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Mean forces, pressures and moments for circular cylindrical structures: finite-length cylinders in uniform and shear flow

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THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item which supersedes ESDU 70014, was monitored and guided by the Wind Engineering Panel which has the following constitution:

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The Panel has benefited from the participation of members from several engineering disciplines. In particular, Dr. A.R. Flint has been appointed to represent the interests of structural engineering as the nominee of the Institution of Structural Engineers.

(Continued on inside back cover)

MEAN FORCES, PRESSURES AND MOMENTS FOR CIRCULAR CYLINDRICAL STRUCTURES: FINITE-LENGTH CYLINDERS IN UNIFORM AND SHEAR FLOW

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MEAN FORCES, PRESSURES AND MOMENTS FOR CIRCULAR CYLINDRICAL STRUCTURES: finite-length cylinders in uniform and shear flow

1. NOTATION AND UNITS

SI Unit

C_{D0}	mean drag coefficient of two-dimensional circular cylinder ($H/D \rightarrow \infty$) whose longitudinal axis is normal to a uniform-velocity flow	
C_{Du}, C_{Ds}	mean overall drag coefficient, based on V_m^2 , of finite-length cylinder in uniform and shear flow respectively; overall drag force per unit length/ $(\frac{1}{2}\rho V_m^2 D_m)$	
C_{Dzu}, C_{Dzs}	mean local-drag coefficient at height z , based on V_z^2 , in uniform and shear flow respectively; local drag force per unit length/ $(\frac{1}{2}\rho V_z^2 D_z)$	
C_{L0}	mean side-force coefficient (defined in Section 8)	
C_p	quasi two-dimensional pressure coefficient based on V_z^2 ; $(p - p_\infty)/(\frac{1}{2}\rho V_z^2)$	
C_{pb}	value of C_p on cylinder surface in wake region (see Sketch 9.1)	
C_{pm}	minimum value of C_p around circumference at particular spanwise section	
C_{ps}	pressure coefficient in shear flow	
C'_{ps}	pressure coefficient in shear flow based on $(V_z')^2$	
$\hat{C}_{ps}(t)$	extreme value of fluctuating pressure coefficient	
D	cylinder diameter	m
D_H, D_0	diameter at tip and base of cylinder respectively	m
D_m	mean diameter: $D_m = \int_0^1 D_z d(z/H)$	m
D_z	local diameter at height z	m
f_H	overall correction factor on C_{D0} for effect of tip; C_{Du}/C_{D0}	
f_{Hz}	local correction factor at height z on C_{Du} for effect of tip; C_{Dzu}/C_{Du}	
f_{Lz}	local correction factor on C_{Dzu} for effect of closed top	
f_{mz}	local correction factor at height z on C_{pm} for effect of shear flow	

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f_{oz}	correction factor on C'_{ps} ; $(V'_z)^2/V_z^2$	
f_s	additional correction factor on C_{Du} for overall effect of shear flow; C_{Ds}/C_{Du}	
f_{sz}	additional local correction factor at height z on C_{Dzu} for local effect of shear flow; C_{Dzs}/C_{Dzu}	
f_t	correction factor on C_{Du} for effect of taper	
H	height or length of surface-mounted structure	m
L	length of cylindrical body with two free ends	m
M_L	local bending moment	N m
M_T	moment about base	N m
p	local static pressure on cylinder surface	N/m ²
p_∞	free-stream static pressure	N/m ²
Re	Reynolds number, $V_H D_H/\nu$	
Re_e	effective Reynolds number; see Section 4.1	
V_H, V_z	mean free-stream velocity at heights H and z respectively	m/s
V'_z	local velocity in shear flow just ahead of stagnation point	m/s
V_m^2	mean value of V_z^2 over appropriate length of structure; see Equation (4.1)	m ² /s ²
X_L	local shear force acting along x -axis	N
X_T	overall shear force (equal to total drag force)	N
Y_T	overall mean side force acting along y -axis	N
x, y, z	system of rectangular cartesian co-ordinates with x -axis aligned with mean flow direction and z -axis vertical	
z	height measured from ground plane	m
z'	distance measured from tip of cylinder towards ground plane	m
z_{cpu}, z_{cps}	centre of pressure position in uniform and shear flow respectively; point at which total drag force can be considered to act	m
z_i	height measured from lower level of i th section	m

$\bar{\alpha}$	shear flow parameter (see Section 4.2)	
$\bar{\alpha}_e$	effective value of $\bar{\alpha}$ for non-standard shear flow (see Section 7)	
ε	effective surface-roughness height of cylinder (see Reference 1)	m
θ	angular location of point (see Sketch 9.1)	degree
θ_b	value of θ where C_p first equals C_{pb}	degree
θ_m	value of θ where $C_p = C_{pm}$	degree
ν	kinematic viscosity of free-stream fluid	m ² /s
ρ	density of free-stream fluid	kg/m ³
σ_{Cps}	standard deviation of C_{ps}	

Subscripts

H	denotes value at $z = H$
i	refers to value for i th step or section
s	relates to value in shear flow
u	relates to value in uniform flow

2. PURPOSE, SCOPE AND USE OF THIS DATA ITEM

2.1 Purpose

The purpose of this Data Item is to provide data for estimating the mean forces, pressures and moments induced by flow around a finite-length cylindrical structure of circular section. It supersedes the data presented in Item No. 70014 and considerably extends the range of information previously given.

