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Contribution of wing planform to derivatives of yawing moment and sideforce due to roll rate at subsonic speeds, $(N_p)_w$ and $(Y_p)_w$

Associated software: ESDUpac A8114



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Mr F.W. Stanhope	— Rolls-Royce Ltd, Derby
Mr H. Vogel	— British Aerospace, Weybridge-Bristol Division.

* Corresponding Member

The members of staff who undertook the technical work involved in the initial assessment of the available information and the construction and subsequent development of the Item were

Mr L.K. Morris	— Engineer
Mr S.F. Wood	— Principal Engineer.

The person with overall responsibility for the work in this subject area is Mr A.J. Clarke, Head of Aerodynamics Group.

CONTRIBUTION OF WING PLANFORM TO DERIVATIVES OF YAWING MOMENT AND SIDEFORCE DUE TO ROLL RATE AT SUBSONIC SPEEDS, $(N_p)_w$ AND $(Y_p)_w$

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CONTRIBUTION OF WING PLANFORM TO DERIVATIVES OF YAWING MOMENT AND SIDEFORCE DUE TO ROLL RATE AT SUBSONIC SPEEDS, $(N_p)_w$ AND $(Y_p)_w$

1. NOTATION AND UNITS

The derivative notation used is that proposed in ARC R&M 3562 (Hopkin, 1970) and described in ESDU 86021. Coefficients and aero-normalised derivatives are evaluated in aerodynamic-body axes with origin at the aircraft centre of gravity and with the wing span as the characteristic length. The derivatives Y_p and N_p are often written as C_{Yp} and C_{np} in other systems of notation, but attention must be paid to the reference dimensions used. In particular, in forming C_{Yp} and C_{np} differentiation of C_Y and C_n may be carried out with respect to $pb/2V$ not pb/V as implied in the Hopkin system. It is also to be noted that a constant datum value of V is employed by Hopkin.

		SI	British
A	aspect ratio, b^2/S		
B	Prandtl-Glauert compressibility factor, $\left(1 - M^2 \cos^2 \Lambda_{1/4}\right)^{1/2}$		
b	wing span	m	ft
C_D	drag coefficient, $D/1/2\rho V^2 S$		
C'_D	viscous drag coefficient, $(C_D - C_L^2/\pi A)$		
C_L	lift coefficient, $L/1/2\rho V^2 S$		
C_l	rolling moment coefficient, $\mathcal{L}/1/2\rho V^2 S b$		
C_n	wing yawing moment coefficient, $\mathcal{N}/1/2\rho V^2 S b$		
C_Y	sideforce coefficient, $Y/1/2\rho V^2 S$		
D	drag	N	lbf
F	factor in Equation (5.1)		
L	lift	N	lbf
$\mathcal{L}$	rolling moment	N m	lbf ft
L_p	aero-normalised rolling moment derivative due to roll rate $(\partial \mathcal{L}/\partial p)/1/2\rho V S b^2$		
M	free-stream Mach number		
$\mathcal{N}$	yawing moment about origin	N m	lbf ft
N_p	aero-normalised yawing moment derivative due to roll-rate $(\partial \mathcal{N}/\partial p)/1/2\rho V S b^2$		
p	angular velocity in roll	rad/s	rad/s

S	gross wing area	m^2	ft^2
V	aircraft velocity relative to air	m/s	ft/s
x_{ac}	longitudinal distance rearward from coordinate origin (yawing axis) to wing aerodynamic centre	m	ft
Y	sideforce	N	lbf
Y_p	aero-normalised sideforce derivative due to roll rate ($\partial Y/\partial p$)/ $\frac{1}{2}\rho V S b$		
α	angle of attack	degree	degree
$\Lambda_{1/4}$	quarter-chord sweep of wing planform	degree	degree
λ	ratio of tip chord to centre-line chord (taper ratio)		
ρ	density of air	kg/m^3	slug/ft^3

Subscripts

w	denotes wing contribution
M	denotes value in compressible flow
0	denotes value in incompressible flow
$\Lambda_{1/4} = 0$	denotes unswept wing contribution

2. INTRODUCTION

This Item presents a method of estimating the planform contribution to the yawing moment due to roll-rate, $(N_p)_w$, and the sideforce due to roll rate, $(Y_p)_w$, of swept wings at subsonic speeds. The analysis is made for wings without camber, dihedral, or twist, but the effects of sweep angle, aspect ratio, taper ratio and location of yawing axis are considered.

