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Lift and drag of two staggered lifting surfaces at low speeds

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LIFT AND DRAG OF TWO STAGGERED LIFTING SURFACES AT LOW SPEEDS

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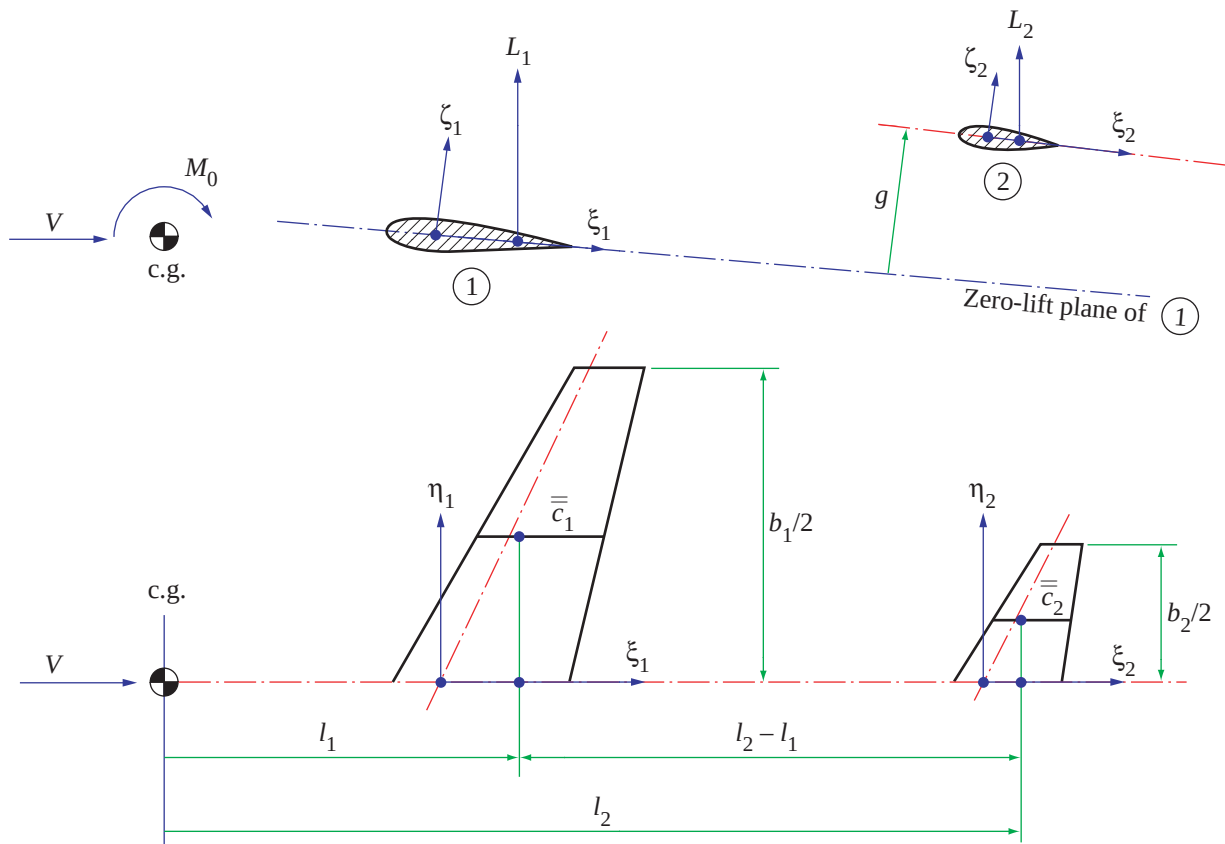
LIFT AND DRAG OF TWO STAGGERED LIFTING SURFACES AT LOW SPEEDS

