



# ESDU 81032

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## Estimation of rolling moment derivative due to sideslip for complete aircraft at subsonic speeds

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## ESTIMATION OF ROLLING MOMENT DERIVATIVE DUE TO SIDESLIP FOR COMPLETE AIRCRAFT AT SUBSONIC SPEEDS

### CONTENTS

|                                                                | Page |
|----------------------------------------------------------------|------|
| 1. NOTATION AND UNITS                                          | 1    |
| 1.1 Notation                                                   | 1    |
| 1.2 Longitudinal Body Reference Axis                           | 2    |
| 2. INTRODUCTION                                                | 2    |
| 3. COMPARISONS WITH EXPERIMENTAL DATA                          | 4    |
| 3.1 Overall Accuracy                                           | 4    |
| 3.2 Variation of $L_v$ with $\alpha$ and $C_L$                 | 5    |
| 3.3 Effect of Aircraft Configuration                           | 6    |
| 4. DERIVATION AND REFERENCES                                   | 8    |
| 4.1 Derivation                                                 | 8    |
| 4.2 References                                                 | 10   |
| 5. EXAMPLE                                                     | 12   |
| 5.1 Calculation of Wing Planform Parameters                    | 14   |
| 5.2 Calculation of Dihedral Component, $(L_v)_\Gamma$          | 15   |
| 5.3 Calculation of Planform Component, $(L_v)_w$               | 16   |
| 5.4 Calculation of Isolated Body Component, $(L_v)_b$          | 17   |
| 5.5 Calculation of Wing-body Interference Component, $(L_v)_h$ | 18   |
| 5.6 Calculation of Nacelle Contribution, $(L_v)_n$             | 19   |
| 5.7 Calculation of Fin Contribution, $(L_v)_F$                 | 20   |
| 5.8 Calculation of Trailing-Edge Flap Contribution, $(L_v)_f$  | 21   |
| 5.9 Total Value of $L_v$                                       | 23   |

## ESTIMATION OF ROLLING MOMENT DERIVATIVE DUE TO SIDESLIP FOR COMPLETE AIRCRAFT AT SUBSONIC SPEEDS

### 1. NOTATION AND UNITS

The derivative notation used is that proposed in ARC R&M 3562 (Hopkin, 1970) and described in Item No. 86021. Coefficients and aeronormalised derivatives are evaluated in aerodynamic body axes with origin at the aircraft centre of gravity and with the wing span as the characteristic length. The derivative  $L_v$  is often written as  $\partial C_l / \partial \beta$  or  $C_{l\beta}$  in other systems of notation, but attention must be paid to the reference dimensions used and it is to be noted that a constant datum value of  $V$  is employed in the Hopkin system.

This Item makes use of several other Items which have been produced at different times over a period of many years. Although the nomenclature in these Items is consistent for the important parameters such as stability derivatives, it involves some variation and duplication for the less significant parameters. Because of this, and to avoid repetition, the Notation given here is limited to the major quantities appearing in the main text of this Item and to quantities not appearing in other Items. When referred to the method in another Item the user should consult the Notation at the front of that particular Item before carrying out any calculations.

The computer program ESDUpac A0025, associated with Item No. 00025, calculates the stability derivatives treated in this Item.

#### 1.1 Notation

|               |                                                                                                                          | SI             | British         |
|---------------|--------------------------------------------------------------------------------------------------------------------------|----------------|-----------------|
| $b$           | wing span                                                                                                                | m              | ft              |
| $b_T$         | tailplane span                                                                                                           | m              | ft              |
| $C_L$         | lift coefficient, $L / \frac{1}{2} \rho V^2 S$                                                                           |                |                 |
| $C_l$         | rolling moment coefficient $\mathcal{L} / \frac{1}{2} \rho V^2 S b$                                                      |                |                 |
| $L$           | lift                                                                                                                     | N              | lbf             |
| $\mathcal{L}$ | rolling moment                                                                                                           | N m            | lbf ft          |
| $L_v$         | aeronormalised rolling moment derivative due to sideslip, $(\partial \mathcal{L} / \partial v) / \frac{1}{2} \rho V S b$ |                |                 |
| $S$           | wing planform area                                                                                                       | m <sup>2</sup> | ft <sup>2</sup> |
| $S_T$         | tailplane planform area                                                                                                  | m <sup>2</sup> | ft <sup>2</sup> |
| $V$           | velocity of aircraft relative to air                                                                                     | m/s            | ft/s            |
| $v$           | sideslip velocity                                                                                                        | m/s            | ft/s            |
| $\alpha$      | angle of attack of longitudinal body axis (see definition of reference axis in Section 1.2)                              | degree         | degree          |

