with surface finish effects can lead to considerable variation in η_e about the value obtained from Figure 3. This is particularly so when $e < 0.1$, that is, over the dashed portion of the curve. For this reason the dashed portion of the curve should be considered tentative. At the other end of the scale Figure 3 shows that conditions similar to point loading apply at clearances greater than about 10 per cent.

2.4 Waisted Round-Ended Lugs, $t/d < 0.5$

Figures 5, 6 and 7 give the elastic stress concentration factors for the particular waisted lug geometries shown in the respective figures. They are based on the photoelastic tests of Derivation 3 supported by data from Derivation 1 in particular. While limited to $d/W = 0.5$ they give guidance on the effect that the geometrical variations shown will have on the lug stress concentrations K_w and K_{wf} for a range of a/W , and K_s for $a/W = 0.5$ (for which alone data were available).

In Figures 5 and 6 the stress concentrations K_w and K_{wf} are respectively related to $K'_{0.27}$ for round-ended lugs, where $K'_{0.27}$ is taken from the tests in the same derivation for consistency. The difference between values of $K'_{0.27}$ from Derivation 3 and values of $K'_{0.27}$ obtained using Figures 2 and 3 of this Item are shown in Figure 8. Figure 8 is provided only to support the application of the factors given in Figures 5 and 6 to the values of K'_e obtained from Figures 2 and 3 which have a more sound basis statistically. In Figure 7 values of the stress concentration K_s due to the introduction of a long slot (slot with length to width ratio ≥ 2.5) in the waisted section are related to values of K_{wf} from Figure 6 to which they may be directly applied to obtain the stress concentration for a slotted, waisted lug since

$$\frac{K_s}{K'_{0.27}} = \frac{K_s}{K_{wf}} \times \frac{K_{wf}}{K'_{0.27}}.$$

For waisted lugs (with and without a slot) with a clearance e of 0.27 per cent the position of f_{max} is close to the Point 1 of Sketch 1.1a and for large a/W may even move beyond it towards the waisted section. For looser fit pins the position of f_{max} will move towards Point 2 of Sketch 1.1a, as in the case of plain lugs. While the data relate strictly to $d/W = 0.5$ and $e = 0.27$ tests for other values of d/W and e in Derivations 1 and 3 indicate that effects similar to those shown are likely to occur within the ranges $0.3 < d/W < 0.6$ and $0.2 < e < 1.5$.

2.5 Effect of Lug Thickness $t/d > 0.5$

Figure 4 provides curves of K''_e/K'_e plotted against t/d for lugs with small clearance between pin and hole and with negligible clearance between the loading fork ends and the sides of the lug. These data strictly apply to square-ended lugs with $d/W = 0.45$ and $a/W = 0.67$ but may be used as a guide to the effect of t/d on both round- and square-ended lugs with d/W between about 0.3 and 0.6. The solid line relates to assemblies with $E_p/E_l = 1.0$. Only one data point exists for $E_p/E_l = 3.0$, at $t/d = 2.24$. The dashed line, which is of the same form as the solid line, has been drawn through this single data point and should therefore be considered tentative.

The major effect of increasing t/d is to induce pin bending which increases the loading at the ends of the hole (cross sections 4-4 on Sketch 1.1c) and relieves it over the central portion of the hole (cross section 5-5 on Sketch 1.1c). Note that below t/d of about 0.5 the stress distribution through the lug is sensibly uniform; hence the limits stated in the titles of Sections 2.1, 2.2, 2.3 and 2.4.

The use of radiused corners for the lug hole or the introduction of clearance between the faces of the lug and the loading fork will allow further pin bending thus increasing K''_e/K'_e above the value shown on Figure 4. The only data available on this effect relate to an assembly with $E_p = E_l$, $t/d = 2.24$ and corner radii of $0.18d$ (equal to $0.083t$) which gave $K''_e/K'_e = 2.6$, compared with 2.07 where no radius existed. Thus, the effect of the corner radii and loading fork clearances should be taken into account when estimating K''_e .

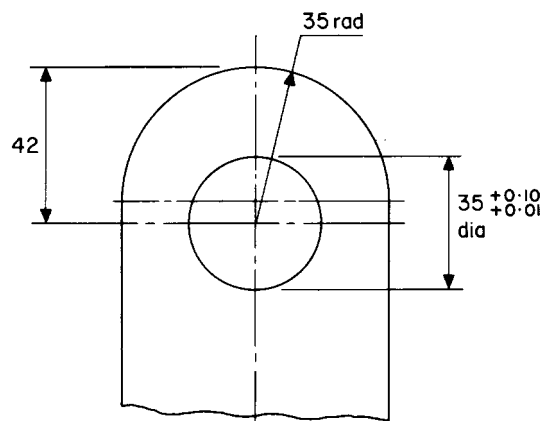
The data may be used as a guide to the asymmetric stress concentration factor in fork ends loading a lug. However, as Figure 4 shows, pin bending has a significant effect on the stress concentration and for the asymmetric loading experienced by one side of a fork this effect may be expected to be more severe.

3. DERIVATION

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4. EXAMPLES

- 4.1 It is required to estimate the maximum stress concentration factor acting at the lug shown in Sketch 4.1. The pin is to be $35_{-0.01}^{+0.08}$ mm diameter.



Sketch 4.1 All dimensions in mm

The maximum clearance will be $0.10 + 0.08 = 0.18$ mm.

Thus,

$$e = \frac{0.18 \times 100}{35} = 0.51.$$

Also,

$$\frac{d}{W} = \frac{35}{35 \times 2} = 0.5$$

and $\frac{a}{W} = \frac{42}{35 \times 2} = 0.6.$

Thus, from Figure 2 for round-ended lugs, $K'_{0.2} = 2.68$ and $K'_{100} = 3.61$.

From Figure 3, $\eta_e = 0.22$ so that, from the definition of η_e given in the notation, the maximum stress concentration is estimated as

$$K'_e = 2.68 + 0.22(3.61 - 2.68) = 2.88.$$

Repeating the calculation for the minimum clearance ($e = 0.06$) gives a minimum stress concentration of $K'_e = 2.46$.

- 4.2** It is required to know the likely effects of waisting and slotting on the maximum elastic stress concentration factor for the lug considered in example 4.1. The dimensions given in that example remain the same, dimensions relating to the waisting and slotting are those given in the diagrams in Figures 5, 6 and 7, where $L = 17.5$ mm.

Figures 5, 6 and 7 are valid for a clearance of $e = 0.27$ which falls between the minimum and maximum clearance of the lug, $e = 0.06$ and $e = 0.51$, respectively. The three forms of lug geometry shown in Figures 5, 6 and 7 will be considered in turn.

- (a) Figure 5 lug form, waisted lug with no additional side flat or slot.

From Figure 5, when $\frac{a}{w} = 0.6$,

$$\frac{K_w}{K'_{0.27}} = 1.26.$$

This means an increase of 26 per cent in the elastic stress concentration should be expected. While $e = 0.27$ in this expression, a better estimate of the possible maximum stress concentration factor will be obtained using $K'_{0.51}$ from Example 4.1.

Hence $K_{w(max)} = 2.88 \times 1.26 = 3.63.$

- (b) Figure 6 lug form, waisted lug with additional side flat but no slot.

From Figure 5, when $\frac{a}{w} = 0.6$,