1. NOTATION AND UNITS (see Sketch 1.1)

		SI	British
A	aspect ratio, b^2/S		
b	span of lifting surface	m	ft
C_D	drag coefficient, D/qS		
C_L	lift coefficient, L/qS		
C_m	pitching moment coefficient, $M/qS\bar{c}$		
C_{m0}	zero-lift pitching moment coefficient, $M_0/qS\bar{c}$		
$\bar{c}$	aerodynamic mean chord of lifting surface	m	ft
D	drag	N	lbf
G	gap parameter, g/b_1 when $b_2 \leq b_1$, g/b_2 when $b_2 \geq b_1$		
g	gap between aerodynamic centres of lifting surfaces measured normal to zero-lift plane of leading surface (see Sketch 1.1)	m	ft
H	dimensionless streamwise distance between aerodynamic centres of lifting surfaces, $(l_2 - l_1)/(b_1/2)$ when $b_2 \leq b_1$, $(l_2 - l_1)/(b_2/2)$ when $b_2 \geq b_1$		
H_{pe}	downwash parameter for elliptic loading (Equation (3.19))	radian	radian
$\bar{H}_{pe}$	average over span of H_{pe} at quarter-chord line	radian	radian
k	dummy variable in Equation (3.19)		
L	lift	N	lbf
l	distance from centre of gravity to aerodynamic centre of lifting surface, positive aft (see Sketch 1.1)	m	ft
M	pitching moment	N m	lbf ft
M_0	pitching moment at zero lift	N m	lbf ft
m	$\tan \Lambda_{1/4}$ in Equation (3.19)		
q	kinetic pressure, $\frac{1}{2}\rho V^2$	N/m ²	lbf/ft ²

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S	planform area of lifting surface	m^2	ft^2
V	free-stream velocity	m/s	ft/s
x, y, z	orthogonal co-ordinates with origin at quarter-chord point of lifting surface centre-line chord, x -axis located on intersection of plane of symmetry and zero-lift plane of lifting surface, positive aft; y -axis normal to plane of symmetry, positive to starboard; z -axis normal to zero-lift plane of lifting surface, positive upwards	m	ft
α	angle of attack	degree	degree
$\bar{\epsilon}$	average downwash angle over lifting surface, assumed positive if downwash, negative if upwash	degree	degree
$\Lambda_{1/4}$	sweepback of quarter-chord line	degree	degree
ξ, η, ζ	$2x/b, 2y/b, 2z/b$ (see Sketch 1.1)		
λ	ratio of lifting surface tip chord to centre-line chord		
μ	span ratio, b_2/b_1 when $b_2 \leq b_1$, b_1/b_2 when $b_2 \geq b_1$		
ρ	free-stream density	kg/m^3	$slug/ft^3$
σ	biplane interference factor		
<i>Subscripts</i>			
f	relating to foreplane		
min	denotes condition for minimum vortex drag		
t	relating to tailplane		
v	relating to vortex drag		
w	relating to wing		
0	relating to zero-lift		
$1, 2$	relating to lifting surface 1 or 2		
$[]_1$	denotes parameter for combination based on span or on planform area and/or aerodynamic mean chord appropriate to surface 1		



Sketch 1.1 Notation for general configuration

2. INTRODUCTION

One of the important factors to be considered in the very early design stages of a project is the choice of layout for the lifting surfaces involved. The choice is wide and can vary between two extremes. At one extreme, the most common, there is the use of two lifting surfaces, of widely different areas and spans, the larger one (the wing) providing most of the lift and the smaller one (the horizontal stabiliser) providing trim in pitch. At the other extreme there is the use of two lifting surfaces of very similar sizes, both contributing significantly to lift with the trim in pitch being provided by controls on one surface. When the choice has been made of which philosophy to adopt there still remains the further choice of whether pitch control shall be provided by the leading or trailing surface. The choice of layout and the relative size and disposition of one lifting surface with respect to the other and the centre of gravity will affect the way in which the lift of the combination is divided between the two surfaces; but perhaps more significantly the total drag of the combination will be affected. The opportunity therefore arises in the early design stages of varying the layout in order, for example, to minimise the drag of the combination, trimmed in pitch with a given lift coefficient and static margin.

Obviously a proper optimisation process is a complex one involving not only modifications to a datum layout of two lifting surfaces by varying their relative dispositions and planforms but also modifications to their camber and twist. Reference 8 provides a method, using vortex-lattice theory, of minimising the vortex drag of two lifting surfaces trimmed in pitch, but even that method is an approximation of the real situation since it does not account for viscous effects or for the presence of a body. Nor is the resulting combination necessarily stable since static margin is not an input parameter in that method.

