

A METHOD OF ESTIMATING A SEPARATION BOUNDARY FOR TWO-DIMENSIONAL AEROFOIL SECTIONS IN TRANSONIC FLOW

SUMMARY

A method is given of estimating a separation boundary for two-dimensional aerofoils in transonic flow in terms of lift coefficient and freestream Mach number. In this context transonic flow denotes the range of subsonic freestream Mach numbers for which the flow around an aerofoil embodies one or more local supersonic regions which may be terminated by shock waves. The method is limited to conditions where the shock interacts with a turbulent boundary layer. The boundary referred to is identified in experiment by a decrease in trailing-edge pressure coefficient (with increasing lift coefficient or freestream Mach number) of 0.05 relative to its trend in attached flow. The flow separations defining this boundary occur on the aerofoil upper surface and are due to the combined effects on the boundary layer of the shock waves terminating the local regions of supersonic flow and adverse pressure gradients associated with the pressure recovery to the trailing edge.

The method has been derived from a considerable range of experimental data but its validity has not been checked extensively against experimental data other than those from which it was derived. However, there is some evidence, from an attempt to adapt the method herein to the prediction of flow separation on finite wings¹³, that, under the circumstance where a wing design is such that the wing can support a long bubble downstream of the shock without the trailing edge pressure being significantly disturbed, the method provides a considerable underestimate of the observed separation condition as defined here (Section 2.2). It is conceivable that this situation would carry across to aerofoils also, if the aerofoil designs were such as to support a similar condition.

The small amount of scatter in the correlations that constitute the criteria suggest that in the majority of cases the method should predict the separation boundary to an accuracy of ± 0.03 in C_L in the region of a boundary where $C_{L\text{ sep}}$ varies relatively slowly with Mach number and to an accuracy of ± 0.005 in M_∞ in the region of a boundary where $M_{\infty\text{ sep}}$ varies relatively slowly with lift coefficient.

1. INTRODUCTION

A knowledge of conditions at which wing section buffet onset will be encountered in transonic flow is frequently a crucial factor influencing the choice of aerofoil profile and wing size for both civil and military aircraft. In the early stages of aircraft design, therefore, it is often necessary, for two-dimensional aerofoils representative of the wing, to provide estimates of a boundary corresponding to significant flow separation. A knowledge of this boundary identifies the start of a region in which the flow no longer leaves the trailing edge smoothly and buffet may be encountered.

The separation in question occurs on the upper surface of a two-dimensional aerofoil when the, still subsonic, freestream Mach number is sufficiently high for the flow past the aerofoil to develop one or more local supersonic regions terminated by a shock wave. Separations particular to the lower surface of an aerofoil are not considered here.

The various types of shock-induced separation that can occur at subsonic freestream Mach numbers have been classified by Pearcey, Haines and Osborne^{1, 14}. They may be grouped into two broad categories.

- (a) Types A and B1, in which separation is first manifest as a bubble at the foot of the shock wave and then spreads rearwards as incidence or stream Mach number is increased either to reach the trailing edge (Type A) or to link up with a rear separation that forms subsequently to (and is provoked by) the formation of the bubble (Type B1).
- (b) Types B2 and B3, in which the flow separates, first, at the trailing edge (rear separation) either provoked by a shock wave (Type B2) or by the severity of the pressure gradient even in the absence of a shock wave (Type B3). With increasing incidence and/or stream Mach number the separation spreads forward from the trailing edge to interact with a shock wave (Types B2b and B3b) that in the case of Type B3b has developed subsequent to the rear separation, or to interact with a bubble separation (Types B2a and B3a) that in both types has developed subsequent to the rear separation.

Thus in these two broad categories the dominant features are that in

- (a) separation first occurs at the foot of the shock
- whilst in
- (b) separation first occurs immediately forward of the trailing edge.

In the development of the method of predicting separation given here it has been found that these two categories correspond roughly to two separation criteria with separation being predicted to occur when either one is first invoked as lift coefficient or Mach number is increased. These criteria are given in Section 3. They correspond to a condition on the upper surface of a two-dimensional aerofoil for which the shock strength and the adverse pressure gradient, for the range of experimental data considered, are sufficiently severe for the trailing edge pressure coefficient, relative to its trend with increasing lift coefficient or Mach number in attached flow, to have, in the real viscous partly-separated flow, decreased by 0.05.

