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Linearised two-dimensional oscillatory airforce coefficients and load distributions on thin aerofoils in subsonic flow

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LINEARISED TWO-DIMENSIONAL OSCILLATORY AIRFORCE COEFFICIENTS
AND LOAD DISTRIBUTION ON THIN AEROFOILS IN SUBSONIC FLOW

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LINEARISED TWO-DIMENSIONAL OSCILLATORY AIRFORCE COEFFICIENTS AND LOAD DISTRIBUTION ON THIN AEROFOILS IN SUBSONIC FLOW

1. NOTATION AND UNITS

		SI	British
a	speed of sound	m/s	ft/s
c	aerofoil chord	m	ft
i	$\sqrt{-1}$		
M	free-stream Mach number, V/a		
$-\mathbf{Q}'$	aerodynamic stiffness matrix		
$-\mathbf{Q}''$	aerodynamic damping matrix		
Q_{ij}	complex generalised airforce coefficient		
Q'_{ij}	in-phase part of Q_{ij} , see Equation (3.2)		
Q''_{ij}	in-quadrature (90° out of phase) part of Q_{ij}/v , see Equation (3.2)		
V	free-stream velocity	m/s	ft/s
x, z	rectangular Cartesian coordinates relative to mean position of aerofoil chord with origin at leading edge; x positive rearwards, z positive upwards	m	ft
z_i	upward displacement mode	m	ft
$\bar{\gamma}_j(\xi)$	dimensionless chordwise loading, lift per unit area $/(\rho V^2)$		
$\gamma'_j(\xi)$	in-phase part of $\gamma_j(\xi)$		
$\gamma''_j(\xi)$	in-quadrature part of $\gamma_j(\xi)$		
ζ	z/c		
$\zeta_i(\xi)$	z_i/c		
v	frequency parameter, $\omega c/V$		
ξ	x/c		
ρ	free-stream density	kg/m ³	slug/ft ³

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ω circular frequency rad/s rad/s

Subscripts

a denotes general pitching axis

h denotes hinge line

i integer (1 for lift, 2 for pitching moment, 3 for trailing-edge control hinge moment)

j integer (1 for heave, 2 for pitch, 3 for trailing-edge control motion)

Dressings

prime (') as in Q' denotes real part

double prime (") as in Q'' denotes imaginary part

2. INTRODUCTION

This Item presents results, for use in flutter calculations, obtained from a recently developed FORTRAN computer program (Derivation 1). The program embodies a method for evaluating the forces, moments and pressure distributions on a thin aerofoil oscillating harmonically with small amplitude in subsonic flow. The calculations were made for two modes of oscillation of the aerofoil, in heave and in pitch, and for a third mode in which a trailing-edge control oscillated about its hinge-line at 0.8 of the aerofoil chord. The results are presented in the form of tabulated airforce coefficients (Section 3) and graphs of the chordwise load distribution (Section 4) over a frequency parameter range from 0.05 to 2.0 and a Mach number range from 0 to 0.90.

Currently there are alternative forms in which data for flutter calculations are presented, *i.e.* airforce coefficients and flutter derivatives. Appendix A provides a set of conversion equations to obtain flutter derivatives from airforce coefficients. These equations have been used to derive tables of flutter derivatives from the airforce coefficient data given herein. These data for flutter derivatives, calculated using a precise computer program, supersede the corresponding data obtained over thirty years ago in Reference A2. Also given in Appendix A is an example of the use of the conversion equations to illustrate the effect of a variation in the location of the pitching axis.

The data would typically be used in flutter calculations using strip theory. In flutter calculations the use of two-dimensional strip theory (see Appendix B and References 2 and 3) has advantages over three-dimensional lifting-surface theory due to the ease with which wind-tunnel data may be incorporated and the effects of parametric variations may be studied. Moreover, for initial calculations, the use of an approximate strip theory offers a saving in time and cost of computation with acceptable loss of accuracy.

The data given in this Item are strictly applicable to infinitesimal displacements of flat plate aerofoils in inviscid subsonic flow. Nevertheless, the data may be applied to aerofoils of moderate thickness and camber, at incidence, provided that the flow remains attached and subsonic over the whole aerofoil.