The aim of the present Item is to provide a much simplified first-order method (based on Derivation 3) using lifting-line principles. The method provides a means of estimating the effect of the variation in geometry and disposition of two straight tapered lifting surfaces with respect to each other and the centre of gravity on the lift and drag of the combination, assumed trimmed in pitch and having a given static margin. The method makes the assumption of elliptic loading on the lifting surfaces. This makes the problem more tractable and enables the Item to be simple and self-contained and is not felt to be too much of a limitation in practice since most practical designs would probably employ planforms giving a reasonable approximation to an elliptic loading.

The resulting geometries established from an analysis using this simple method would then probably be analysed at a later stage in the design process using more sophisticated methods. The method is also felt to be suitable for incorporation into multivariate analysis programs since it provides an adequate means of evaluating the relative performance of a large number of configurations with little effort.

An outline of the method is given in Section 3 while a fuller development up to Equation (3.12) is given in Appendix B. An assessment of the applicability and accuracy of the method is given in Section 5 and two examples illustrating possible uses are given in Section 7. Details of a computer program for the method are given in Appendix A, see also Section 4.

3. LIFT AND DRAG OF TWO STAGGERED LIFTING SURFACES AT LOW SPEEDS

3.1 General Considerations

The drag of the combination of lifting surfaces illustrated in Sketch 1.1 may be considered as the sum of the zero-lift drag and the lift-dependent drag (assumed inviscid here) arising from the trailing vortex system, *i.e.*

$$[C_D]_1 = [C_{D0}]_1 + [C_{Dv}]_1. \quad (3.1)$$

It should be reiterated here that throughout this Item the use of the notation $[]_1$ denotes a parameter for the combination based on span or on planform area and/or aerodynamic mean chord appropriate to surface 1, *i.e.* b_1 or S_1 and/or $\bar{c}_1$.

If it is assumed that the zero-lift drag coefficients of the two lifting surfaces are equal, *i.e.* $C_{D01} = C_{D02}$, then

$$[C_{D0}]_1 = C_{D01} \left(1 + \frac{S_2}{S_1} \right). \quad (3.2)$$

For two staggered lifting surfaces Prandtl and Munk (References 4 to 6) derived the vortex drag of the combination, assuming elliptic loading on each surface, as

$$[C_{Dv}]_1 = \frac{[C_L]_1^2}{\pi A_1} \frac{1}{(1 + L_2/L_1)^2} \left[1 + 2\sigma \frac{b_1}{b_2} \frac{L_2}{L_1} + \left(\frac{b_1}{b_2} \frac{L_2}{L_1} \right)^2 \right] \quad (3.3)$$

in which σ is the Prandtl biplane interference factor given in Figure 1 as a function of μ and $|G|$. A good approximation (within about ± 0.005) to σ over the range $0.05 \leq |G| \leq 0.4$ is given by

$$\sigma = \left\{ 1 - 2|G| \left[1 + (2|G|)^2 \right]^{-1/2} \right\} (\mu - K) + K \left(\frac{1 - 0.66|G|}{1.05 + 3.7|G|} \right) \quad (3.4)$$

where
$$K = \mu^{3.2} - (\mu^{3.2} - \mu^5) e^{-800|G|^3}. \quad (3.5)$$

For $G = 0$, $\sigma = \mu$ and Equation (3.4) gives errors greater than those quoted for $0.05 \leq |G| \leq 0.4$ when $\mu \geq 0.6$.

The minimum vortex drag is obtained when

$$\frac{L_2}{L_1} = \frac{b_2/b_1 - \sigma}{b_1/b_2 - \sigma} \quad (3.6)$$

giving
$$[C_{Dv}]_{1min} = \frac{[C_L]_1^2}{\pi A_1} \left[\frac{1 - \sigma^2}{1 - 2\sigma b_2/b_1 + (b_2/b_1)^2} \right] \quad (3.7)$$

in which $[C_L]_1^2 / \pi A_1$ is the vortex drag coefficient for an isolated lifting surface (1) having the same lift coefficient as the combination.

The total drag coefficient of the combination is given by the sum of Equations (3.2) and (3.3) as

$$\frac{[C_D]_1}{[C_L]_1^2 / \pi A_1} = \frac{C_{D01}}{[C_L]_1^2 / \pi A_1} \left(1 + \frac{S_2}{S_1} \right) + \frac{1}{(1 + L_2/L_1)^2} \left[1 + 2\sigma \frac{b_1}{b_2} \frac{L_2}{L_1} + \left(\frac{b_1}{b_2} \frac{L_2}{L_1} \right)^2 \right]. \quad (3.8)$$

3.2 Practical Constraints

In practice, for trimmed, stable, straight and level flight, a lift coefficient and static margin for the combination need to be specified.