Data are presented in Item No. 80025¹ for two-dimensional flow conditions when the effects of flow around the free end (or ends) can be ignored, as with very long cylinders. The data provided by this Item are in the form of correction factors to the two-dimensional values to account for

- (i) the effect of the free end (or ends),
- (ii) the effect of a shear flow (*i.e.* a non-uniform approaching flow where, as in the atmospheric wind, V_z varies along the length of the structure).

The scope of the information presented is summarised in Section 2.2. The main applications for the data lie in calculating the mean wind loading acting on ground-based structures such as stacks, towers, silos, storage tanks and chemical processing equipment but the data are also relevant in the calculation of the aerodynamic forces on the bodies of aerospace vehicles and missiles* and the excrescence drag of ancillaries such as aerials.

This Data Item provides the means for estimating the mean forces, moments and pressures induced by the flow of air, or other fluid such as water, around the structure. For design purposes, fluctuating components of loading, arising from buffeting due to turbulence or induced by vortex shedding, must also be considered; Item No. 87035³ provides methods for estimating the maximum design loading due to buffeting by atmospheric turbulence (along the wind direction) and Item No. 96031⁴ deals with the across-flow response due to vortex shedding.

2.2 Scope

The following Table outlines the scope of the information presented in the Item.

The numbers indicate the relevant Sections	Uniform Flow or Shear Flow				
	Shear forces: X		Moments: M		Pressures:
	Overall	Local:	Base M_T	Local M_L	Local: $C_{ps}, \dot{C}_{ps}(t)$
Parallel-sided cylinder Tapered cylinders Stepped cylinders Cylinder with part-span roughness (e.g. strakes) }	5 6.1.1, 6.1.2 6.2, 6.2.1 6.3	5 6.1.1 6.2	5 6.1.1, 6.1.2 6.2, 6.2.1 6.3	5 6.1.1 6.2	9.1, 9.2, 9.3
Top open or closed	– see Section 5.1				
Effect of efflux from top	– see Section 5.1				
Application to other flow velocity profiles	– see Section 7				
Velocity flow field around cylinder	– see Section 9.4				

The effects of parameters such as Reynolds number, surface roughness, flow turbulence and full-span protrusions (such as pipes, plates and ladders) are accounted for in the estimation of C_{D0} obtained from Item No. 80025¹.

* The effects of compressible flow around finite-length circular cylindrical bodies will be considered in future Data Items to be included in the Aerodynamics Sub-series.

2.3 Notes on the Use of the Data Item

- (i) To make the best use of this Data Item use the Table in Section 2.2 as a guide to the appropriate Sections of interest. Select the appropriate calculation sheet from Table 12.1 to 12.6. Read the general background notes in the appropriate Sections of interest (Section 10 contains two worked examples and Appendix A provides a general description of the features of the flow around finite-length cylinders of circular cross section).
- (ii) Take note of the general applicability and uncertainty of the data (see Section 3). The use of dashed lines in the graphical data indicates extrapolated values or values with a higher uncertainty than normal.
- (iii) In order to estimate the wind- or other fluid-loading for a finite-length circular cylindrical structure using this Item, data for the two-dimensional flow case around an “infinitely long” equivalent cylinder are required from Item No. 80025¹ (see Section 4.1). Calculation sheets are also provided in that Item to assist in making this estimate easily and conveniently.
- (iv) For repeated calculations many of the data in this Item can be programmed using the equations in the text and in Appendix B. Appropriate equations are also given in Item No. 80025.

3. UNCERTAINTIES, LIMITATIONS AND APPLICABILITY

The estimated uncertainties in the data are, in general, noted on the appropriate Figures.

The limitations on the use of the data are either implied by comments in the text or by the range of experimental data used to derive the various Figures (summarised in Table C1.1 in Appendix C). Where possible the data have been interpolated and extrapolated (using semi-theoretical models) to practical conditions other than those for which experimental data are available.

The applicability of the data, with particular reference to the various approaches to estimating static wind loading, is described in Sections 3.1 and 3.2.

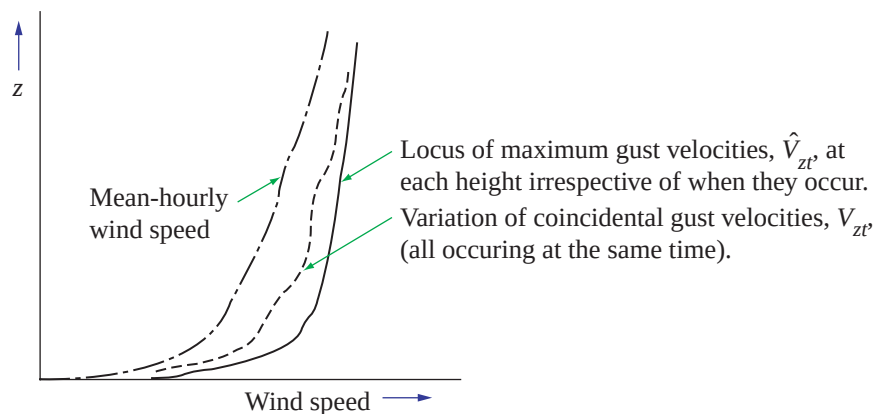
3.1 Application to Wind Loading Problems

There are two main approaches to estimating wind loads on structures; these are the so-called “discrete gust loading” approach (Section 3.1.1) and the “statistical” approach (Section 3.1.2). The data presented in this Item can be used in both approaches. There are, however, implications in the discrete gust loading approach which make it less reliable than the (preferred) statistical approach for the reasons discussed in Section 3.1.1.