3. GENERALISED AIRFORCE COEFFICIENTS

Derivation 1 provides a method for calculating the aerodynamic forces on a two-dimensional thin aerofoil oscillating harmonically, with small amplitude, in an inviscid subsonic flow. The unknown chordwise load distribution is related, via an integral equation, to the known upwash on the aerofoil which is dependent on the mode of oscillation. Derivation 1 solves the integral equation for the loading, which is represented approximately as a finite series of terms. The resulting load distribution is used to define the generalised airforce coefficients on a thin aerofoil,

$$Q_{ij} = \int_0^1 \zeta_i(\xi) \bar{\gamma}_j(\xi) d\xi, \quad (3.1)$$

in which $\zeta_i(\xi)$ and $\bar{\gamma}_j(\xi)$ are the dimensionless displacement and loading at the chordwise position $x = \xi c$. The subscripts i and j define respectively the force and oscillatory motion considered. In this Item, i takes the values 1, 2 or 3 according to whether lift, pitching moment or control hinge-moment is considered, and j takes the values 1, 2 or 3 according to whether the motion considered is heaving, pitching or control oscillation. Thus Q_{23} , for example, is the airforce coefficient for pitching moment due to control oscillation. For lift $\zeta_1 = 1$, for pitching moment $\zeta_2 = \xi - \xi_a$, where ξ_a is the pitching axis location, while for trailing-edge control hinge-moment $\zeta_3 = 0$, for $0 \leq \xi \leq \xi_h$ and $\zeta_3 = \xi - \xi_h$, for $\xi_h \leq \xi \leq 1$, where ξ_h is the hinge-line location.

In flutter calculations Q_{ij} is split into its real and imaginary parts, *i.e.*,

$$Q_{ij} = Q'_{ij} + i\nu Q''_{ij}, \quad (3.2)$$

where the real part provides the aerodynamic stiffness matrix $-Q'$ and the imaginary part (without the factor ν) provides the aerodynamic damping matrix $-Q''$. The airforce coefficients Q'_{ij} and Q''_{ij} presented in this Item relate to aerofoil oscillations in heave and in pitch with the pitching axis taken at the quarter-chord point ($\xi_a = 0.25$) and oscillations of a trailing-edge control with its hinge-line at 0.8 of the aerofoil chord. The calculations were for all combinations of Mach number and frequency parameter covering the values $M = 0, 0.50, 0.65, 0.75, 0.80, 0.85, 0.88$ and 0.90 and $\nu = 0.05, 0.10, 0.20$, and then in steps of 0.2 to 2.0 . The results for ascending Mach number are tabulated in Tables 6.1 to 6.8.

4. CHORDWISE LOADING

As noted in Section 3 the program of Derivation 1 also calculates the two-dimensional chordwise load distribution corresponding to any mode of oscillation.

The results for the real and imaginary parts of the loading $\bar{\gamma}_j(\xi)$, given by

$$\bar{\gamma}_j(\xi) = \gamma'_j(\xi) + i\gamma''_j(\xi) \quad (4.1)$$

have been calculated for oscillations in heave, pitch and control rotation for the following combinations of M and ν :

M	0.5	0.8	0.8	0.8	0.9
ν	1.0	0.4	1.0	2.0	1.0

All calculations were carried out with 80 chordwise terms.

These examples, spanning the practical ranges of M and ν , are given to illustrate the effects of Mach number and oscillation frequency on the chordwise loading and to highlight particular features.

The results of the calculations are presented graphically to show the effect of Mach number and frequency parameter on the real and imaginary parts of the loading. Figures 1 and 2 show the effect of Mach number in heaving and pitching respectively, while Figures 3 and 4 show the effect of frequency parameter in the pitching and control rotation modes, respectively. Fluctuations are seen to develop as M or ν increases. In particular, for the oscillating trailing-edge control in Figure 4, the fluctuations relate to the rate of propagation of disturbances upstream of the hinge. As discussed in Section 4.3 of Reference 4, over the stationary part of the aerofoil there is a strong tendency for the peaks and troughs in γ_3' to coincide with zeros in γ_3'' and vice-versa.

