

METHODS FOR ESTIMATING THE PRESSURE DISTRIBUTION ON A TWO-DIMENSIONAL AEROFOIL IN VISCOUS TRANSONIC FLOW

SUMMARY

The Memorandum outlines the principal features of two methods (developed at the RAE, Farnborough) for calculating the pressure distribution, force and moment coefficients and boundary-layer and wake characteristics for a two-dimensional aerofoil in supercritical viscous flow when the boundary layer is fully attached over the aerofoil surface.

The methods are the Garabedian and Korn and Transonic Small Perturbation methods extended to take account of the boundary layer and wake. Viscous terms are added in the first case (the VGK method) using the transpiration model which allows the aerofoil ordinates to be retained as the surface about which the flow is calculated. In the second method (VISTRAN) a calculated displacement thickness is added iteratively to the aerofoil ordinates and the flow is calculated about the displacement surface. In both procedures Thwaites' method is used to calculate the laminar portion of the boundary layer and the lag-entrainment method of Green, Weeks and Brooman to calculate the turbulent portion.

Also VISTRAN has been extended to take into account the effects of tunnel constraints.

Computer programs in standard FORTRAN IV have been written for the methods. Comparisons are presented between the two methods in free-air, with Firmin's first-order method in near-critical flow in both free-air and in constrained flow and between VISTRAN and experimental data in wind-tunnel constrained flow. These comparisons form a small part, and are typical, of an extensive evaluation of the methods which is referenced. In this evaluation it was found that VISTRAN, with a few particular exceptions, yielded results in very good agreement with experiment but it was concluded that if a constrained-flow version of VGK were available it would probably provide even better agreement with experiment.

Thus for free-air calculations the use of VGK is recommended whilst VISTRAN has been shown to be a generally reliable method of obtaining constrained-flow results for direct comparison with, or extension of, wind-tunnel results. Finally the comment is made that although VGK cannot readily be extended to constrained-flow conditions good agreement between the method and experimental results has been obtained by making the calculations for the corrected experimental values of stream Mach number and lift coefficient to account for blockage and (if necessary) modifying the aerofoil camber line to account in some degree for wall-induced streamline curvature effects.

1. INTRODUCTION

Methods for calculating the pressure distribution on a two-dimensional aerofoil in attached viscous transonic flow have been developed at the RAE, Farnborough, a summary of that work (up to 1975) being presented in Reference 1.

Two methods are now established. They are extensions that take account of the boundary layer and wake, to two inviscid transonic flow methods^{2, 3}; that of Garabedian and Korn^{4, 5} (G & K) and a solution to the Transonic Small Perturbation (TSP) equation⁶⁻⁸.

Wind-tunnel constraint terms⁹⁻¹² have been incorporated into the viscous version of the TSP method and this form has been the subject of an extensive evaluation reported in Reference 13 performed by comparison with uncorrected experimental data. The evaluation also includes comparisons between the two methods in free-air conditions.

A summary of the inviscid methods (G & K and TSP) is given in Section 3, some brief details of the application of tunnel constraint terms follow in Section 4 and the methods of allowing for viscous effects are outlined in Section 5. Details of the viscous flow computer program (VGK and VISTRAN) are given in Sections 6 and 7 whilst in Section 8 a comment is made on the different procedures used to calculate drag coefficient. A summary of the evaluation of the methods¹³ is presented in Section 9, a few typical examples are given in Section 10, and conclusions including recommendations regarding the method to be preferred are provided by Section 11.

2. NOTATION

All linear quantities made non-dimensional are based on aerofoil chord.

a	speed of sound
C_{DF}	drag coefficient due to skin friction
C_{DP}	pressure drag coefficient
C_{DT}	total drag coefficient = $C_{DF} + C_{DP}$, or $C_{DV} + C_{DW}$
C_{DV}	drag coefficient due to viscous layers
C_{DW}	drag coefficient due to shock wave
C_L	lift coefficient
$C_{m\frac{1}{4}}$	pitching-moment coefficient about $\frac{1}{4}$ chord (positive nose up)
C_N	normal force coefficient
C_p	pressure coefficient based on M_∞
C_p^*	pressure coefficient for local Mach number of unity