3.1.1 The discrete gust loading approach

In the discrete gust loading approach the assumed increase in the design wind speed ($\hat{V}_{zt}$) with height is broadly equivalent to that exhibited by a gust sufficiently large to envelop the structure (or the relevant element of a structure) under consideration so that all points are loaded by the same gust coincidentally. The size or spatial influence of a gust is related to the time over which the wind speed is averaged. For example, a 5 second averaging time is appropriate for structures with a maximum dimension in the range 20 m to 50 m whilst 15 seconds is a more appropriate time when the dimension is larger than 50 m.

The discrete gust loading approach suffers from several disadvantages. Firstly, although the way in which the averaging times are derived allows, to some extent, for the fact that the spatial influence of a t -second gust is limited to certain dimensions, this influence or “correlation” tends to be overestimated for relatively small eddies such as represented by a 3, 5 or 15 second gust. Also, the maximum gust speeds, $(\hat{V}_{zt})$, do not occur simultaneously at each level. A summation of local forces, derived using local values of (V_{zt}) , will thus tend to give an overestimate of the overall loading.



Sketch 3.1

Secondly, the local forces generated depend partly on the magnitude of the local shear (defined as the rate of change of velocity with height). In the real situation the rate of change with height of the coincident mean gust velocity, V_{zt} (i.e. the local mean shear measured coincidentally at all heights) is very irregular compared with that exhibited by the mean-hourly wind speed, as illustrated in Sketch 3.1. Thus the use of force coefficient data appropriate to shear flow profiles which are monotonic in form (as assumed in this Data Item and elsewhere) will introduce further uncertainties into the gust loading calculation. These uncertainties cannot be quantified but in view of the *generally* conservative nature of the discrete gust loading approach they are ignored.

Finally, the discrete gust loading approach provides only a static loading estimate; it cannot be extended to estimate dynamic effects.

These drawbacks are overcome if the statistical approach (Section 3.1.2) is used.

3.1.2 The statistical approach

The statistical approach is based on estimating a mean loading due to the mean-hourly wind speed, V_z , onto which is superimposed a component to account for fluctuations in the wind speed about the hourly mean (turbulence). This approach uses statistical relationships to account for correlation effects in a more realistic way than the gust loading approach. In addition, the mean-hourly wind speed increases monotonically with height so that the effect of the local velocity gradient on the local mean forces can be more rationally accounted for from wind-tunnel data than is possible in a gust loading estimate.

The complete statistical method is more rational but more complicated to use from first principles than the discrete gust loading approach; however, in Item No. 87035³ the approach has been reduced to a simple form which in most situations is no more difficult to apply than the discrete gust loading method. The statistical approach has the advantage that the concepts may be extended, through the use of spectral data, to deal with dynamic effects due to turbulence in an equally simple and rational way as again provided by Item No. 87035³.

3.2 Application to Other Problems

The data in this Item are also applicable to situations when the loading is induced by the flow of a fluid other than air, *e.g.* water flow. In the case of liquid flow the main restrictions are that free-surface effects are not included and the data will be inapplicable if cavitation in the wake flow takes place.

In relation to gas flows, the data are applicable only for incompressible flow (*i.e.* free-stream Mach numbers less than about 0.3). The effects of compressible flow around finite-length circular-cylindrical bodies will be considered in future Data Items to be included in the Aerodynamics Sub-series.

4. INPUT DATA ACCOUNTING FOR EFFECT OF FLOW PROPERTIES AND CYLINDER ROUGHNESS

The methods of estimating loadings on various types of finite-length structure are described in Sections 5 to 8. In all these cases a knowledge of the basic force coefficient for the equivalent two-dimensional condition is required and also, for estimating overall forces and moments, an appropriate mean flow velocity is needed. The sources of these data are described in Sections 4.1 and 4.2.

4.1 Two-Dimensional Force Coefficients

The flow-induced forces acting on a finite-length circular cylinder can be related to those acting on an infinitely long (two-dimensional) cylinder in uniform flow. The flow pattern around a two-dimensional circular cylinder is controlled by the separation of the boundary layer from each side of the cylinder. This separation is affected by the turbulence characteristics of the approaching flow (the free stream), the surface roughness of the cylinder and the flow Reynolds number. The flow-induced forces appropriate to a finite-length cylinder are similarly affected by these characteristics; in practice their effect can be accounted for via their influence on C_{D0} , the drag coefficient for the equivalent cylinder with two-dimensional flow when tip effects are not present ($H/D \rightarrow \infty$).

In this Item the drag coefficient associated with a parallel-sided finite-length cylinder is related to C_{D0} through various correction factors to account for the flow around the tip and the effect of shear flow. Values of C_{D0} can be estimated using Item No. 80025¹ for flow conditions and cylinder surface roughness appropriate to $z = H$. Reference 1 provides calculation sheets for this purpose which have specific procedures for including the effects of free-stream turbulence, surface roughness and spanwise surface-protrusions (such as strakes, pipes and plates). The value of C_{D0} is determined by ε/D and by an effective Reynolds number, $Re_e = \lambda_T \lambda_R Re$, in which the factors λ_T and λ_R account for the effect of free-stream turbulence and cylinder surface roughness on the Reynolds numbers corresponding to the various flow regimes.