The trim constraints on lift and pitching moment lead to the expression

$$\frac{L_2}{L_1} = \frac{-l_1/\bar{c}_1 + [C_{m0}/C_L]_1}{l_2/\bar{c}_1 - [C_{m0}/C_L]_1}. \quad (3.9)$$

The lifting surface moment arms $l_1/\bar{c}_1$ and $l_2/\bar{c}_1$ in Equation (3.9) are given by the trim and stable flight constraints as

$$\frac{l_1}{\bar{c}_1} = - \left[\frac{dC_m}{dC_L} \right]_1 - \frac{(l_2 - l_1)}{\bar{c}_1} \frac{F}{1 + F} \quad (3.10)$$

$$\text{and} \quad \frac{l_2}{\bar{c}_1} = - \left[\frac{dC_m}{dC_L} \right]_1 + \left(\frac{l_2 - l_1}{\bar{c}_1} \right) \frac{1}{1 + F} \quad (3.11)$$

$$\text{in which} \quad F = \frac{S_2}{S_1} \frac{dC_{L2}/d\alpha_2 (1 - d\bar{\varepsilon}_1/d\alpha_1)}{dC_{L1}/d\alpha_1 (1 - d\bar{\varepsilon}_2/d\alpha_2)} \quad (3.12)$$

In Equation (3.12) $d\bar{\varepsilon}_1/d\alpha_1$ is the average downwash gradient over lifting surface 2 due to lifting surface 1 and $d\bar{\varepsilon}_2/d\alpha_2$ is the average downwash gradient (actually an upwash gradient) over lifting surface 1 due to lifting surface 2.

The moment arms in Equation (3.9) to (3.11) may, if required, be expressed in terms of the semi-span of lifting surface 1 via the expression

$$\frac{b_1/2}{\bar{c}_1} = \frac{3A_1(1 + \lambda_1)^2}{8(1 + \lambda_1 + \lambda_1^2)} \quad (3.13)$$

For example, the parameter $(l_2 - l_1)/\bar{c}_1$ may be expressed as

$$\frac{l_2 - l_1}{\bar{c}_1} = \frac{3A_1(1 + \lambda_1)^2}{8(1 + \lambda_1 + \lambda_1^2)} [H]_1 \quad (3.14)$$

$$\text{where} \quad [H]_1 = \frac{l_2 - l_1}{b_1/2} \quad (3.15)$$

For a wide range of planforms the expression

$$\frac{dC_L}{d\alpha} = \frac{2\pi A}{2 + \sqrt{4 + (A/\cos\Lambda_{1/4})^2}} \quad (3.16)$$

for lift-curve slope, obtained from Derivation 2, is in good agreement (within about ± 5 per cent) with values derived from lifting-surface theory.

The average downwash gradient, $d\bar{\varepsilon}/d\alpha$, compatible with the rest of the theory, *i.e.* assuming elliptic loading and a planar vortex wake, can be determined from the equation

$$\frac{d\bar{\varepsilon}}{d\alpha} = \frac{2}{\pi} \frac{dC_L/d\alpha}{A} \bar{H}_{pe} \quad (3.17)$$

in which $\bar{H}_{pe}$ is the average value of the downwash parameter H_{pe} over the lifting surface,

$$\text{i.e.} \quad \bar{H}_{pe} = \int_0^1 H_{pe} d\eta \quad (3.18)$$

where

$$H_{pe} = \frac{1}{2\pi} \int_0^1 \sum_{k=-1}^{+1} \left\{ \left[\frac{\xi - km\eta}{(\xi - km\eta)^2 + \zeta^2(1+m^2)} \right] \left[\frac{m\xi + k\eta}{\sqrt{\xi^2 + \eta^2 + \zeta^2}} + \frac{(1+m^2)\eta' - m\xi - k\eta}{\sqrt{(\xi - m\eta')^2 + (\eta - k\eta')^2 + \zeta^2}} \right] \right. \\ \left. + \left[\frac{k\eta + \eta'}{(\eta + k\eta')^2 + \zeta^2} \right] \left[1 + \frac{\xi - m\eta'}{\sqrt{(\xi - m\eta')^2 + (\eta' + k\eta)^2 + \zeta^2}} \right] \right\} \frac{\eta'}{\sqrt{1-\eta'^2}} d\eta' \quad (3.19)$$

where $\xi = 2x/b$, $\eta = 2y/b$ and $\zeta = 2z/b$ (see Sketch 1.1) and η' is the variable of integration.