5. DERIVATION AND REFERENCES

5.1 Derivation

The Derivation lists selected sources that have assisted in the preparation of this Item.

1. SALMOND, D.J. Evaluation of two-dimensional subsonic oscillatory airforce coefficients and loading distributions. RAE tech. Rep. 79096, 1979.

5.2 References

The References list selected sources of information supplementary to that given in this Item.

2. THEODORSEN, T. General theory of aerodynamic instability and the mechanism of flutter. NACA Rep. 496, 1934.
3. THEODORSEN, T. Mechanism of flutter, a theoretical and experimental investigation of the flutter problem. NACA Rep. 685, 1940.
4. GARNER, H.C. A phenomenon of subsonic aerodynamic loading from rapid aileron oscillations. RAE tech. Rep. 80025, 1980.

6. TABLES OF AIRFORCE COEFFICIENTS

TABLE 6.1 Generalised Airforce Coefficients of an Oscillating Aerofoil with Trailing-edge Control ($\xi_a = 0.25$, $\xi_h = 0.80$, $M = 0$)

$M = 0$																		
ν	Q'_{11}	Q''_{11}	Q'_{12}	Q''_{12}	Q'_{13}	Q''_{13}	Q'_{21}	Q''_{21}	Q'_{22}	Q''_{22}	Q'_{23}	Q''_{23}	Q'_{31}	Q''_{31}	Q'_{32}	Q''_{32}	Q'_{33}	Q''_{33}
0.05	-0.0117	-2.9981	-3.0045	3.1969	-1.6494	2.6790	0.0005	0	0.0002	-0.3927	-0.3200	-0.0986	-0.00002	-0.00953	-0.00954	-0.00308	-0.01822	0.00056
0.10	-0.0332	-2.8557	-2.8743	1.8911	-1.5731	1.9324	0.0020	0	0.0007	-0.3927	-0.3200	-0.0986	-0.00004	-0.00908	-0.00910	-0.00723	-0.01797	-0.00182
0.20	-0.0769	-2.6136	-2.6598	0.6143	-1.4447	1.1819	0.0079	0	0.0030	-0.3927	-0.3198	-0.0986	0.00002	-0.00831	-0.00831	-0.01129	-0.01755	-0.00420
0.40	-0.1114	-2.2858	-2.3729	-0.4468	-1.2729	0.5327	0.0314	0	0.0118	-0.3927	-0.3193	-0.0986	0.00071	-0.00727	-0.00698	-0.01467	-0.01696	-0.00627
0.60	-0.0553	-2.0891	-2.1874	-0.8910	-1.1705	0.2490	0.0707	0	0.0265	-0.3927	-0.3185	-0.0986	0.00221	-0.00664	-0.00568	-0.01608	-0.01655	-0.00717
0.80	0.0880	-1.9634	-2.0451	-1.1192	-1.1045	0.0984	0.1257	0	0.0471	-0.3927	-0.3173	-0.0986	0.00452	-0.00624	-0.00423	-0.01680	-0.01622	-0.00765
1.00	0.3119	-1.8785	-1.9189	-1.2512	-1.0589	0.0088	0.1964	0	0.0736	-0.3927	-0.3158	-0.0986	0.00761	-0.00597	-0.00255	-0.01722	-0.01593	-0.00793
1.20	0.6115	-1.8184	-1.7953	-1.3339	-1.0253	-0.0487	0.2827	0	0.1060	-0.3927	-0.3140	-0.0986	0.01148	-0.00578	-0.00060	-0.01749	-0.01565	-0.00812
1.40	0.9834	-1.7743	-1.6674	-1.3888	-0.9990	-0.0878	0.3849	0	0.1443	-0.3927	-0.3118	-0.0986	0.01611	-0.00564	0.00166	-0.01766	-0.01535	-0.00824
1.60	1.4250	-1.7409	-1.5311	-1.4271	-0.9774	-0.1155	0.5027	0	0.1885	-0.3927	-0.3093	-0.0986	0.02148	-0.00554	0.00422	-0.01778	-0.01504	-0.00833
1.80	1.9348	-1.7151	-1.3839	-1.4547	-0.9588	-0.1359	0.6362	0	0.2386	-0.3927	-0.3065	-0.0986	0.02761	-0.00545	0.00710	-0.01787	-0.01470	-0.00839
2.00	2.5116	-1.6947	-1.2243	-1.4752	-0.9421	-0.1513	0.7854	0	0.2945	-0.3927	-0.3033	-0.0986	0.03448	-0.00539	0.01031	-0.01794	-0.01434	-0.00844