In the case where surface protrusions (such as strakes) do not extend over the whole length of the cylinder, Section 6.3 provides specific guidance on how to deal with this situation. Also, if a surface protrusion causes the flow around the cylinder to be asymmetric, side forces will be generated; Item No. 80025 provides side force data for two-dimensional flow conditions and Section 8 explains how these data may be used for finite-length cylinders.

4.2 Mean Flow Velocity

In most cases, the maximum design flow speed (and its variation with height) will be assumed for the calculation. However, if V_H is such that the effective Reynolds number (see Section 4.1) lies in the range $3 \times 10^5 < Re_e < 6 \times 10^5$ it is recommended that a range of design flow speeds is considered for the following reasons. Firstly, in this Reynolds number range transition from laminar to turbulent boundary-layer flow takes place near the separation point (see Appendix A of Reference 1). This transition region is characterised

by a rapid fall in C_{D0} as Re_e increases and the drag coefficient is subject to higher uncertainties than at other values of Re_e . Secondly, on a relatively smooth finite-length cylinder, transition in this Re_e region does not necessarily occur simultaneously at all spanwise sections, thus further increasing the uncertainty. Additionally, where C_{D0} is falling rapidly as Re (and V_H) increases, the actual drag force ($\propto C_{D0} V_H^2$) can decrease as V_H increases. For these reasons if, at the design flow speed, the corresponding value of Re_e falls in the range $3 \times 10^5 < Re_e < 6 \times 10^5$ it is advisable to investigate the range of flow speeds represented by the extent of this Re_e range and take the highest loading as being the design case, irrespective of the originally assumed maximum flow speed.

The effect on the overall loading of a *non-uniform* velocity profile, of the type illustrated in Sketch 5.1, is twofold. Firstly, the local flow velocities in the shear flow are smaller than those in a corresponding uniform flow and these induce correspondingly smaller forces. Secondly, spanwise velocities along the cylinder axis are induced by the three-dimensional nature of the flow (see Appendix A) and the local drag coefficient values are thus different from those occurring in uniform flow, being smaller near the tip and larger near the base for the reasons described in Section 5.4.

In practice it is found that, for a cylindrical structure in a velocity gradient of the type illustrated in Sketch 5.1, a summation of the *local* values of $\frac{1}{2} \rho V_z^2 DC_{Dzs}$ over the appropriate cylinder length ($z_2 - z_1$) gives approximately the same *overall* force per unit length as the quantity $\frac{1}{2} \rho V_m^2 DC_{Du}(z_2 - z_1)$ appropriate to a cylinder in an equivalent uniform flow (with the same properties at $z = H$) of velocity V_m . The velocity V_m is an integrated mean velocity over the appropriate cylinder length defined from

$$V_m^2 = \frac{1}{z_2 - z_1} \int_{z_1}^{z_2} V_z^2 dz. \quad (4.1)$$

The approximation to the overall force in shear flow can be refined by a small additional correction (f_s), dependent on the nature of the shear flow and the value of H/D as described in Section 5.2, but in most cases it is approximately unity.

Equation (4.1) can be evaluated from a knowledge of the particular design flow conditions. In the case of wind flow, local mean wind speeds can be evaluated from sources such as Item No. 82026² which recommends the use of the log-law type of relationship and ground roughness parameter z_0 to relate the variation of V_z with z . For simplicity, a power law approximation to the approaching velocity profile is sometimes used in the form

$$\frac{V_z}{V_H} = \left(\frac{z}{H} \right)^{\bar{\alpha}} \quad (4.2)$$

$$\text{from which} \quad V_m^2 / V_H^2 = 1 / (2\bar{\alpha} + 1) \quad (4.3)$$

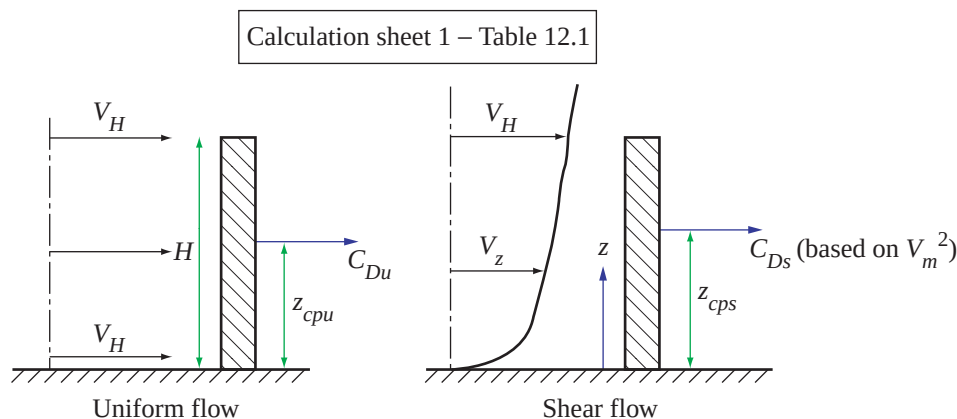
$$\text{and} \quad \bar{\alpha} = \frac{1}{2} (V_H^2 / V_m^2 - 1). \quad (4.4)$$

The parameter $\bar{\alpha}$ is used in the present methods to correlate the effects of shear flow on the drag forces. A value of $\bar{\alpha}$ is therefore required in calculating the load effect and the appropriate value should be evaluated via the use of Equations (4.1) and (4.4). However, some typical values of $\bar{\alpha}$ for mean-hourly wind speeds (used in the statistical approach to wind loading – see Section 3.1.2) are given in Table 4.1.