Equation (3.19) was obtained from Derivation 1 for a system of swept horseshoe vortices.

The ξ -axis is located in the planar vortex sheet in the plane of symmetry. For the purposes of this Item it is reasonable to assume that the vortex sheet lies in the zero-lift plane of the lifting surface.

In order to evaluate Equation (3.19) it is necessary to know the relative location of the two quarter-chord lines in the ξ, η, ζ axis system. For this purpose it is a satisfactory approximation, firstly, to assume that the aerodynamic centres are located on the quarter-chord points of the aerodynamic mean chords and, secondly, to assume that their distance apart in the ξ direction is $(l_2 - l_1)$. By this means the location of centre-line quarter-chord points can be established, thus fixing the origins of the ξ, η, ζ axis systems for the two lifting surfaces.

In calculating the average downwash parameter using Equation (3.18) the H_{pe} values are assumed to be those appropriate to the quarter-chord line.

The relationships for lift-curve slope and average downwash gradient for each lifting surface calculated from Equations (3.16)* and (3.17)* are substituted into Equation (3.12) to obtain F . Then, knowing the distance between the aerodynamic centres of the lifting surfaces, $(l_2 - l_1)/\bar{c}_1$, and the required static margin, $[dC_m/dC_L]_1$, together with the value of F , Equations (3.10) and (3.11) can be used to find the lifting surface moment arms, $l_1/\bar{c}_1$ and $l_2/\bar{c}_1$. These are substituted into Equation (3.9) to obtain the lift ratio, which is in turn used in Equations (3.3) and (3.8) to obtain the vortex and total drag coefficients.

3.3 Conversion of Equations for Particular Cases of Wing-tailplane and Wing-foreplane Combinations

The equations in the foregoing sections have been formulated in terms of two lifting surfaces denoted by subscripts 1 and 2, in which 1 is assumed to be the upstream, or leading, surface.

In order to convert the equations for use in the two cases of a wing-tailplane and wing-foreplane combination, apart from the substitution of, say, $w(\text{wing})$ for 1 and $t(\text{tail})$ for 2 in the former case and $f(\text{foreplane})$ for 1 and $w(\text{wing})$ for 2 in the latter case, the following important point should be noted.

The lift, drag and pitching moment coefficients for the combination are based on planform area and aerodynamic mean chord appropriate to **surface 1** (as denoted by $[dC_m/dC_L]_1$ or $[C_{m0}/C_L]_1$ for example). The moment arms are also made dimensionless by means of the aerodynamic mean chord of **surface 1** (e.g. $l_1/\bar{c}_1$). It is common practice in aircraft design to base the force and moment coefficients and lengths for the combination on a reference area and length associated with the wing. This should be borne in mind

* Values obtained from other methods may be used, however, if they are felt to be more applicable to particular cases.

when converting the equations given herein for a **wing-foreplane** combination and, as an example, the conversion of Equations (3.10) and (3.12) for this case will be considered.

$$\frac{l_f}{\bar{c}_f} = - \left[\frac{dC_m}{dC_L} \right]_f - \frac{(l_w - l_f)}{\bar{c}_f} \frac{F}{1 + F} \quad (3.10a)$$

in which

$$F = \frac{S_w}{S_f} \frac{dC_{Lw}/d\alpha_w}{dC_{Lf}/d\alpha_f} \frac{(1 - d\bar{\epsilon}_f/d\alpha_f)}{(1 - d\bar{\epsilon}_w/d\alpha_w)} \quad (3.12a)$$

If the substitution for F from Equation (3.12a) and

$$\left[\frac{dC_m}{dC_L} \right]_f = \left[\frac{dC_m}{dC_L} \right]_w \frac{\bar{c}_w}{\bar{c}_f}$$

are made in Equation (3.10a), the resulting equation for the stabiliser arm

$$\frac{l_f}{\bar{c}_w} = - \left[\frac{dC_m}{dC_L} \right]_w - \frac{(l_w - l_f)}{\bar{c}_w} \frac{F}{1 + F} \quad (3.10b)$$

can be shown to be identical to the stabiliser arm determined from Equation (3.11) when applied to a wing-tailplane combination.