c	aerofoil chord-length
F	wind-tunnel geometry parameter for slotted walls
H	wind-tunnel height ratio ($= h/2c$)
h	wind-tunnel height
K	transonic similarity parameter
M	Mach number
P	wind-tunnel wall porosity parameter
R_c	Reynolds number based on chord length
r	exponent of Mach number in denominator of K
t	aerofoil maximum thickness
u	local velocity component in x -direction
w	local velocity component in z -direction
x	non-dimensional coordinate in chordwise direction
z	non-dimensional coordinate measured from and normal to chord
$\bar{z}$	non-dimensional stretched z -coordinate ($= M_\infty^{2/3}(t/c)^{1/3}z$)
α	incidence
γ	ratio of specific heats of air (taken to be 1.4)
Δ	denotes a change to affixed value
θ	non-dimensional momentum thickness of viscous layer
Φ	velocity potential
φ	perturbation velocity potential
ϕ	transformed perturbation velocity potential ($= M_\infty^{2/3}(t/c)^{-2/3}\varphi$)

Suffixes

L	denotes local value
s	denotes camber value
x, z	denote partial differentials with respect x, z
$\bar{z}$	denotes partial differential with respect to $\bar{z}$
∞	denotes far upstream value in tunnel or free air

Abbreviations

NC	denotes non-conservative version of VGK
PC	denotes partially-conservative version of VGK

3. THE BASIC INVISCID FLOW METHODS

Two numerical methods for solving the potential flow equation for transonic flow were outlined in References 2 and 3. These two methods are the Garabedian and Korn solution and a solution of the Transonic Small Perturbation equation.

The practicability of the methods lies in the facility provided by modern computers to solve with great rapidity sets of non-linear partial differential equations by employing finite difference techniques, the infinite real flow field being represented by a computing plane with finite limits.

It should be noted that the methods are subject to continuing review and References 2 and 3 represent a statement of the current position at their time of issue.

3.1 The Garabedian and Korn Method

The method devised by Garabedian and Korn is described in References 4 and 5. It is referred to here as "G & K". Using a conformal transformation into the interior of a circle[†] and assuming isentropic flow the authors considered the equations of conservation of mass and energy; these may be combined in steady potential flow and expressed exactly in terms of the Neumann problem for the velocity potential, Φ , as

$$(a^2 - u^2)\Phi_{xx} - 2uw\Phi_{xz} + (a^2 - w^2)\Phi_{zz} = 0 \quad (3.1)$$

where $u (= \Phi_x)$, $w (= \Phi_z)$ are the velocity components and the speed of sound, a , is defined by Bernoulli's law as

$$a^2 = M_\infty^{-2} + \frac{1}{2}(\gamma - 1)[1 - u^2 - w^2], \quad (3.2)$$

where the velocities a , u and w are non-dimensionalised with respect to the freestream velocity.

This partial differential equation, with the Neumann boundary condition on the surface and the Dirichlet boundary condition at infinity, is represented by finite difference equations in the manner suggested by Murman and Cole⁶.

Since first becoming available in the UK and during its extension to take account of the boundary layer and wake, the method has been subject to continuing development; three particular developments being the incorporation of the Sells mapping procedure², a modification to the finite difference scheme to yield through a so-called partially-conservative scheme a better representation of shock waves²² and the incorporation of a crude model of the flow behind an aerofoil with a blunt base.

[†] See Figure 1 of Reference 2.

3.2 The Transonic Small Perturbation Method

Murman and Cole's paper⁶ presented a finite difference technique for the solution of the transonic small perturbation approximation of the full inviscid steady flow equations for non-lifting aerofoils. The method was extended and improved by Krupp and Murman⁷ and further improvements were made by Albane *et al*⁸. This latter version is that employed in British Industry and is known as "TSP".

The exact equation of motion for steady isentropic irrotational flow in two dimensions (Equation (3.1)) can be expressed in terms of small perturbations and, neglecting small terms, leads to the second-order partial differential equation

$$(1 - M_\infty^2)\phi_{xx} + \phi_{zz} = M_\infty^2[(\gamma + 1)\phi_x\phi_{xx} + (\gamma - 1)\phi_x\phi_{zz} + 2\phi_x\phi_{xz}] \quad (3.3)$$

where ϕ is the perturbation velocity potential.

For transonic flows the final two terms on the right-hand side of Equation (3.3) may be neglected. It is then shown in Reference 2 that by introducing a transformation of the perturbation velocity potential in terms of freestream Mach number and the aerofoil thickness/chord ratio and a stretched coordinate $\bar{z}$ in the z -direction the standard equation of motion of transonic small perturbation theory is finally obtained. This is

$$[K - (\gamma + 1)\phi_x]\phi_{xx} + \phi_{\bar{z}\bar{z}} = 0 \quad (3.4)$$

$$\text{in which} \quad K = \frac{1 - M_\infty^2}{M_\infty^r (t/c)^{2/3}} \quad (3.5)$$

is a transonic similarity parameter and r is an arbitrary index which may be chosen so that solutions of Equation (3.4) give the best agreement with more-exact theories.