The criteria do not require separated flow to be modelled. In the method of prediction given here they are applied to a flow calculation scheme that, strictly, fails for flows that are close to separation. This arises because:

- (a) the boundary-layer component of the scheme does not include certain terms that become important close to separation², and
- (b) some boundary-layer parameters, over local regions of the aerofoil surface, are allowed artificial values to avoid convergence problems that are found with the scheme as separation is approached.

Thus the problem of the boundary layer method needing to represent the occurrence of separation has been deliberately avoided.

Nevertheless, by “calibrating” results obtained from the scheme against experimental data it has been found possible to derive a method of predicting the onset of significant separation. The calculated pressure distribution corresponding to this condition is not, however, entirely representative of the real flow.

The method, detailed in Section 4, leads to the prediction of a separation boundary with lift coefficient and free-stream Mach number as parameters. In the development of the method, described in detail in Reference 3, the viscous Garabedian and Korn computer program⁴ (VGK) which incorporates the RAE lag entrainment boundary layer theory⁵ has been used and it is recommended that this program should be used in the application of the method.

In addition to its application to define a separation boundary for an individual aerofoil the method should be applicable to families of aerofoils, related through systematic changes to their major parameters, to provide exchange rates at the separation condition between these parameters.

There may be a temptation to use the first criterion in Section 3 in an inverse sense to provide guidance on what local changes or refinements are necessary to improve the separation boundary of a particular aerofoil design. However this possibility has not been investigated and so the use of the criterion in this way cannot be advocated.

2. NOTATION AND DEFINITIONS[†]

2.1 Notation

All linear quantities made non-dimensional are based on aerofoil chord.

Symbols in square brackets are those used in the VGK program output.

C_L	[CL]	lift coefficient
C_p	[CP]	pressure coefficient
$\bar{H}$	[HT]	transformed boundary layer velocity-profile shape parameter
M_∞	[EM]	freestream Mach number
M_L	[EM]	local Mach number on aerofoil surface
R_c	[RE]	Reynolds number based on chord length
R_θ	[RTHETA]	Reynolds number based on momentum thickness
t		non-dimensional aerofoil thickness
x	[X]	non-dimensional co-ordinate in chordwise direction
z_s		non-dimensional aerofoil camber ordinate measured normal to conventional chord line passing through nose and mid point of trailing edge
α	[ALP]	angle of incidence
Δ		denotes an increment in affixed parameter
δ	[DELTA]	non-dimensional thickness of viscous layer [†]
θ	[THETA]	non-dimensional momentum thickness of viscous layer

[†] In the RAE lag entrainment boundary layer method used in VGK δ approximates to the distance normal to the aerofoil surface (non-dimensionalised by aerofoil chord) at which the local streamwise velocity has reached 0.995 of the value at the edge of the boundary layer but with the precise value depending on Mach and Reynolds numbers.

Suffixes

max	denotes maximum value
sep	denotes separation
sh	denotes shock-upstream location
te	denotes trailing edge
tr	denotes transition
$\delta(te)$	denotes one boundary-layer thickness forward of trailing edge, where boundary layer thickness is at trailing edge itself

2.2 Definitions**(i) Observed separation condition**

A condition $C_{L\ sep}$, $M_{\infty\ sep}$ obtained from experimental data corrected to free-air conditions at which the trailing edge pressure coefficient has, with increasing lift coefficient or freestream Mach number, decreased by 0.05 from its trend in attached flow.

(ii) Observed separation boundary

The locus in the C_L , M frame of points $C_{L\ sep}$, $M_{\infty\ sep}$.

(iii) VGK

An extension to viscous flow of the method of Garabedian and Korn for solving the transonic potential flow about two-dimensional aerofoils (see Reference 4).