Section 3 describes the basis of the method.

The Item allows the estimation of the initial linear variation of $(N_p)_w$ and $(Y_p)_w$ with lift coefficient, corresponding to fully attached flow, and, for $(N_p)_w$ only, the subsequent non-linear variation with lift coefficient following the onset of flow separation (see ESDU 66033 (Reference 11) for wings with symmetrical sections). The linear components are based largely on lifting-line considerations modified empirically to account for the effects of tip suction (Section 4.2). The non-linear component of $(N_p)_w$ is attributed to the effect of viscous drag which is approximately accounted for on a semi-empirical basis (Section 4.3). Correction factors to the low-speed estimation of $(N_p)_w$ and $(Y_p)_w$ are presented for first order effects of compressible flow (Section 5).

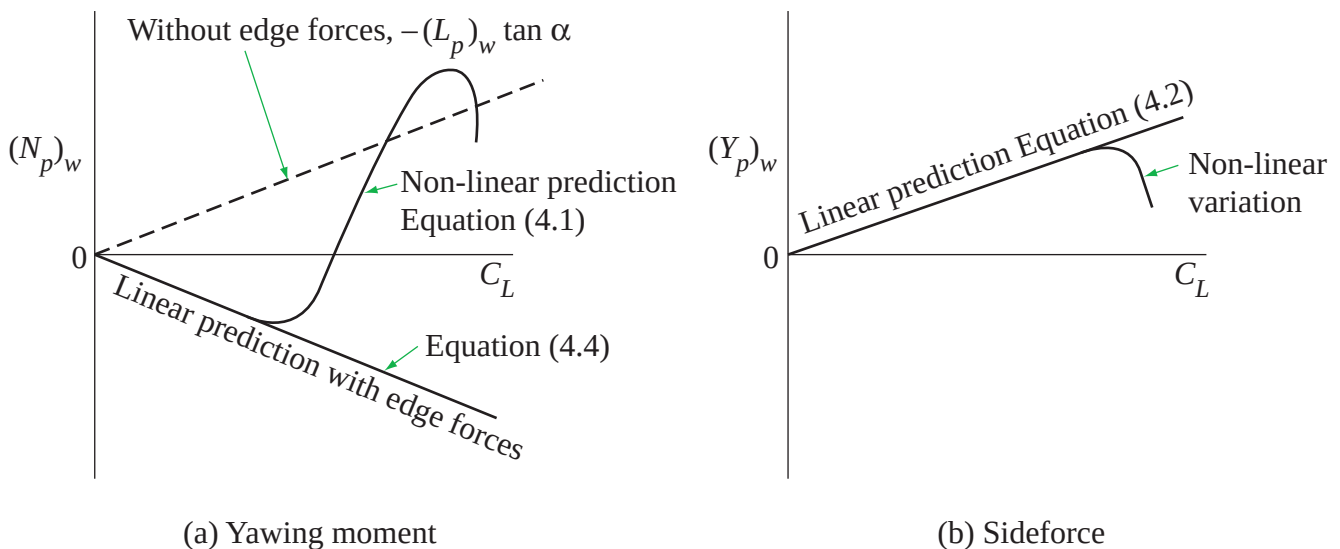
Section 6 summarises the contributions to $(N_p)_w$ and $(Y_p)_w$ due to wing planform, for both fully attached and separated flow. The applicability and accuracy of the method are discussed in Section 7. The Derivation and References are listed in Section 8. Illustrations of typical uses of the Item are given in worked examples

in Section 9. Details of a computer program for the present method, ESDUpac A8114, are provided in Appendix A. Appendix B describes the forces acting on a rolling wing, and explains the effects of flow separation.

3. BASIS OF THE METHOD

For swept wings at subsonic speeds, Sketches 3.1a and 3.1b respectively show typical variations with C_L of the wing planform contribution to yawing moment due to roll rate, $(N_p)_w$, and to sideforce due to roll rate, $(Y_p)_w$. Appendix B describes the various contributions to the forces on a rolling wing, including the effects of flow separation, and explains how these components play a part in the behaviours shown in Sketches 3.1a and 3.1b.