Equation (3.4) is non-linear and of changing form: if the coefficient $K - (\gamma + 1)\phi_x$ is positive then the equation takes an elliptic form, but it is of hyperbolic form when the coefficient is negative.

The derivatives in Equation (3.4) are represented by finite difference approximations, reducing the differential equations to a set of algebraic equations for ϕ . A further development in the extension of the method to take account of the boundary layer and the wake has been the incorporation of a model of the flow behind an aerofoil with a blunt base, similar to the model introduced into VGK.

4. ALLOWANCE FOR TUNNEL INTERFERENCE

Murman⁹ has devised a method for allowing for the boundary conditions at the solid or perforated walls of a wind tunnel when calculating constrained transonic flows. This has been modified and improved by Catherall¹⁰ for use with the TSP method to give solutions also with slotted walls. Full details of the method can be found in Reference 10 and a concise summary is given in Section 2.2 of Reference 1. The method uses, instead of the far field boundary condition at $z \rightarrow \pm\infty$, the standard homogeneous boundary condition for a ventilated wall

$$\phi_x \pm FH\phi_{xz} \pm \frac{1}{P}\phi_z = 0 \quad (4.1)$$

on the walls at $z = \pm H$, where H is the half-height of the tunnel non-dimensionalised by the aerofoil chord-length. F is a function of the wall slot dimensions and P is a wall porosity parameter. (It should be noted that Murman⁹ only considered the case $F = 0$.)

The significance of the parameters F and P are described in Appendix A to Reference 11 and in Reference 12.

It should be recognised that considerable doubt exists as to the validity of the simple boundary conditions for ventilated walls. Calculation methods for wind-tunnel constrained-flow in fact provide a useful technique for evaluating the coefficient P by matching calculated and measured pressure distributions.

No method has been devised (in the UK) for incorporating tunnel constraint effects into the G & K program although some allowance for blockage and wall-induced streamline curvature effects can be made by adopting the artifices referred to in Section 11.4.

The incorporation of viscous effects into the TSP program (see Section 5) neglects the growth of the boundary layer on the tunnel walls and any changes far downstream due to the displacement of the wake.

In the evaluation of the viscous version of the TSP program (known as VISTRAN) reported in Reference 13, comparisons were made between results from the program for constrained flow and uncorrected experimental data. Thus the same standard of constraint correction was applied to each of the cases considered.

5. ALLOWANCE FOR VISCOUS EFFECTS

The incorporation of viscous effects into aerofoil analysis methods was largely neglected until the mid 1960's because, firstly, prior to the advent of fast digital computers, with the facility to provide rapid iterative solutions, accounting for viscosity was time consuming and, secondly, for earlier generations of aerofoils, that did not have extensive rear loading nor were required to generate extensive regions of supercritical flow, a reasonable approximation to the pressure distribution in viscous flow could be obtained from an inviscid flow calculation for the same lift coefficient.

The general principles that are now established for calculating pressure distributions on aerofoils with adequate allowance for viscous effects require the calculation of the development of the viscous layers (the boundary layer and the wake) under a given external pressure distribution and the calculation of the external inviscid flow with given internal boundary conditions. These two calculations are then matched by allowing the existence of a fictitious inviscid flow in the regions occupied by the viscous layers¹.

For the viscous versions of both the TSP and G & K programs (known by the acronyms VISTRAN and VGK respectively) the lag entrainment theory of Green, Weeks and Brooman¹⁴ is employed to predict the turbulent boundary layers and wakes. The laminar boundary layer, extending from the stagnation point[†] to the transition location, is calculated by Thwaites' method¹⁵ extended to compressible flow by the Stewartson-illingworth transformation¹⁶.

To incorporate these viscous effects into the TSP and G & K programs (to give VISTRAN and VGK) two equivalent models have been described by Lock^{1, 17}. Model A, which requires the calculated displacement thickness to be added to the aerofoil, is used with the TSP inviscid method in the VISTRAN program, the application being similar in principle to that used in Firmin's subcritical method¹¹. In this method the aerofoil boundary conditions are applied on the chord line and its downstream continuation.