TABLE 4.1 Typical Values of $\bar{\alpha}$ for Mean-Hourly Wind Speeds

<i>Terrain description</i>	<i>Approx. value of $\bar{\alpha}$</i>
Open sea; large stretches of open water	0.10 – 0.12
Rural areas; open country with few, low obstacles	0.14 – 0.16
Low density built-up areas; small towns; suburbs; open woodlands with small trees	0.20 – 0.24
Town and city centres with high density of buildings; broken country with tall trees	0.26 – 0.30

5. UNIFORM, PARALLEL-SIDED CYLINDER



Sketch 5.1 Parallel-sided cylinder

A “uniform” cylinder in the present context means that the external surface is uniformly smooth or rough over the whole length and that any spanwise protrusions also extend over the whole length. Section 6.3 provides guidance on how to deal with “non-uniform” cylinders.

In many applications the design loading condition will be the overall drag or shear force and bending moment at the base of the structure. For this situation Equations (5.1) to (5.3) apply and these require values of the overall drag coefficient $C_{Ds}(= C_{D0}f_Hf_s)$ and centre of pressure position, z_{cps} .

In other applications local shear forces and moments at points other than the base may be required. For example, in the case of a stack supported at a point (or points) along its length, the stress in the support(s) may be required. Alternatively, in the case where a change in shell thickness occurs, the critical design condition may occur at a cross-section away from the base. In these cases the local shear forces and moments will have to be estimated by summing local loading contributions over the relevant parts of the structure and using, for example, beam theory to estimate local shear forces and moments. Equations (5.4) and (5.7) will be appropriate for the estimation of local loadings on a free-standing structure and these require a knowledge of the local drag coefficient $C_{Dzs}(= C_{D0}f_Hf_zf_s)$. However, as shown in Appendix B, a simpler but slightly more approximate estimation of the local forces and moments is given by Equations (5.6) and (5.8).

In the case of other non-standard cylinders (e.g. stepped cylinders) the shear forces and moments also need

to be built up from a knowledge of the local drag coefficient values as described in Section 6.

The following equations give the overall and local values of shear force and bending moments for a free-standing structure. Table 5.1 indicates where data for each of the factors may be obtained and also the relevant Section which provides additional background information. The calculation sheet in Table 12.4 provides a convenient means of summarising the steps required for these particular estimations.

The overall shear (or drag) force is given by

$$X_T = \frac{1}{2} \rho V_m^2 D H C_{Ds}, \quad (5.1)$$

where $C_{Ds} = C_{Du} f_s = C_{D0} f_H f_s. \quad (5.2)$

The overall moment at the base is given by

$$M_T = X_T z_{cps}. \quad (5.3)$$

The local shear force at $z = z_L$ for a free-standing structure is given by

$$X_L = \frac{1}{2} \rho D \int_{z_L}^H V_z^2 C_{Dzs} dz \quad (5.4)$$

where $C_{Dzs} = (C_{Du}) \frac{C_{Dzu}}{C_{Du}} \frac{C_{Dzs}}{C_{Dzu}} = (C_{D0} f_H) f_{Hz} f_{sz}. \quad (5.5)$

or more approximately (see Section B4 in Appendix B) from

$$\frac{X_L}{X_T} = 1 - \left(\frac{z_L}{H} \right)^{2\bar{\alpha} + 1}, \quad (5.6)$$

The local moment at $z = z_L$ for a free-standing structure is given by

$$M_L = \frac{1}{2} \rho D \int_{z_L}^H V_z^2 C_{Dzs} (z - z_L) dz \quad (5.7)$$

or more approximately (see Section B4 in Appendix B) from

$$\frac{M_L}{M_T} = 1 - \frac{2\bar{\alpha} + 2}{2\bar{\alpha} + 1} \frac{z_L}{H} + \frac{1}{2\bar{\alpha} + 1} \left(\frac{z_L}{H} \right)^{2\bar{\alpha} + 2}. \quad (5.8)$$

TABLE 5.1 The Factors in Equations (5.1) to (5.8)

Factor	Values given by	Further information
V_m^2	Equation (4.1)	Section 4.2
C_{D0}	Item No. 80025 (Ref. 1)	Section 4.1
f_H	Figure 1	Section 5.1
f_s^*	Figure 2	Section 5.2
f_{Hz}	Figure 3 or 4	Section 5.3
f_{sz}^*	Figure 5	Section 5.4
z_{cps}	Figure 6	Section 5.5
$\bar{\alpha}$	Eqns (4.1) and (4.4) or Table 4.1.	Section 4.2

* The corrections (f_s and f_{sz}) to the data for the effect of a non-uniform velocity profile (shear flow) should be made with caution if the approaching velocity profile is other than that in which dV_z/dz decreases monotonically with increasing z as illustrated in Sketch 5.1. The reasons for this are discussed in Sections 3.1.1 and 5.4 and further guidance for non-standard velocity profiles is given in Section 7.

5.1 Overall Correction Factor on C_{Du} for Effect of Free End; f_H (Figure 1)

The factor $f_H (= C_{Du}/C_{D0})$ accounts for the reduction in the drag coefficient due to the induced flow over the free end (or ends) of the cylinder. This has the effect of increasing the pressures in the base region behind the cylinder and thus decreasing the drag coefficient. The extent to which this happens depends on H/D and on C_{D0} as explained in Appendix A.