A knowledge of the lift coefficient at which the change of sign of $(N_p)_w$, shown in Sketch 3.1a, occurs is of particular importance in assessing various aspects of the lateral stability of an aircraft and can be deduced by means of this Item.



Sketch 3.1 Typical variations of $(N_p)_w$ and $(Y_p)_w$ with C_L

4. METHOD FOR INCOMPRESSIBLE FLOW

4.1 General

The wing planform contribution due to the derivatives of yawing moment due to roll rate, $(N_p)_w$, and sideforce due to roll rate, $(Y_p)_w$ at low speeds, are given in Derivation 2 by the following semi-empirical equations which apply to wings without camber, dihedral or twist,

$$\begin{aligned} [(N_p)_w]_0 = C_L \frac{A+4}{A+4\cos\Lambda_{1/4}} \left[1 + 6 \left(1 + \frac{\cos\Lambda_{1/4}}{A} \right) \left(\frac{x_{ac}}{b} \tan\Lambda_{1/4} + \frac{\tan^2\Lambda_{1/4}}{12} \right) \right] \left[\frac{(N_p)_w}{C_L} \right]_{0, \Lambda_{1/4}=0} \\ - \frac{C_L}{4A(1+\lambda)} \left(\frac{2+\lambda}{3} \tan\Lambda_{1/4} + \frac{\lambda}{A} \right) - \frac{C_L}{2A} \frac{x_{ac}}{b} + \frac{(\Delta N_p)_w}{(dC'_D/d\alpha)} \frac{dC'_D}{d\alpha} \end{aligned} \quad (4.1)$$

and

$$[(Y_p)_w]_0 = \frac{C_L}{2} \left(\frac{A + \cos\Lambda_{1/4}}{A + 4\cos\Lambda_{1/4}} \right) \tan\Lambda_{1/4} + \frac{C_L}{2A}. \quad (4.2)$$

4.2 Linear Contribution

4.2.1 Yawing moment due to roll rate, $(N_p)_w$

In Equation (4.1) the first three terms relate to the linear contribution. The first term, which may be written as

$$\left[\frac{(N_p)_w}{C_L} \right]_0 = \frac{A+4}{A+4\cos\Lambda_{1/4}} \left[1 + 6 \left(1 + \frac{\cos\Lambda_{1/4}}{A} \right) \left(\frac{x_{ac}}{b} \tan\Lambda_{1/4} + \frac{\tan^2\Lambda_{1/4}}{12} \right) \right] \left[\frac{(N_p)_w}{C_L} \right]_{0, \Lambda_{1/4}=0}, \quad (4.3)$$

was developed in Derivation 2 from strip theory and simple lifting-line considerations. It arises from interactions between symmetrical forces due to incidence and anti-symmetrical forces due to roll rate (see Section 3). The wing sweep factor was developed for untapered wings but the effects of taper are assumed to be accounted for by using the sweepback relating to the quarter-chord line.

The quantity for unswept wings in Equation (4.3) (i.e. $[(N_p)_w/C_L]_{0, \Lambda_{1/4}=0}$) has been calculated theoretically in Derivation 2 using data from Derivation 1 to allow for the small effect of taper.

The second and third terms relate to the effect of tip suction and were derived in Derivation 4 for untapered wings and modified in Derivation 6 to account for the effect of taper. The indications from experimental data (Derivation 5) are that wing taper has a minor effect on $(N_p)_w$ provided the quarter-chord sweep angle is used. With this assumption Figure 1 presents data for the linear contribution to $(N_p)_w$, obtained from the first three terms in Equation (4.1) with $\lambda = 1$, i.e.

$$\begin{aligned} \left[\frac{(N_p)_w}{C_L} \right]_0 = \frac{A+4}{A+4\cos\Lambda_{1/4}} \left[1 + 6 \left(1 + \frac{\cos\Lambda_{1/4}}{A} \right) \left(\frac{x_{ac}}{b} \tan\Lambda_{1/4} + \frac{\tan^2\Lambda_{1/4}}{12} \right) \right] \left[\frac{(N_p)_w}{C_L} \right]_{0, \Lambda_{1/4}=0} \\ - \frac{1}{8A} \left(\tan\Lambda_{1/4} + \frac{1}{A} \right) - \frac{1}{2A} \frac{x_{ac}}{b}. \end{aligned} \quad (4.4)$$

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Figure 1 covers a range of sweep angles up to 60° and aspect ratios from 2 to 12 for fixed values of $x_{ac}/b = 0.0, 0.1$ and 0.2 . The position of the aerodynamic centre to be used in the calculation of x_{ac}/b may be found from ESDU 70011 (Reference 12).