Pertinent quantities in the VGK method and their recommended values are:

<i>Parameter</i>	<i>Purpose</i>	<i>Recommended Value</i>
EP	Governs the order of the numerical solution. EP = 0 gives a first-order solution in the supersonic region, whilst EP $\rightarrow$ 1 gives a second-order solution.	0.8
QCF	Governs the strength and location of the shock wave between the non-conservative and conservative extremes.	0.25 to 0.4
RES	A measure of convergence of the inviscid part of the solution.	$\leq 0.5 \times 10^{-4}$
VRES	A measure of convergence of the viscous part of the solution.	$\leq 0.1 \times 10^{-2}$
XTE	Governs the flow model for an aerofoil with a blunt base.	$5t_{te}$

3. THE CRITERIA FOR PREDICTING SEPARATION

Two criteria are applied:

- (I) at the shock up-stream location[†],
- (II) one boundary layer thickness ahead of the trailing edge.

$M_{\infty \text{ sep}}$, $C_{L \text{ sep}}$ are the lower pair of values that satisfy the criteria.

The criteria are:

- (I) For a given value of the non-dimensionalised momentum thickness at the shock-upstream location the local Mach number at the same location has the value given by Figure 1a, or
- (II) $\bar{H}_{\delta(te)} = 2.35 - 0.3 \log_{10}[R_c/(6 \times 10^6)]$. (3.1)

(For convenience this equation is given in graphical form in Figure 1b.)

The application of these criteria is described in detail in Section 4.

The greater confidence lies in the shock-upstream criterion for reasons given in Sections 4.2, 5.2 and 5.3. However in the interpretation of results obtained with this criterion it should be remembered that at the predicted separation boundary VGK yields an equivalent pressure distribution (see Section 5.3) that may differ from the pressure distribution that would occur at the separation condition in the real viscous flow. Because of the way separation develops⁶ these differences are likely to be most manifest in the shock strength and location.

Furthermore it should not be inferred from the trailing edge criterion that an aerofoil can be redesigned to give an improved separation boundary simply by making a *localised* change to the geometry near the trailing edge that significantly reduces the values of $\bar{H}_{\delta(te)}$, compared with that given by the original geometry, but leaves the values of $\bar{H}$ away from $x = 1 - \delta_{te}$ substantially unchanged. A further comment concerning the application of the trailing-edge criterion, depending on the shape of the $\bar{H}$ vs x distribution as the trailing edge is approached, is contained in Section 4.1.2.

[†] The shock-upstream location is defined in step (iii) of Section 4.1.1.

4. THE METHOD

The method requires the aerofoil geometry and the flow conditions[†] to be specified for which the aerofoil pressure distribution and associated boundary layer development are to be calculated.

The criteria of Section 3 have been determined for a particular calculation scheme employing VGK. However the ideas leading to the criteria could be applied to any computational scheme because the concept (Section 5.3) basic to the criteria is one of calibrating a calculation method against experimental data. Where an inviscid flow and/or boundary layer theory alternative to those in VGK is preferred, and adequate experimental data are available, the process of calibration (fully described in Reference 3) could be repeated and criteria obtained that are appropriate to the preferred methods of calculation.

However, in the application of the procedures given here the required pressure distribution should be calculated using VGK employing the standard fine grid (160×30) because this was used in the development of the method. If another method is used there may be some loss in accuracy.

The separation boundary determined by the procedure that follows is formed from the locus of the points $M_{\infty \text{ sep}}, C_{L \text{ sep}}$ and will normally be associated with the shock wave and boundary layer development on the upper surface of the aerofoil. For cambered aerofoils at small positive or at negative values of lift coefficient the boundary may be associated with the shock wave and/or boundary layer development on the lower surface but this circumstance has not been investigated in the development of the method.

4.1 Procedure

The procedure requires that the surface pressure distribution and the boundary layer development on the aerofoil be determined along a series of calculation paths starting from points for which any shock waves on the upper surface are of only modest strength and the adverse pressure gradients are not severe.

It is recommended that the calculation paths should be at constant freestream Mach number where $C_{L \text{ sep}}$ is expected to have its higher values but at constant incidence where $M_{\infty \text{ sep}}$ is expected to be high. (In the latter case calculations at constant C_L would be an alternative, but see Appendix A.)