Issued October 1981

With Amendments A to D, October 2000 – 25 pages

This page Amendment D

|            |                                                                                                                                                                                       |        |        |
|------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------|--------|
| $\alpha_w$ | angle between wing zero-lift line and longitudinal body axis, so that angle of attack of wing zero-lift line is $\alpha + \alpha_w$ (see definition of reference axis in Section 1.2) | degree | degree |
| $\beta$    | sideslip angle, $\sin^{-1}(v/V)$                                                                                                                                                      | radian | radian |

## Additional Symbols

|              |                                                                                                                    |
|--------------|--------------------------------------------------------------------------------------------------------------------|
| $( )_b$      | denotes component of $L_v$ due to isolated body                                                                    |
| $( )_F$      | denotes component of $L_v$ due to fin (or fin and undeflected rudder), allowing for presence of tailplane and body |
| $( )_f$      | denotes component of $L_v$ due to deployment of trailing-edge flaps                                                |
| $( )_h$      | denotes component of $L_v$ due to wing-body interference arising from vertical position of wing on body            |
| $( )_n$      | denotes component of $L_v$ due to nacelles                                                                         |
| $( )_w$      | denotes component of $L_v$ due to wing planform                                                                    |
| $( )_\Gamma$ | denotes component of $L_v$ due to wing dihedral                                                                    |

## 1.2 Longitudinal Body Reference Axis

The angle of attack of the body may be expressed in terms of any longitudinal axis which lies in the aircraft plane of symmetry and is fixed in the body, provided that the same axis is used consistently in all calculations. Because the stability derivative  $L_v$  is defined in terms of a set of aerodynamic body axes with their origin at the aircraft centre of gravity (moment reference centre) the calculation of the fin contribution to  $L_v$  is considerably simplified if a body reference axis which passes through the aircraft centre of gravity is chosen. Aircraft dimensions are customarily defined with respect to some geometrically convenient “horizontal datum” axis which lies in the aircraft plane of symmetry and is fixed in the body but which does not necessarily pass through the centre of gravity position. Therefore, in practice, the best longitudinal body axis to choose for the calculation of  $L_v$  is one which is parallel to the “horizontal datum” axis but which is displaced vertically so as to pass through the centre of gravity position. Such a reference axis is assumed in this Item.

## 2. INTRODUCTION

The rolling moment derivative due to sideslip,  $L_v$ , of an aircraft is customarily estimated by calculating the effects of the major components of the aircraft separately and adding together the part derivatives so obtained. This Item demonstrates how to combine the Data Items dealing with the major components and illustrates the overall accuracy of prediction by comparison with wind-tunnel data for complete aircraft models. It should be noted that the methods in the various Data Items are applicable over the range of aircraft incidences for which there is a linear variation of rolling moment coefficient with angle of sideslip and of lift coefficient with angle of attack. They apply at subsonic speeds, for Mach numbers below that at which the aerodynamic characteristics start to diverge rapidly with Mach number.

