

THE STABILITY AND RESPONSE OF LINEAR SYSTEMS

PART VI: SELECTION OF AN ASSESSMENT METHOD

1. INTRODUCTION

This Item is the sixth in a group dealing with methods of assessing the stability and response of linear systems. Part I, Item No. 74019, introduces the mathematical description of linear systems, both with and without feedback, and considers the assessment of stability by the use of the Routh-Hurwitz Criteria. Part II, Item No. 74020, considers methods of graphical presentation that provide means of displaying the effects on the characteristics of the system of varying some of its parameters. Part III, Item No. 74021, treats the methods of system assessment that are based on frequency response. Part IV, Item No. 80002, considers the specification and measures of performance that may be used in conjunction with these methods of assessment, and Part V, Item No. 80018, deals with the function of the different system elements and with design techniques used to construct a system with the required characteristics.

The present Item considers the factors that, in particular circumstances, affect the selection of a specific method of system assessment. The notes on the selection criteria are drawn directly from Parts IV and V, and the text on the methods of assessment is from Parts II, III and V. Section 2 explains how an initial method of system assessment can be chosen, and subsequently how this choice may change as the system is designed. In Sections 3 and 4, the advantages and disadvantages of the root and logarithmic/polar frequency response methods are detailed. These advantages and disadvantages (when applied to assessment and design problems) are discussed further in Section 5 where outline design procedures are compared. The advantages and disadvantages of synthesis methods are not set out in this Item. However, some introductory notes on these methods are given in Section 4.1.5 of Part V.

2. SELECTING A METHOD OF ASSESSMENT[†]

The initial choice of method must be made by ascertaining how far each method can reveal what is required to be known about the system and how quickly the task can be completed. The task of system design or synthesis will often be simplified by using different methods at various stages in the investigation, controller specification, and controller design cycle. In the first instance, the method with the best investigative characteristics is required.

The three basic requirements of the assessment method are:

- (i) that it should be compatible with the presentation of the mathematical model (see Part V);
- (ii) that it should conveniently display the required measures of system performance (see Part IV);
- and (iii) that it should at the same time show the consequences of varying any system parameters (generally the forward and/or feedback path gains).

[†] Users unfamiliar with the control of dynamic systems should read Section 3 of Part V on The Elements of Control Systems before proceeding.

First basic requirement

From the first requirement, when the open-loop transfer function is defined by a factored rational function there are two methods suitable for the initial assessment of the system; the root locus sketching rules which will determine the closed-loop characteristics, or the Bode straight line approximations which may be used to plot the open-loop frequency response. Where the frequency response method of analysis is selected, a decision must be made as to how to determine the closed-loop characteristics for those systems which do not have a single pure-gain feedback path (for simple systems either the Nichols or inverse Nyquist diagrams are the most expedient). If this feedback path transfer function is fixed, that is that control is to be effected by modifications to the forward path only, then use of the Nyquist diagram is appropriate. To analyse systems with complicated measurement/shaping networks that are not pre-determined, closed-loop Bode diagrams should be constructed for the family of systems. Should there be several distinct measurement/shaping systems which cannot be considered to form one loop, or should the measurement system transfer function(s) be defined and it is required to determine the best shaping function(s), reference to Sections 3 and 4.1.3 of Part V should be made.

Where the open-loop transfer function is defined by a rational polynomial, the system design objective is not intrinsically changed but if there is no computer available the method of system investigation is. (Where a computer is available this form of the transfer function can and should be factorised.) There may be several reasons for the transfer function to be presented in this form, each requiring a different approach. The two sets of circumstances which occur most frequently are as follows:

- (a) Where the transfer function has been derived from the differential state equation model of the plant (see Section 3.2.2 of Part V), the natural course is to use a frequency response method. The frequency response may be plotted on any of the stability assessment diagrams but, if it appears that the open-loop system is itself unstable, or that there is a stability problem with the closed-loop system, one of the polar assessment methods is to be preferred. Should it be necessary to investigate the stability of the open-loop system, then the Routh-Hurwitz criteria should be employed and, if possible, the stability boundaries for the function should be sketched. (See Parts I and II.)
- (b) Where the open-loop transfer function has been derived through a process of block diagram reduction, the frequency response of the system and detailed knowledge of its characteristics must already be available. Thus, the polynomial transfer function is only being used to check the estimated frequency response against the exact response. The assessment diagram will thus be that used to plot the estimated response.