TABLE 6.2 Generalised Airforce Coefficients of an Oscillating Aerofoil with Trailing-edge Control ($\xi_a = 0.25$, $\xi_h = 0.80$, $M = 0.5$)

$M = 0.50$																			
ν	Q'_{11}	Q''_{11}	Q'_{12}	Q''_{12}	Q'_{13}	Q''_{13}	Q'_{21}	Q''_{21}	Q'_{22}	Q''_{22}	Q'_{23}	Q''_{23}	Q'_{31}	Q''_{31}	Q'_{32}	Q''_{32}	Q'_{33}	Q''_{33}	
0.05	-0.0187	-3.3985	-3.4086	5.7714	-1.8690	4.3353	0.0007	-0.0007	-0.0004	-0.5222	-0.3699	-0.1288	-0.00003	-0.01083	-0.01085	0.00073	-0.02094	0.00319	
0.10	-0.0523	-3.1766	-3.2056	3.6415	-1.7476	3.1077	0.0029	-0.0021	-0.0010	-0.5158	-0.3707	-0.1248	-0.00007	-0.01016	-0.01020	-0.00583	-0.02057	-0.00058	
0.20	-0.1205	-2.8237	-2.8950	1.6054	-1.5547	1.8979	0.0111	-0.0055	-0.0015	-0.5065	-0.3728	-0.1184	-0.00001	-0.00914	-0.00916	-0.01199	-0.02001	-0.00422	
0.40	-0.1928	-2.4011	-2.5397	0.0173	-1.3225	0.9158	0.0421	-0.0128	0.0020	-0.4976	-0.3775	-0.1100	0.00082	-0.00796	-0.00764	-0.01674	-0.01937	-0.00708	
0.60	-0.1690	-2.1862	-2.3632	-0.6032	-1.2006	0.5187	0.0925	-0.0204	0.0113	-0.4961	-0.3825	-0.1051	0.00261	-0.00742	-0.00630	-0.01864	-0.01906	-0.00820	
0.80	-0.0618	-2.0756	-2.2691	-0.9122	-1.1323	0.3194	0.1626	-0.0291	0.0254	-0.4987	-0.3882	-0.1016	0.00535	-0.00720	-0.00488	-0.01969	-0.01891	-0.00875	
1.00	0.1179	-2.0256	-2.2198	-1.0911	-1.0930	0.2065	0.2532	-0.0396	0.0433	-0.5042	-0.3946	-0.0989	0.00904	-0.00719	-0.00327	-0.02040	-0.01884	-0.00905	
1.20	0.3619	-2.0158	-2.1997	-1.2065	-1.0711	0.1376	0.3648	-0.0527	0.0640	-0.5118	-0.4019	-0.0966	0.01368	-0.00733	-0.00144	-0.02098	-0.01882	-0.00923	
1.40	0.6634	-2.0360	-2.2025	-1.2875	-1.0603	0.0936	0.4982	-0.0690	0.0866	-0.5211	-0.4101	-0.0944	0.01930	-0.00763	0.00061	-0.02150	-0.01884	-0.00934	
1.60	1.0153	-2.0808	-2.2265	-1.3477	-1.0567	0.0651	0.6533	-0.0893	0.1097	-0.5318	-0.4192	-0.0921	0.02591	-0.00808	0.00286	-0.02201	-0.01889	-0.00940	
1.80	1.4095	-2.1466	-2.2717	-1.3942	-1.0579	0.0466	0.8298	-0.1141	0.1319	-0.5436	-0.4293	-0.0897	0.03350	-0.00870	0.00527	-0.02252	-0.01896	-0.00943	
2.00	1.8361	-2.2312	-2.3398	-1.4305	-1.0618	0.0350	1.0266	-0.1441	0.1517	-0.5562	-0.4401	-0.0872	0.04207	-0.00950	0.00782	-0.02303	-0.01905	-0.00944	