The individual steps in the procedure follow. They are given first for the shock-upstream criterion and then for the trailing-edge criterion.

4.1.1 Application of the shock-upstream criterion

A diagram illustrating the application of the shock-upstream criterion is provided in Figure 2, to which the sketch numbers introduced below refer.

- (i) Select pairs of values of (C_L, M_{∞}) for which it is anticipated that the shock wave on the upper surface of the aerofoil being considered will be of only modest strength. These are the starting points in Sketch 1. Here, knowledge of the pressure distribution and separation boundary of an aerofoil of similar thickness and camber to that being evaluated may be helpful.
- (ii) At each starting point selected in step (i) and at two or three other points at the same M_{∞} or α (as may be appropriate) as these starting points obtain converged solutions, employing the fine grid (160×30), for the values of Reynolds number and transition locations for which the separation boundary is required. Calculations with the same value of M_{∞} or α constitute the calculations along a calculation path.

[†] The flow conditions include the Reynolds number and the upper and lower surface transition locations for which the separation boundary is required.

- (iii) For each converged calculation identify the two surface grid points in the vicinity of the shock wave on the upper surface between which the greatest decrease in local Mach number occurs. Label these two points as locations 5 and 4, moving in the direction of decreasing x . Then counting forward in the same direction identify location 2 (Sketch 2). This is taken as the shock-upstream location and the values of local Mach number and non-dimensional boundary layer momentum thickness at this location are designated M_{Lsh} and θ_{sh} .
- (iv) For each calculation path plot M_{Lsh} against θ_{sh} and determine the values of M_{Lsh} and θ_{sh} for which these plots intersect the curve in Figure 1a (Sketch 3).

These values are designated $(M_{Lsh})_{sep}$ and $(\theta_{sh})_{sep}$.

Note that if the calculations have not been extended to sufficiently high values of incidence or freestream Mach number for these plots to intersect the curve of Figure 1a some modest extrapolation of these plots will not usually lead to significant error.

- (v) (a) For each calculation path at constant M_∞ , plot θ_{sh} against lift coefficient (*i.e.* the value of C_L output by the calculations) as shown by Sketch 4. Determine the value of C_L that corresponds to $(\theta_{sh})_{sep}$. This value of C_L is $C_{L\ sep}$, (Sketch 4) whilst the value of M_∞ for which the calculations have been made is, of course, $M_{\infty\ sep}$.

An alternative is to plot M_{Lsh} vs C_L and then the value of $C_{L\ sep}$ is that which corresponds to $(M_{Lsh})_{sep}$.
- (b) For each calculation path at constant α , plot θ_{sh} against freestream Mach number as shown by Sketch 5. Determine the value of M_∞ that corresponds to $(\theta_{sh})_{sep}$ (Sketch 5). Then for each calculation path plot lift coefficient (*i.e.* the value of C_L output by the calculations) against freestream Mach number and, for the value of M_∞ just determined, determine the corresponding value of C_L (Sketch 6). These values of freestream Mach number and lift coefficient are $M_{\infty\ sep}$ and $C_{L\ sep}$.

Here an alternative is to plot M_{Lsh} vs M_∞ and then the value of $M_{\infty\ sep}$ is that which corresponds to $(M_{Lsh})_{sep}$.
- (iv) Collect together the pairs of values of $M_{\infty\ sep}$ and $C_{L\ sep}$ for each calculation path (Sketch 10(a)) and proceed to Section 4.1.2.

4.1.2 Application of the trailing-edge criterion

The trailing-edge criterion has been derived from aerofoils for which the $\bar{H}$ vs x distribution reached its maximum value (apart possibly from an excursion of small chordwise extent immediately downstream of the shock) at the trailing edge or at some value of x just forward of the trailing edge where $\bar{H}_{max}$ exceeded $\bar{H}_{te}$ by only a small amount (less than 0.1). If the $\bar{H}$ vs x distribution is of neither of these forms the applicability of the trailing-edge criterion will be in doubt.

A diagram illustrating the application of the trailing-edge criterion is provided in Figure 3, to which the sketch numbers introduced below refer.