The major contributions to  $L_v$  and the Data Items from which they may be estimated are listed in Table 2.1. A brief description of these contributions is contained in the introductory Data Item on rolling moment derivatives, Item No. Aero A.06.01.00 (Derivation 9), which may usefully be consulted in conjunction with Table 2.1. The Data Items dealing specifically with  $L_v$  will often require the introduction of additional information from Data Items of a more general nature. These are also identified in Table 2.1. Comparisons between the total values of  $L_v$  predicted by the Data Item methods and the wind-tunnel data in Derivations 20 to 32, which cover a variety of different aircraft types, are discussed in Section 3. The Derivation and References are given in Section 4. Section 5 contains a detailed worked example which demonstrates the calculation of the various components of  $L_v$  for a particular aircraft for both the cruise (clean) and the landing configurations. The example devotes a separate subsection to the calculation of each component of  $L_v$  and describes at which stages in the calculation the additional Data Items listed in Table 2.1 may be needed to complement those dealing specifically with  $L_v$ . It also provides guidance and information on particular points which are not covered in other Data Items. It is therefore useful to refer to the appropriate subsection in the example when each Data Item dealing with the separate components of  $L_v$  is used.

**TABLE 2.1**

| <i>Component*</i> | <i>Due to</i>                                                         | <i>Calculated from Item No.</i>                                         | <i>Possible additional Item Nos</i>                                                                                                                                                                                                    |
|-------------------|-----------------------------------------------------------------------|-------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $(L_v)_\Gamma$    | wing dihedral                                                         | Aero A.06.01.03 <sup>7</sup> and 09 <sup>8</sup>                        | Aero W.01.01.01 <sup>3</sup> and 05 <sup>4</sup><br>76003 <sup>16</sup>                                                                                                                                                                |
| $(L_v)_w$         | wing planform                                                         | 80033 <sup>17</sup>                                                     | 70011 <sup>10</sup><br>76003 <sup>16</sup>                                                                                                                                                                                             |
| $(L_v)_b$         | body                                                                  | 73006 <sup>11</sup>                                                     | —                                                                                                                                                                                                                                      |
| $(L_v)_h$         | wing-body interference                                                | 73006 <sup>11</sup>                                                     | 76003 <sup>16</sup>                                                                                                                                                                                                                    |
| $(L_v)_n$         | nacelles                                                              | 73006 <sup>11</sup>                                                     | —                                                                                                                                                                                                                                      |
| $(L_v)_F$         | fin sideforce (in presence of body and tailplane), rudder undeflected | 82010 <sup>19</sup><br>Aero C.01.01.01 <sup>5</sup> and 02 <sup>6</sup> | 70011 <sup>10</sup>                                                                                                                                                                                                                    |
| $(L_v)_f$         | trailing-edge flap deployment                                         | 80034 <sup>18</sup>                                                     | Aero F.01.01.08 <sup>2</sup> and 09 <sup>1</sup><br>Aero W.01.01.01 <sup>3</sup> and 05 <sup>4</sup><br>70011 <sup>10</sup><br>74009 <sup>12</sup> , 11 <sup>13</sup> , 12 <sup>14</sup><br>75013 <sup>15</sup><br>76003 <sup>16</sup> |

\* Item No. Aero A.06.01.00<sup>9</sup> gives a brief introduction to each component and its related Data Item.

(Derivation numbers are given as indices.)

The total value of  $L_v$  for an aircraft is obtained by evaluating each of the components in Table 2.1, for the same angle of attack, and summing the results. It should be noted that no contribution is estimated for the tailplane (although there is an indirect contribution through  $(L_v)_F$  because the tailplane affects the magnitude and centre of pressure position of the sideforce on the fin). In practice there are antisymmetric spanwise loadings on the tailplane similar to those on the wing, together with an additional antisymmetric spanwise loading induced by the fin. These give rise to rolling moments about the plane of symmetry of the aircraft and hence contribute to the total value of  $L_v$ . The contributions are small, however, and for the purposes of aircraft stability calculations sufficiently accurate estimates of  $L_v$  are obtained if they are ignored. The tailplane loadings are of more importance when the structural loading of the tailplane itself