4.2.2 Sideforce due to roll rate, $(Y_p)_w$

The first term of Equation (4.2), which may be written as

$$\left[\frac{(Y_p)_w}{C_L} \right]_0 = \frac{A + \cos \Lambda_{1/4}}{A + 4 \cos \Lambda_{1/4}} \frac{\tan \Lambda_{1/4}}{2},$$

was developed in Derivation 2 from strip theory and simple lifting-line considerations. The equation was originally developed for untapered wings but the effects of taper are assumed to be accounted for by using the sweepback relating to the quarter-chord line. The second term in Equation (4.2) relates to the effect of tip suction. It was derived empirically in Derivation 4 as the zero sweep contribution for untapered wings, and is considered here to apply to tapered wings by referring the sweepback to the quarter-chord line, *i.e.*

$$\left[\frac{(Y_p)_w}{C_L} \right]_{\Lambda_{1/4}=0} = \frac{1}{2A}.$$

It should be noted that this term has been used in Derivation 4 to derive the tip suction terms in Equation (4.1) for $(N_p)_w$.

Figure 2 gives the ratio $(Y_p)_w/C_L$ in incompressible flow for a range of sweep angles up to 60° and aspect ratios from 2 to 12, obtained from

$$\left[\frac{(Y_p)_w}{C_L} \right]_0 = \frac{1}{2} \left(\frac{A + \cos \Lambda_{1/4}}{A + 4 \cos \Lambda_{1/4}} \right) \tan \Lambda_{1/4} + \frac{1}{2A}. \quad (4.5)$$

4.3 Non-linear Contribution to $(N_p)_w$

The fourth term in Equation (4.1)

$$\frac{(\Delta N_p)_w}{(dC'_D/d\alpha)} \frac{dC'_D}{d\alpha},$$

relates to the non-linear contribution to $(N_p)_w$.

It arises from the effects of flow separations over the wing at moderate to high lift coefficients. Derivation 4 suggests that the non-linear increment in $(N_p)_w$ can be correlated in terms of the rate of change of viscous drag coefficient with angle of attack, *i.e.* $dC'_D/d\alpha$, where C'_D is assumed to be given by $C_D - C_L^2/\pi A$. The result of the analysis of Derivation 4, modified in Derivation 7 to extract a wing sweep effect, is shown in Figure 3. It should be noted that in order to calculate $(\Delta N_p)_w$ via Figure 3, it is necessary to have drag data for the wing under consideration at sufficiently close intervals in α for a satisfactory determination of $dC'_D/d\alpha$ to be made, since $(\Delta N_p)_w$ is very sensitive to changes in $dC'_D/d\alpha$.

5. METHOD FOR COMPRESSIBLE FLOW

Derivation 3 gives an approximate method for determining the first order effects of compressible flow. Equation (4.4) together with the Prandtl-Glauert rule gives the correction factor due to compressibility for $(N_p)_w$ as

$$\frac{[(N_p)_w/C_L]_M}{[(N_p)_w/C_L]_0} = \frac{A + 4\cos\Lambda_{1/4}}{AB + 4\cos\Lambda_{1/4}} \cdot \frac{AB + 6F(AB + \cos\Lambda_{1/4})}{A + 6F(A + \cos\Lambda_{1/4})}, \quad (5.1)$$

where $B = (1 - M^2 \cos^2 \Lambda_{1/4})^{1/2}$ and $F = \left(\frac{x_{ac}}{b} \tan \Lambda_{1/4} + \frac{\tan^2 \Lambda_{1/4}}{12} \right)$.