4. PROGRAM

Even though the method is based on the simplest possible principles consistent with practicality it is still relatively complex to use, mainly due to the determination of $d\bar{\epsilon}/d\alpha$. In addition there are a considerable number of parameters which can be varied in using the method. This being so the method has been programmed as described in Appendix A.

5. APPLICABILITY AND ACCURACY

The method of this Item is based on lifting-line considerations applied to two staggered lifting surfaces having elliptic spanwise loadings, trimmed in pitch and with a specified static margin. No effects of viscosity have been taken into account. In addition no effects due to the presence of a body have been included so that the method might be expected to apply only to configurations having bodies which are small in diameter compared with the span of the larger lifting surface (less than 10 per cent, say).

The method makes the assumption of a flat vortex sheet. This was necessary to solve the problem over the complete range of possible configurations, but it is a reasonable assumption in most cases since, within the range of lift coefficients for fully attached flow over a lifting surface the vortex wake remains substantially flat for at least a semi-span downstream. Rolling up of the vortex wake is therefore only likely to be of significance for configurations in which the span of the leading surface is considerably less than that of the trailing surface. For such configurations the average downwash over the trailing surface is likely to be less significant with regard to overall lift. For swept wings it could be significant with regard to pitching moment because of the aerodynamic twist induced by the flow field of the leading surface.

The method is a first-order one in which the lift distribution on each surface is assumed to remain elliptic in the presence of the other. In this respect the downwash gradient for the trailing surface is calculated assuming that the lift on the leading surface is unaffected by the upwash from the trailing surface and vice versa. This is, however, only likely to be of any significance for very close-coupled configurations, for which the simple lifting-line theory used would not be expected to be very accurate anyway. In spite of this the minimum vortex drag coefficients calculated by the Prandtl theory used in this Item for a particular low aspect ratio ($A_w = 2.5, A_f = 2.6$) close-coupled wing-foreplane configuration have been shown in Derivation 3 to agree well (within 0.0002) for a range of trim points, with those determined by the vortex-lattice method of Reference 8.

The method of calculating the average downwash gradient ($d\bar{\epsilon}/d\alpha$) presented in this Item (Equations (3.17) to (3.19)) was checked for two configurations using the lifting-surface method of Reference 7. Agreement for a swept wing-tailplane combination ($A_w = 8, A_t = 4$) was within 0.001 while for an unswept wing-foreplane combination $A_w = A_f = 5$ the wing upwash gradient ($d\bar{\epsilon}_w/d\alpha_w$) and foreplane downwash gradient ($d\bar{\epsilon}_f/d\alpha_f$) were predicted to be 0.01 and 0.03, respectively, less than the lifting-surface calculations. This level of agreement was felt to be satisfactory in view of the simplicity of the method given herein.

6. DERIVATION AND REFERENCES

6.1 Derivation

The Derivation lists selected sources that have assisted in the preparation of this Item.

1. SPREITER, J.R.
SACKS, A.H. The rolling up of the trailing vortex sheet and its effect on the downwash behind wings. *J. aeronaut. Sci.*, Vol. 18, No. 1, p. 21, January 1951.
2. LOWRY, J.G.
POLHAMUS, E. A method for predicting lift increments due to flap deflection at low angles of attack in incompressible flow. NACA tech. Note 3911, 1957.
3. McLAUGHLIN, M.D. Calculations, and comparison with an ideal minimum, of trimmed drag for conventional and canard configurations having various levels of static stability. NASA tech. Note D-8391, 1977.

6.2 References

The References list selected sources of information supplementary to that given in this Item.

4. PRANDTL, L. Application of modern hydrodynamics to aeronautics. NACA Rep. 116, 1921.
5. MUNK, M.M. General biplane theory. NACA Rep. 151, 1922.
6. PRANDTL, L. Induced drag of multiplanes. NACA tech. Note 182, 1924.
7. DAVIES, D.E. Calculations of generalised airforces on two parallel lifting surfaces oscillating harmonically in subsonic flow. ARC R&M 3749, 1972.
8. LAMAR, J.E. A vortex-lattice method for the mean camber shapes of trimmed non-coplanar planforms with minimum vortex drag. NASA tech. Note D-8090, 1976.