is being considered, and the contributions to  $L_v$  can be of use when considering the bending moment at the tailplane root chord. The contribution of the tailplane due to its dihedral and planform can be evaluated in the same way as for an isolated wing and the results factored by  $S_T b_T / Sb$  to convert them to the standard reference area and length. Unless the tailplane has a large dihedral angle (more than  $5^\circ$  say) or a high lift coefficient these contributions will be negligible. The tailplane lift coefficient is small in cruise but may be more significant during landing and take-off, especially in the case of all-moving or trimming tailplanes, which can be at a relatively high angle of attack. Similarly, for tailplanes fitted with elevator controls the effect of elevator deflection can be treated in the same way as the effect of wing trailing-edge flaps, although this contribution is again very small. Lastly, there is a contribution from the tailplane that has no analogous wing contribution, but arises because the presence of the fin induces an antisymmetric spanwise loading across the tailplane, and this generates a contribution to  $L_v$ . Methods for estimating this effect are not readily available although some theoretical studies are reported in References 40 to 43, and these have been put into a tentative generalised form in Reference 44. This contribution is customarily ignored during the calculation of aircraft stability derivatives. The sign of the fin-induced contribution is negative for tailplanes mounted high on the fin, positive for tailplanes mounted low on the fin or on the body, and the effect is zero for tailplanes at or near the mid-point of the fin. Section 5.9 of the example discusses these points further and gives some typical values.

## 3. COMPARISONS WITH EXPERIMENTAL DATA

### 3.1 Overall Accuracy

The accuracy with which the methods in the Data Items listed in Section 2 predict  $L_v$  for complete aircraft has been assessed by comparing predicted values of  $L_v$  with experimental values measured in the low-speed wind-tunnel tests reported in Derivations 20 to 32. Comparisons have been made for thirty different aircraft models.

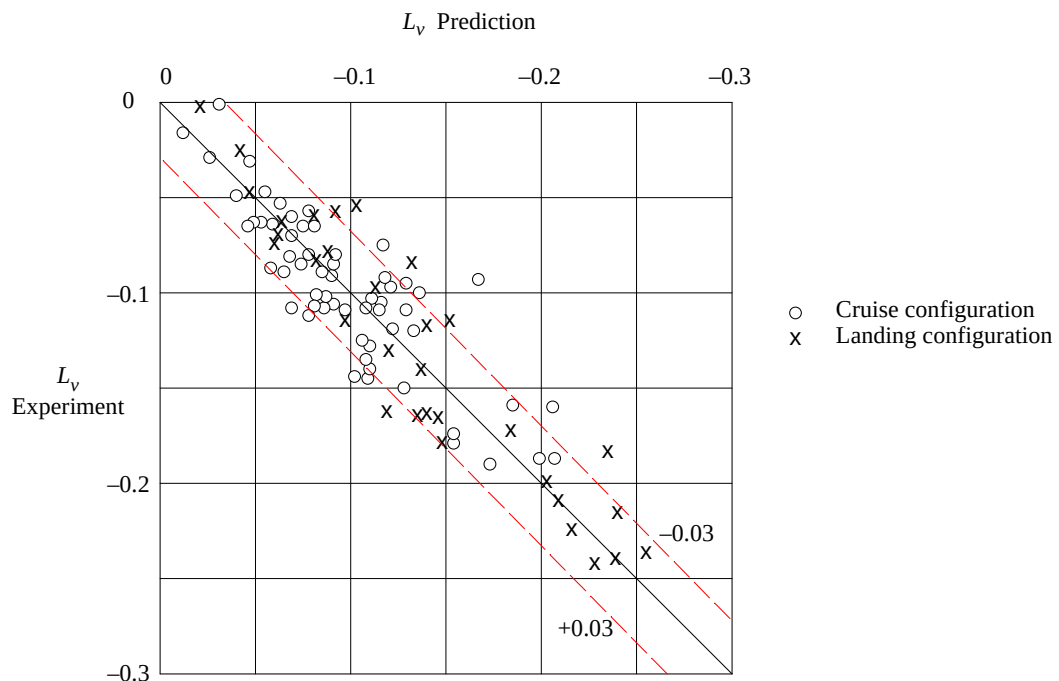
The results of these comparisons indicate that for the cruise (clean) configuration the predicted values provide a good initial estimate of both the magnitude of  $L_v$  and its variation with angle of attack,  $\alpha$ , and lift coefficient,  $C_L$ , for values of  $\alpha$  and  $C_L$  within the range of practical values for cruise and where the experimental variation of  $C_L$  with  $\alpha$  is essentially linear. The agreement between predicted and experimental data becomes less satisfactory as  $\alpha$  and  $C_L$  increase to values where the flow over the wing is not fully attached. The derivative  $L_v$  is very sensitive to any separation effects and can suffer significant departures from the predicted variation with  $\alpha$  and  $C_L$  well before stalling conditions are reached (see Section 3.2 for example). In such situations the Data Item methods are not strictly applicable and they should be used with caution.