The parameter x_{ac}/b has a negligible effect on the correction equation and may be assumed to be zero in Equation (5.1), i.e.

$$\frac{[(N_p)_w/C_L]_M}{[(N_p)_w/C_L]_0} = \frac{A + 4\cos\Lambda_{1/4}}{AB + 4\cos\Lambda_{1/4}} \cdot \frac{AB + \frac{1}{2}(AB + \cos\Lambda_{1/4})\tan^2 \Lambda_{1/4}}{A + \frac{1}{2}(A + \cos\Lambda_{1/4})\tan^2 \Lambda_{1/4}}. \quad (5.2)$$

Similarly, Derivation 3 gives the correction factor due to compressibility for the sideforce derivative, $(Y_p)_w$, as

$$\frac{[(Y_p)_w/C_L]_M}{[(Y_p)_w/C_L]_0} = \frac{A + 4\cos\Lambda_{1/4}}{AB + 4\cos\Lambda_{1/4}} \cdot \frac{AB + \cos\Lambda_{1/4}}{A + \cos\Lambda_{1/4}}. \quad (5.3)$$

Figures 4 and 5 give the compressibility corrections in carpets of Mach number and quarter-chord sweep angle obtained from Equations (5.2) and (5.3) for a range of aspect ratios. It should be noted that although the data were derived from a method neglecting tip suction effects, it may be assumed that Figures 4 and 5 apply to the whole of the linear contribution at least. In the absence of further information the data of Figure 4 may also be tentatively applied to the non-linear component. The data of Figures 4 and 5 apply to Mach numbers up to that at which the aerodynamic characteristics start to change rapidly. This restriction means that in practice the actual effects of Mach number on $(N_p)_w$ and $(Y_p)_w$ will be comparatively small.

6. SUMMARY OF WING PLANFORM CONTRIBUTIONS

6.1 Fully Attached Flow

For fully attached flow, the wing planform contributions to the derivatives of yawing moment and sideforce due to roll rate, $(N_p)_w$ and $(Y_p)_w$ respectively, consist of linear components only.

The linear incompressible flow component of $(N_p)_w$ is found from Equation (4.4) (or Figure 1), and the linear compressible flow component is obtained from Equation (5.2) (or Figure 4). The total contribution for fully attached flow, comprising both incompressible and compressible effects, is therefore given by the combination of values obtained from these Equations (or Figures), as given in Equation (6.1).

$$[(N_p)_w]_M = C_L \left[\frac{(N_p)_w}{C_L} \right]_0 \frac{[(N_p)_w/C_L]_M}{[(N_p)_w/C_L]_0} \quad (6.1)$$

Similarly, the linear incompressible flow component of $(Y_p)_w$ is found from Equation (4.5) (or Figure 2), whilst the linear compressible flow component is obtained from Equation (5.3) (or Figure 5). The total contribution for fully attached flow, comprising both incompressible and compressible flow components, is therefore given by Equation (6.2).

$$[(Y_p)_w]_M = C_L \left[\frac{(Y_p)_w}{C_L} \right]_0 \frac{[(Y_p)_w/C_L]_M}{[(Y_p)_w/C_L]_0} \quad (6.2)$$

A calculation for a case with fully attached compressible flow is presented in Example 1, see Section 9.1.

6.2 Separated Flow

For separated flow, beyond the linear region there are non-linear components to be added to the linear components of the wing planform contributions to the derivatives of yawing moment and sideforce due to roll rate.

The linear incompressible flow component of $(N_p)_w$ is found by use of Equation (4.4) (or Figure 1), whilst the linear compressible flow component is obtained from Equation (5.2) (or Figure 4). The non-linear contribution is given in Section 4.3 (via Figure 3). As indicated in Section 5, the ordinate value from Figure 4 can be applied with caution as a compressibility correction on the non-linear contribution. The total contribution to $(N_p)_w$ for separated flow, comprising both linear and non-linear components, is therefore given in Equation (6.3).

$$[(N_p)_w]_M = \left(C_L \left[\frac{(N_p)_w}{C_L} \right]_0 + \frac{(\Delta N_p)_w}{(dC'_D/d\alpha)} \frac{dC'_D}{d\alpha} \right) \frac{[(N_p)_w/C_L]_M}{[(N_p)_w/C_L]_0} \quad (6.3)$$

In Example 2, calculations are presented for separated flow, for both incompressible and compressible flow cases, see Section 9.2.

As yet, there is no method for the estimation of the non-linear contribution to $(Y_p)_w$.