For ground-based structures of height H there is only one free end but in some situations, such as in the case of aerospace and missile bodies, the cylindrical body has two free ends. In this case the factor f_H is given by Figure 1 taking H/D equal to $1/2 L/D$ where L is the body length. This follows from the theoretical consideration that the mirror image of the cylinder in an end plane does not affect the flow and the plane can be considered as the dividing streamline in the flow around an isolated cylinder of length $2H$. However, this assumption is only strictly true for a uniform free stream when boundary-layer effects are not large.

When H/D is greater than about 4 the drag coefficient is larger when the free end is not closed, as with a chimney stack; Figure 1 takes this into consideration. However, the effect of an efflux from the open end is to decrease the drag coefficient to approximately that appropriate to a closed end; the design condition for a stack (in relation to loading in the mean wind direction) is therefore with the top open and no efflux.

5.2 Overall Correction Factor on C_{Du} for Effect of Shear Flow; f_s (Figure 2)

The factor f_s accounts for the additional effect of a shear flow on the drag coefficient that it is not accounted for through the use of V_m^2 in Equation (4.1). It is defined as

$$f_s = C_{Ds}/C_{Du}$$

where C_{Ds} and C_{Du} relate to shear flow and uniform flow respectively and are based on V_m^2 .

Figure 2 provides values of f_s as a function of H/D and a shear flow parameter $\bar{\alpha}$ which characterises the variation of V_z with z over the structure height through the power law expression given by Equation (4.2). The power law relation is only an approximate empirical expression but it is sufficiently representative and

simple in form to enable a generalised expression for f_s to be derived analytically (as described in Section B1 of Appendix B) from a limited number of experimental data. The factor f_s is not particularly sensitive to variations in $\bar{\alpha}$ and in practice, for H/D greater than about 6, $f_s \approx 1$.

5.3 Correction Factor Giving Local C_{Dzu} ; f_{Hz} (Figures 3 and 4)

The variation of $f_{Hz} = C_{Dzu}/C_{Du}$ along the length of a parallel-sided cylinder in uniform flow is given by Figure 3 for low Reynolds numbers ($Re_e < 3 \times 10^5$) and by Figure 4 for high Reynolds numbers ($Re_e > 10^6$). The values of f_{Hz} in the range ($3 \times 10^5 < Re_e < 10^6$) are very uncertain for the reasons explained at the beginning of Section 4.2. In general it will be safer to assume that the values given by Figure 4 (for $Re_e > 10^6$) apply down to ($Re_e = 3 \times 10^5$).

The “peaky” distribution* near the free end is associated with the formation of the tip vortex, as described in Appendix A, which dominates the top two or three diameters. For this reason the distribution of local drag coefficient in the tip region tends to be independent of H/D for H/D greater than 6. For H/D less than about 6 the tip vortex system collapses and the distribution of C_{Dz} becomes more uniform along the cylinder length.

For a closed top the local drag coefficients for $Re_e > 10^6$ near the tip are reduced (by a factor f_{Lz} in Figure 4) from those appropriate to the “top open” case. For lower Reynolds numbers ($Re_e < 4 \times 10^5$) there does not appear to be any significant difference in f_{Hz} between the “top open” and “top closed” cases.

5.4 Local Correction Factor on C_{Dzu} for Effect of Shear Flow; f_{sz} (Figure 5)

In a shear flow the existence of the velocity gradient tends to reduce the intensity of the tip vortex. In addition, pressure gradients are induced along the length of the cylinder which, in particular, cause the formation of a strong bound vortex system at the base of the cylinder near the ground plane. Thus, compared with uniform flow, the local drag coefficient values (based on V_z^2) are decreased near the tip and (in most cases) increased near the ground.

The extent of these modifications depends on the local shear flow gradient. For shear flow profiles of the boundary-layer type (in which dV_z/dz decreases monotonically with increasing z as illustrated in Sketch 5.1), Figure 5 provides values of $f_{sz} = C_{Dzs}/C_{Dzu}$ as a function of z/H and H/D . These values are based on data for $\bar{\alpha} = 0.2$ and high Re ($Re_H > 1.5 \times 10^6$) but, in the light of limited evidence^{9, 17} relating to shear flow effects on a cylinder with no tip effect at subcritical Re , it is reasonable to assume that they are applicable to all cases where Re_H is greater than about 10^4 .

For forms of shear flow profile other than the boundary-layer type the data in Figure 5 are not strictly applicable but, in the absence of other information, the guidance in Section 7 may be of some help.

5.5 Centre of Pressure Position; z_{cps} (Figure 6)

The centre of pressure position is the point at which the overall drag force can be considered to act when estimating the overall moment at the base of the cylinder, as illustrated in Sketch 5.1. For a parallel-sided cylinder the centre of pressure position, in uniform flow or in a shear flow of the type illustrated in Sketch 5.1, is given by Figure 6 as a function of H/D and the shear flow parameter $\bar{\alpha}$.

The values in Figure 6 are based on equations derived in Section B3 of Appendix B which are in good agreement with the limited number of experimental data for both uniform and shear flow. It may be assumed that the values apply for all cases where Re_H is greater than about 10^4 .

* Due to this “peaky” distribution, the assumption (often made) that the local drag coefficient is constant over the length of the structure can lead to significant underestimates of the local shear forces and bending moments.