For configurations with trailing-edge flaps deployed for landing or take-off the Data Item methods usually provide a good estimate of the size of  $L_v$ , but the variation of  $L_v$  with  $\alpha$  and  $C_L$  is not so well predicted as for cruise conditions (see Section 3.2 for examples). Since the predicted effect of trailing-edge flap deployment on  $L_v$  does not vary greatly with  $\alpha$ , and this is supported by many experimental data, the poor agreement for the variation of  $L_v$  may again be associated with flow separation effects on the wing.

In all cases it should be remembered that the Data Item methods assume that there is a linear variation of lift coefficient with angle of attack and of rolling moment coefficient,  $C_l$ , with angle of sideslip,  $\beta$ . For most practical flight conditions the linear variation of  $C_L$  with  $\alpha$  is particularly well observed experimentally. The experimental variation of  $C_l$  with  $\beta$  is, however, less well behaved. It generally shows an essentially linear variation at low values of  $\beta$ , but then slowly diverges from this as  $\beta$  increases. The extent of the divergence and the point at which it becomes significant vary from aircraft to aircraft, but it has usually appeared when  $\beta$  has reached  $10^\circ$ . For this reason, values of  $L_v$  are deduced from

experimental data on the assumption of a linear variation of  $C_l$  over a small range of sideslip angles (typically  $\beta \leq 5^\circ$ ). The Data Item methods predict values of  $L_v$  which are appropriate to this definition of the derivative.

The overall accuracy obtained in predicting the magnitude of  $L_v$  is demonstrated in Sketch 3.1, where predicted and experimental values of  $L_v$  are plotted against each other. Where sufficient data are available, each of the aircraft configurations that has been studied is represented in the sketch by two points chosen at values of  $\alpha$  and  $C_L$  typical of cruise conditions, and by a further two points chosen at values of  $\alpha$  and  $C_L$  typical of landing conditions with trailing-edge flaps deployed. Not every configuration was tested with flaps deployed and these data are therefore less numerous than the data for cruise conditions. The sketch indicates that 85 per cent of the data are predicted to within  $\pm 0.03$ . The data are evenly distributed to either side of the line of perfect agreement, showing that there is little or no bias in the predicted values. There is no significant difference between the accuracy of prediction for the cruise and landing configurations in terms of total magnitudes.



**Sketch 3.1 Comparison of magnitudes of predicted and experimental values of  $L_v$**

### 3.2 Variation of $L_v$ with $\alpha$ and $C_L$

Examples of the agreement between the predicted and experimental variations of  $L_v$  with  $\alpha$  and  $C_L$  are demonstrated in Sketch 3.2, which shows representative values for three civil transport type aircraft configurations with different wing sweep angles. Data are shown for both the cruise configuration and the landing configuration.