The nature of the G & K program does not allow Model A to be used efficiently because the inviscid method involves mapping the aerofoil onto the interior of a circle, which would have to be repeated each time the boundary layer was recalculated. Model B, however, retains the aerofoil ordinates throughout the calculation procedure but allows a non-zero normal component of velocity at the aerofoil surface and along a line representing the wake and is therefore known as the "transpiration" model.

Full details of the application of viscous effects in the VISTRAN and VGK programs are given in References 1, 18, 19 and 20. The application of the methods is restricted to cases where the boundary layer is fully attached over the aerofoil surface.

6. THE VGK COMPUTER PROGRAM

Full details of the VGK computer program are given in References 20 and 21.

The inviscid solution (G & K), apart from the limitations of the numerical method, is exact in subcritical flow, but in supercritical flow approximations are introduced by the assumption of isentropic flow. The use of a partially-conservative system (see Reference 22) enables the shock pressure rise to match very closely that observed in experiment. The VGK program probably offers the most accurate viscous transonic flow method currently available.

The method, however, does have limitations due to the use of conformal mapping techniques which mean that it is not readily extended to include wind-tunnel constraint effects.

The program is run in two parts, firstly to obtain a mapping of the aerofoil into the circle plane (using the method of Catherall, Foster and Sells²³) and secondly to calculate the flow solution. The mapping result can be stored permanently on magnetic tape for use in subsequent flow calculations; similarly the flow result can also be stored to allow the calculation to be continued at a later date to an improved level of convergence or to be used as initial input to a calculation for new values of Mach number, incidence or other parameters.

[†] The TSP method does not yield a well-defined stagnation point, hence for VISTRAN the stagnation point is assumed to be at the leading edge and the pressures in this region are modified appropriately.

6.1 Computer Requirements

The computer program has been written in standard FORTRAN IV and the flow solution is divided into coarse-grid (80×15) flow calculations and fine-grid (160×30) flow calculations.

On the ICL 1906S computer the mapping requires 24K of core store and takes about 100 seconds of CPU (Central Processor Unit) time. The flow calculations require 59K of core store and take about 75 seconds, 300 seconds and 375 seconds respectively for 100 coarse-grid, fine-grid or boundary-layer iterations.

A typical initial flow calculation requires 150 coarse-grid, 150 fine-grid and 100 boundary-layer iterations to reach the recommended convergence levels, while a typical continuation run, with changed flow parameters but the same convergence level, from a previously converged result requires 50 coarse-grid, 100 fine-grid and 50 boundary-layer iterations.

6.2 Program Input

The input to the program is basically the same as that for the inviscid (G & K) program (see References 2 to 5) but the more-recent developments require three additional parameters to be specified¹³ in a first (inviscid flow) namelist. These provide for the selection of an inviscid or viscous flow solution, the degree of “conservatism” to be used (see Reference 22) and the location of the downstream closing point for an aerofoil with non-zero trailing-edge thickness.

A second namelist provides for the allocation of values to the input parameters that govern the boundary-layer calculation, such as Reynolds number, chordwise location of transition[†] and frequency of the interaction between the potential flow and boundary-layer calculations.

Also, a facility exists to run the program to achieve a specified value of lift coefficient. This requires the use of three additional namelist parameters¹³ to provide for C_L (rather than α) to be specified, to input the required value of C_L and to input the relaxation factor to be used in the approach to the required value of C_L .

Full details of input to the program and comments on aids to convergence may be found in References 13 and 21. In regard to convergence it has been found¹³ advantageous in reducing total computing time to employ a simple “marching” procedure, with the boundary layer recalculated after every third flow calculation.

6.3 Program Output

Results from the flow calculation are given in two line-printer files and on a magnetic tape file which constitutes the source file for continuation runs. One line-printer file contains the complete calculation record whilst the other contains the converged chordwise pressure coefficient distribution in a form suitable for use with a graphics system.

At each iteration details of the flow convergence level are output to the main file and after each boundary-layer calculation similar details for the viscous convergence level are given. On reaching convergence in either inviscid or viscous modes, or on reaching the iteration limit, the full calculation results are printed. These results consist of the pressures and local Mach numbers for the whole aerofoil and the wake, followed by listings of the boundary-layer parameters. Values of C_{DP} , C_L , $C_{m^{1/4}}$, C_{DV} , C_{DF} and C_{DT} are also output.

[†] There is provision for the locations to be different on upper and lower surfaces of the aerofoil and, if required, for the momentum thicknesses to be increased by specified amounts at the transition locations.