The final point to note concerning the first basic requirement is that models of existing systems may also be presented as frequency response data which have been obtained experimentally. For this form of the model the Bode diagram must be used to calculate the modified frequency response plots and so the logarithmic methods will be the principal analytical tools. Considerable insight into the system characteristics can be gained by factoring the frequency response plot (see Part IV, on the dominant modes of response) but any factored transfer function obtained in this way can never be good enough to be used for design. Additionally, where the data points are sparse close to a resonant frequency, the Nyquist plot may be used to sketch in the doubtful areas of the response plot.

Second basic requirement

The second requirement (for a convenient means of displaying the measures of system performance) may be used to discriminate between the root[†] and the different frequency response methods of assessment. The measures of system performance which can be obtained from the frequency response methods of assessment

[†] Note that when the method of curvilinear squares is applied to the Nyquist diagram to determine the predominant pair of complex conjugate poles, the method may be construed as a root or frequency response method.

are listed in Table 4.2 of Section 4.2 in Part IV, and the estimation of the characteristics of a system from its root diagram is discussed in Section 4.3 of the same Item. (Sections 5 and 7 of Part IV further explain, and give examples of, the estimation of system performance.)

Third basic requirement

The third requirement of the assessment method (clear presentation of the effects of parameter change) is slightly different from the first two in that it requires knowledge of the means of control, and thus could be left until after the uncontrolled characteristics of the system have been investigated. The principal point to note at this stage is that, where Proportional plus Derivative feedback is specified or appears to be a suitable means of control, either the root diagram or inverse Nyquist diagram methods have much to recommend them. If more complicated feedback paths are envisaged then Section 4.1.3 of Part V should be consulted.

It is conceivable that only one method of assessment can yield the design information required and be used to design the controlled system. In this instance the method has been selected by default, but more often there will either be several possible methods that can be used, or no single method will provide an adequate solution. Where the use of a specific method has been suggested, this does not preclude the use of another (possibly more familiar) method, but represents the means of solution considered most expedient in the circumstances. Where there is uncertainty in the indicated choice of method then Sections 3, 4 and 5 of this Item should be considered together with Parts IV and V of this Series.

Sections 3 and 4 discuss the advantages and disadvantages of the individual methods of assessment and Section 5 shows the outline design procedures for a common system where the forward path transfer function is $KG(s)$ and the feedback path transfer function is $H(s)$.

Suppose that the initial system assessment shows that some additional form of control is needed. The form of control adopted will generally dictate the method of assessment used to design the controller. The form of control to be employed (that is, feedback, additional forward path controller elements, or filtering) can be deduced from the notes on the characteristics of the different controllers given in Part V in conjunction with the initial assessment method and this in turn will dictate the method of assessment used. For these three forms of control the following observations apply:

- (i) Feedback control – Where the proposed measurement/shaping system is of first-order, either the root diagram or inverse Nyquist diagram offer the most convenient means of choosing the shaping gains and assessing the performance of the controlled system. However, where the proposed measurement/shaping function is of higher order, only the root or Bode diagram controller design methods are applicable (see Section 4.1.3 of Part V).
- (ii) Additional forward path controllers – It is possible to estimate the required changes to the characteristics of the open-loop system that will yield the required closed-loop characteristics from the Nyquist diagram, the Bode/Nichols diagrams, the root diagram and even the inverse Nyquist diagram. To use the frequency response methods it must be possible to prescribe or deduce the required crossover frequency and phase or gain margins from the information given in the performance specification. This task is not always trivial and is discussed in Section 5. To use the root diagram methods all that is required is a knowledge of the required dominant pole(s).

There are, however, only two procedures for designing the characteristics (time constants) of the controlling element. These procedures use the root and Bode diagrams to design controllers in the complex and frequency domains respectively (see Part V, Section 4.1).

- (iii) Filters – Only the Bode diagram displays the frequency-dependent characteristics of the system needed to specify the required filtering characteristics. Low-pass filters (or simple lag controllers)

may be designed by trial and error on the Bode diagram or the cut-off frequency may be chosen by the approximation indicated in Section 4.2.1, point 5 of Part V. If it is decided to employ a higher-order filter, the gain characteristics may be chosen in the same way but the filtering network will have to be designed using one of the standard procedures. (See Part V, Section 3.4.2.) The specification, design and realisation of optimal or Weiner filters for systems with a noise input is also considered in Section 4.2.1 of Part V.