Considering firstly the data for the cruise configuration, Sketch 3.2 shows that the predicted values of  $L_v$  vary approximately linearly with  $\alpha$  and  $C_L$ . The gradient of this variation is very largely determined by the relative magnitudes of the wing planform contribution to  $L_v$ , which increases with  $\alpha$  (or, more properly, with  $C_L$ ), and the fin contribution to  $L_v$ , which decreases with  $\alpha$  (see Section 5). The sketch shows that the predicted gradient of the variation of the total values of  $L_v$  can change from positive to negative depending on the aircraft geometry. The decrease in the gradient with increasing wing sweep angle which is illustrated in the sketch reflects the increasing dominance of the wing planform component over the fin

component. The experimental data in the sketch show that the measured values of  $L_v$  often behave in a somewhat less linear manner than that predicted, but that they generally follow the trend of the prediction over the ranges of  $\alpha$  and  $C_L$  of interest. The data in Sketch 3.2c demonstrate how the measured values of  $L_v$  can depart from the predicted values if, as  $\alpha$  increases, the flow over the wing begins to detach.

Considering the data for the landing configuration, Sketch 3.2 shows that the variation of the predicted values of  $L_v$  with  $\alpha$  and  $C_L$  is again nearly linear, the effect of flap deployment being to change the cruise values of  $L_v$  by an amount which is almost independent of  $\alpha$ . It can be seen, however, that the overall agreement between the experimental and predicted variations of  $L_v$  is less satisfactory for the configurations with flaps deployed than for the cruise configurations, even though the total magnitudes are predicted quite well in both cases. (It is also worth noting that the data in Sketch 3.2 demonstrate that the sign of the trailing-edge flap contribution to  $L_v$  can be positive or negative, depending on the geometry of the aircraft. For aircraft with swept wings the flap contribution is usually negative, as demonstrated in Sketches 3.2b and 3.2c, but for aircraft with unswept wings of medium to high aspect ratio it is usually positive as demonstrated in Sketch 3.2a.)

It is emphasised that the predicted values of  $L_v$  presented in Sketch 3.2 are only intended to illustrate the possible magnitudes and variations of  $L_v$ . To estimate  $L_v$  for a particular aircraft configuration a detailed calculation such as that set out in Section 5 must be performed.

### 3.3 Effect of Aircraft Configuration

From an examination of a large number of plots such as those shown in Sketch 3.2, it is concluded that the Data Item prediction methods for  $L_v$  are most accurate when they are applied to aircraft configurations typical of civil transports, where the body has a long cylindrical mid-section and the fin and tailplane are well clear of the wing. Wind-tunnel tests of this type of aircraft supplied the major part of the experimental data studied.

The prediction methods have to be used with more caution when the body cross-section is not circular or elliptical or when it varies along the body length, particularly in the vicinity of the wing-root junction, as may be the case with light and general aviation aircraft or high-performance military aircraft. Problems may also arise when the fin and tailplane are situated close behind the wing because the interference between those surfaces is likely to be more complex than that allowed for in Item No. 82010. Only a small number of experimental data were available for examining these effects. The values of  $L_v$  for three general aviation aircraft were predicted within the overall level of accuracy but their magnitudes were always under-predicted by about 0.02. In the case of two high-performance military aircraft the magnitudes of  $L_v$  were over-predicted by about 0.02, and subject to greater uncertainty when trailing-edge flaps were deployed. There was also some indication that the experimental values of  $L_v$  for these military aircraft decreased in magnitude and departed from the predicted trends at lower values of  $C_L$  than for more conventional aircraft. There were, however, too few data on which to base any empirical modifications to the basic prediction method for either of these categories of aircraft.

It should also be noted that although the overall accuracy of prediction for configurations with trailing-edge flaps deployed was comparable to that for configurations with flaps retracted, the flap contribution to  $L_v$  depends on an accurate prediction of the lift increment due to flap deflection. The experimental data examined have shown that sufficiently satisfactory estimates for this purpose can usually be obtained from the Data Items dealing with flap effects, except in the case of low aspect-ratio triangular wings where there is a large spanwise variation of the ratio of the local flap chord to wing chord, which is well outside the intended range of application of the Data Items dealing with flap effects.

Apart from these observations it was not possible to identify any correlation between the accuracy of prediction and either particular aspects of aircraft geometry or particular components of  $L_v$ . This suggests that the individual Data Items are well matched to each other and are of comparable reliability.