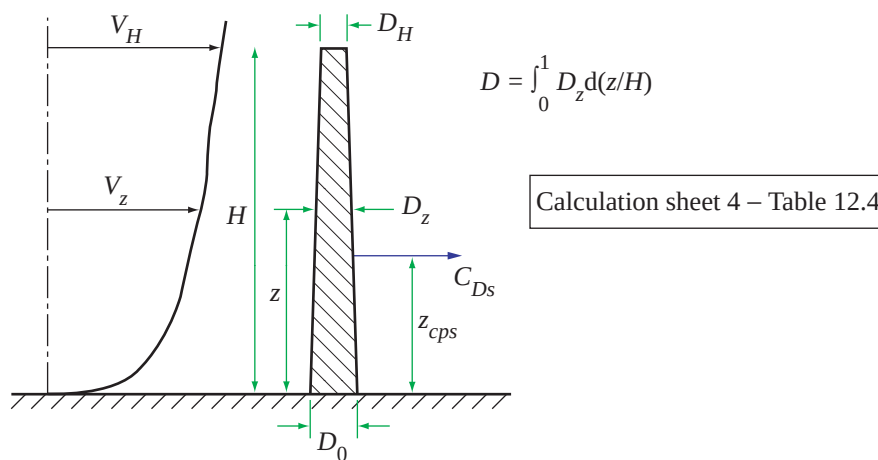
It is often assumed that the centre of pressure position is at $z/H = 0.5$ but, as can be seen from Figure 6, this can lead to significant errors in some circumstances.

6. NON-UNIFORM CYLINDERS

6.1 Tapered Cylinders

Section 6.1.1 deals with cylindrical structures with a linear taper. Section 6.1.2 provides guidance for those structures with lengths of varying taper.

6.1.1 Cylinder with straight taper



Sketch 6.1 Tapered cylinder

For a straight, or nearly straight, tapered cylinder, limited evidence¹⁷ suggests that the local drag coefficient (based on V_z^2 and D_z) is approximately constant along the cylinder length. Assuming (as found to be approximately true for parallel-sided cylinders) that $C_{Du} = C_{Ds}$ when based on V_m^2 the total drag (or shear) force is given by

$$\begin{aligned} X_T &= \frac{1}{2} \rho V_m^2 D_m H (C_{Du} f_t) \\ &= \frac{1}{2} \rho V_m^2 D_m H (C_{D0} f_H f_t) \end{aligned} \quad (6.1)$$

where V_m^2 is given by Equation (4.1) and C_{D0} and f_H (Sections 4.1 and Figure 1) are evaluated for conditions appropriate to a parallel-sided cylinder of $D = D_m$ where

$$D_m = \int_0^1 D_z d(z/h) \quad (6.2)$$

The factor f_t (given by Figure 7) accounts for the reduction in C_{Du} due to the taper which acts to reduce the intensity of the tip vortex system.

The overturning moment, M_T , is given by

$$M_T = X_T z_{cps} \quad (6.3)$$

where z_{cps}/H is given by Figure 7 as a function of the taper parameter $(D_0 - D_H)/D_m$ and the shear flow parameter $\bar{\alpha}$. The derivation of Figure 8 is explained in Appendix B.

Local shear forces and moments can be obtained from an integration of the local loadings but as explained in Appendix B these can be estimated directly, to a close approximation, from

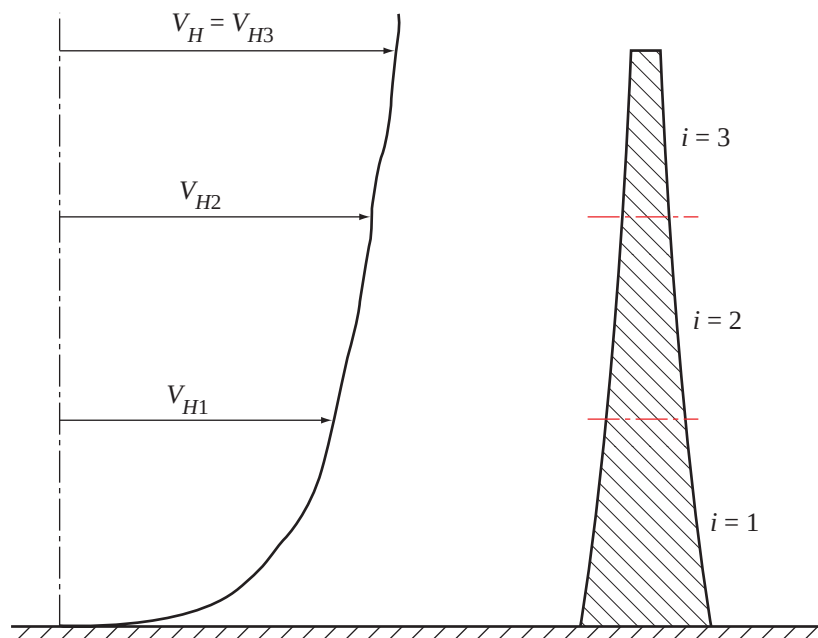
$$\frac{X_L}{X_T} = 1 - \left(\frac{z_L}{H}\right)^{2\bar{\alpha} + 1} \left\{ \frac{1 - (1 - (D_H/D_0)) \frac{2\bar{\alpha} + 1}{2\bar{\alpha} + 2} \frac{z_L}{H}}{1 - (1 - (D_H/D_0)) \frac{2\bar{\alpha} + 1}{2\bar{\alpha} + 2}} \right\}, \quad (6.4)$$