General Remarks

In addition to the techniques outlined above, many problems may be better understood, and thus solved with less difficulty, if a sensitivity analysis is performed (see Section 5 of Part V). From the sensitivity coefficients obtained it is possible to evaluate the merits of the different means of control and/or the significance of the coefficients in the plant model, by inspection.

Not infrequently several different controller designs evolve as a system is developed and is pursued to prototype stage. The methods of assessment used at each stage must reflect the experience gained and should be chosen to highlight the problems encountered. In particular, the exclusive use of root or frequency response methods can deny insights into the system characteristics which would otherwise have been obtained.

3. ADVANTAGES AND DISADVANTAGES OF THE ROOT METHODS OF SYSTEM ASSESSMENT

3.1 Routh-Hurwitz Criteria (see Part I)

When the Routh-Hurwitz criteria are applied to the characteristic polynomial they will determine whether there are any zeros which possess positive real parts. The primary use of the criteria is to determine the stability boundaries of low-order polynomial characteristic equations in parametric form, without the need for factorisation. The application of this method of analysis to the inertia cross-coupling problem of Item No. 67006 illustrates the benefits and problems associated with it. Additionally, the criteria may be used to calculate the contours of constant damping and/or frequency for plotting the root parameter diagram (see Part II).

The disadvantages of the criteria are that the relative stability margins cannot be quantified and that the number of operations required to complete the determinants for equations over sixth-order is prohibitive.

3.2 Root Diagrams (see Part II)

3.2.1 Generalised root diagram

The root diagram gives the poles of the system for any value of the chosen parameter. When the zeros of the system are superimposed on the plot, the system response to any input for a given set of initial conditions may be found, or the dominant modes of response may be used to estimate the system performance. The root diagram may be generated directly by numerical solution of the characteristic equation for different values of the chosen parameter or it may be constructed by following the root locus sketching rules. The advantages and disadvantages particular to the root locus sketching rules are discussed in Section 3.2.2.

A root diagram may be constructed from a factorised system model derived from frequency response experiments to gain insight into the system characteristics. However, in general, the factorised model and thus the root diagram will not be accurate enough to design the control elements themselves.

Advantages

- (a) The root diagram is applicable to any system and is easily generated by using numerical methods to determine the eigenvalues of the characteristic equation for any value of any chosen parameter.
- (b) The stability of the system may be deduced immediately. When, for a given value of the variable parameter, there are any zeros of the characteristic equation in the right half of the complex plane, the system is unstable. Similarly, if there are no zeros wholly in the right half plane but only on the imaginary axis, the system is neutrally stable.
- (c) The root diagram shows every pole of the system and how each pole varies with the change in value of a parameter. Thus the parameter value can be chosen to give the required dominant mode of response (see Part IV).
- (d) If required, the response to a specified input may be found by superimposing the zeros of the system on the diagram and using this vectorial system representation to evaluate the coefficients of the partial fraction expansion.
- (e) The design of a controller to bring the system closer to its performance specification may be effected graphically. This procedure is shown in Part V and is generally faster and simpler than the design procedures used with the other methods of assessment.
- (f) Pole sensitivities may be estimated or judged from the root diagram, or calculated directly if a computer is being used to find the eigenvalues of the characteristic equation.

3.2.2 Constructing the root diagram by the root locus sketching rules

It is only possible to construct the root locus by the sketching rules when the open-loop transfer function $KG(s)H(s)$ is a factored rational polynomial, and as K varies from 0 to ∞ . The difficulty of the analysis does not necessarily increase when a high-order system is considered and is independent of the stability of the closed and open-loop systems. Constructing the root diagram by the use of the sketching rules, and then using the general root diagram assessment and design rules, is both fast and effective.

The only difficulty that might be encountered when applying the root locus sketching rules is the root finding problem which arises from the rules that yield the stability boundaries and non-asymptotic breakaway points. However, this problem is always avoidable by using the procedures given in Part II. It must be noted that, in general, the whole sketching procedure must be repeated to plot the root loci of the compensated system.

4. ADVANTAGES AND DISADVANTAGES OF THE FREQUENCY RESPONSE METHODS OF SYSTEM ASSESSMENT

4.1 Logarithmic Methods of Assessment (see Part III)

The assessment methods of Bode and Nichols are complementary. The Bode method is a convenient computational tool and means of displaying system information, whilst the Nichols method is a convenient means of obtaining the closed-loop frequency response from the Bode plot of the open-loop transfer function.