7. EXAMPLES

The method presented in this Item can be used in so many ways that it is impossible in one or two examples to illustrate all possible uses. Therefore just two examples will be given to illustrate some of the more important aspects of possible uses.

- 7.1 The method of this Item has been used to calculate the effect of varying the span ratio and static margin on the ratio of the surface lifts, the zero-lift, vortex and total drag coefficients for a combination of two unswept lifting surfaces trimmed in pitch. The geometrical parameters for the two surfaces are as follows:

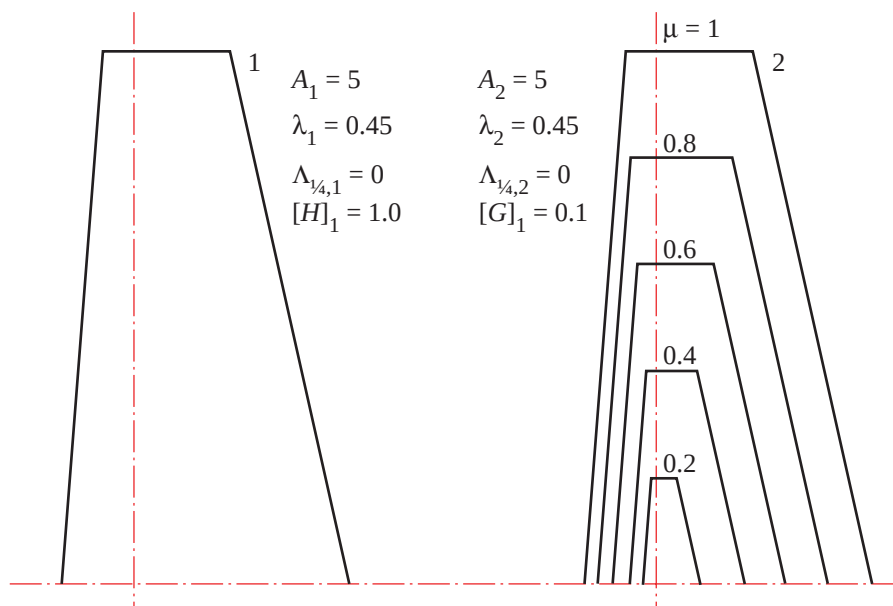
$$A_1 = A_2 = 5, \quad \lambda_1 = \lambda_2 = 0.45, \quad \Lambda_{1/4,1} = \Lambda_{1/4,2} = 0,$$

$$\left. \begin{array}{l} l_2 - l_1 = 0.5b_1, \text{ i.e. } [H]_1 = 1.0 \\ g = 0.1b_1, \text{ i.e. } [G]_1 = 0.1 \end{array} \right\} \text{ when } b_2 \leq b_1 \quad \left. \begin{array}{l} l_2 - l_1 = 0.5b_2, \text{ i.e. } [H]_2 = 1.0 \\ g = 0.1b_2, \text{ i.e. } [G]_2 = 0.1 \end{array} \right\} \text{ when } b_2 \geq b_1.$$

(Note, for the present purpose it will be assumed that $(l_2 - l_1)$ is also the distance between the quarter-chord lines in the direction of the ξ -axis.)

The span ratio μ ($= b_2/b_1$ when $b_2 \leq b_1$, $= b_1/b_2$ when $b_2 \geq b_1$) is varied between 0 and 1.

Sketch 7.1 shows a series of planform combinations for $\mu = 0.2, 0.4, 0.6, 0.8$ and 1.0 when $b_1 \geq b_2$.



Sketch 7.1 $b_2 \leq b_1$

The aerodynamic data are as follows:

$$C_{D01} = C_{D02} = 0.01, \quad [C_{m0}]_1 = [C_{m0}]_2 = 0, \quad [C_L]_1 = [C_L]_2 = 0.5$$

and $[dC_m/dC_L]_1 = [dC_m/dC_L]_2$ takes the values 0, -0.1, -0.2, -0.3 and -0.4.