$$\begin{aligned} \frac{M_L}{M_T} = 1 - \frac{2\bar{\alpha} + 3}{2\bar{\alpha} + 1} \frac{1 + (2\bar{\alpha} + 1)D_H/D_0}{1 + (2\bar{\alpha} + 2)D_H/D_0} \frac{z_L}{H} + \frac{2\bar{\alpha} + 3}{2\bar{\alpha} + 1} \frac{1}{1 + (2\bar{\alpha} + 2)D_H/D_0} \left(\frac{z_L}{H}\right)^{2\bar{\alpha} + 2} \\ - \frac{1 - D_H/D_0}{1 + (2\bar{\alpha} + 2)D_H/D_0} \left(\frac{z_L}{H}\right)^{2\bar{\alpha} + 3} \end{aligned} \quad (6.5)$$

where X_T and M_T are given by Equations (6.1) and (6.3).

6.1.2 Cylinder with varying taper

Calculation sheet 5 – Table 12.5



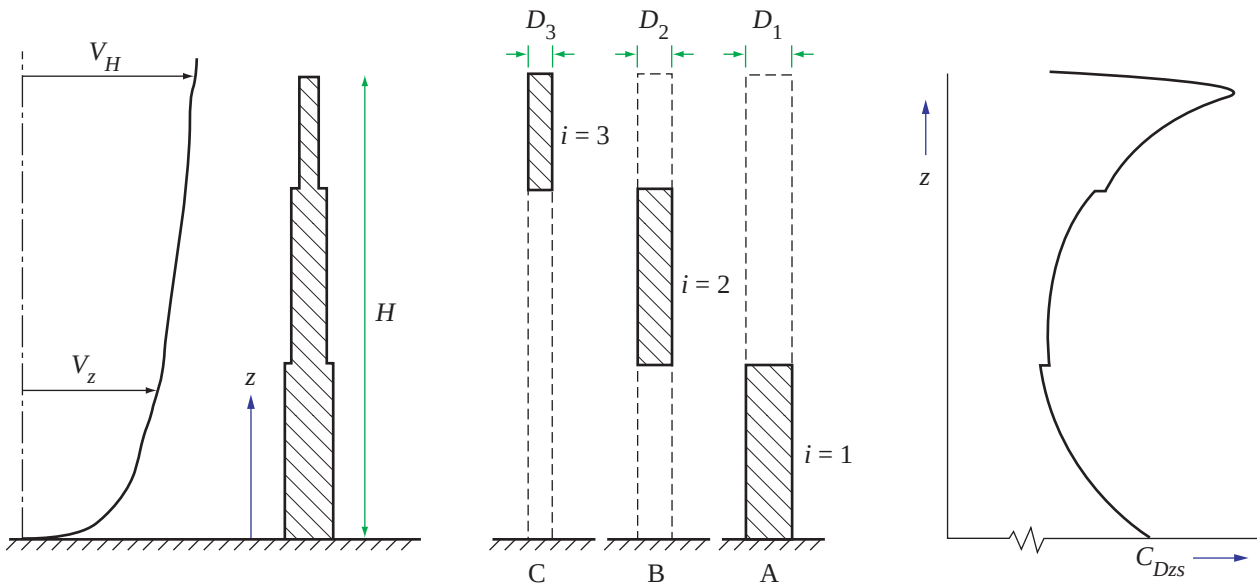
Sketch 6.2 Cylinder with varying taper

For a circular cylindrical structure with a varying amount of taper along its length it is recommended that the structure is divided into components $i = 1, 2, 3, \dots$ such that the taper of each component is approximately linear. The overall data described in Section 6.1.1 can then be applied to each component i by using equivalent values of the appropriate parameters evaluated for each component. The overall loading

is then obtained by summing the contributions from each component. The detailed procedure is set out in the Calculation Sheet in Table 12.5.

6.2 Stepped Cylinder

Calculation sheet 3 – Table 12.5



Sketch 6.3 Stepped cylinder

No simple equation can be given to cover all possible combinations of stepped cylinders but the following procedure, illustrated in Sketch 6.3, can be used to estimate shear forces and bending moments. The procedure assumes two step changes in cylinder diameter but obviously more, or fewer, steps can be included. The Calculation Sheet in Table 12.3 summarises the complete procedure and a calculated example is given in Section 10.2.

- (1) Separate the stepped structure into various components $i = 1, 2$, and 3 of complete but separate cylinders A, B and C of diameters D_1, D_2 and D_3 and each of height H .
- (2) Estimate C_{D0} (see Section 4.1) for each cylinder, A, B and C corresponding to flow conditions at the top of each i th component.
- (3) Obtain values of $f_H = C_{Du}/C_{D0}$ (assuming $f_s \approx 1$) for each cylinder A, B and C from Figure 1 using $H/D_1, H/D_2$ and H/D_3 and calculate C_{Du} for each cylinder.
- (4) Obtain values of $f_{Hz} = C_{Dzu}/C_{Du}$ from Figure 3 or 4 for various values of z/H along each component i assuming that each component is part of the respective cylinder A, B or C.
- (5) Obtain values of $f_{sz} = C_{Dzs}/C_{Dzu}$ from Figure 5 for the same values of z/H as used in step 4.
- (6) Evaluate C_{Dzs} (based on V_z^2 and D_z) for each value of z (or z/H).

$$C_{Dzs} = C_{Du} f_{Hz} f_{sz} \quad (6.6)$$

step (3) (4) (5)