TABLE 6.3 Generalised Airforce Coefficients of an Oscillating Aerofoil with Trailing-edge Control ($\xi_a = 0.25$, $\xi_h = 0.80$, $M = 0.65$)

$M = 0.65$																		
ν	Q'_{11}	Q''_{11}	Q'_{12}	Q''_{12}	Q'_{13}	Q''_{13}	Q'_{21}	Q''_{21}	Q'_{22}	Q''_{22}	Q'_{23}	Q''_{23}	Q'_{31}	Q''_{31}	Q'_{32}	Q''_{32}	Q'_{33}	Q''_{33}
0.05	-0.0269	-3.7866	-3.8011	8.8852	-2.0807	6.3338	0.0011	-0.0021	-0.0017	-0.6829	-0.4223	-0.1638	-0.00005	-0.01210	-0.01213	0.00520	-0.02373	0.00627
0.10	-0.0733	-3.4640	-3.5047	5.6072	-1.9009	4.4288	0.0040	-0.0060	-0.0045	-0.6627	-0.4247	-0.1510	-0.00010	-0.01119	-0.01125	-0.00456	-0.02323	0.00063
0.20	-0.1633	-2.9883	-3.0846	2.6022	-1.6333	2.6236	0.0147	-0.0148	-0.0098	-0.6348	-0.4303	-0.1313	-0.00002	-0.00992	-0.00996	-0.01322	-0.02254	-0.00450
0.40	-0.2633	-2.4786	-2.6635	0.4388	-1.3380	1.2671	0.0532	-0.0324	-0.0156	-0.6100	-0.4422	-0.1074	0.00100	-0.00869	-0.00830	-0.01932	-0.02193	-0.00810
0.60	-0.2648	-2.2493	-2.4955	-0.3382	-1.1915	0.7599	0.1138	-0.0509	-0.0176	-0.6050	-0.4549	-0.0924	0.00313	-0.00829	-0.00699	-0.02166	-0.02179	-0.00933
0.80	-0.1931	-2.1484	-2.4417	-0.6996	-1.1077	0.5185	0.1968	-0.0729	-0.0197	-0.6088	-0.4685	-0.0806	0.00634	-0.00830	-0.00566	-0.02299	-0.02187	-0.00983
1.00	-0.0681	-2.1190	-2.4549	-0.8907	-1.0529	0.3876	0.3018	-0.1007	-0.0257	-0.6172	-0.4829	-0.0699	0.01063	-0.00861	-0.00427	-0.02396	-0.02209	-0.01003
1.20	0.0923	-2.1349	-2.5158	-0.9957	-1.0107	0.3108	0.4271	-0.1359	-0.0391	-0.6272	-0.4973	-0.0593	0.01599	-0.00921	-0.00283	-0.02478	-0.02238	-0.01007
1.40	0.2698	-2.1806	-2.6143	-1.0488	-0.9719	0.2627	0.5686	-0.1792	-0.0633	-0.6364	-0.5107	-0.0486	0.02236	-0.01010	-0.00142	-0.02551	-0.02270	-0.01002
1.60	0.4458	-2.2442	-2.7412	-1.0658	-0.9304	0.2301	0.7199	-0.2305	-0.1008	-0.6429	-0.5216	-0.0379	0.02963	-0.01128	-0.00010	-0.02615	-0.02302	-0.00992
1.80	0.6033	-2.3151	-2.8855	-1.0555	-0.8830	0.2053	0.8720	-0.2883	-0.1522	-0.6446	-0.5285	-0.0276	0.03764	-0.01273	0.00110	-0.02668	-0.02330	-0.00978
2.00	0.7306	-2.3834	-3.0337	-1.0245	-0.8289	0.1834	1.0156	-0.3498	-0.2159	-0.6405	-0.5303	-0.0183	0.04615	-0.01440	0.00216	-0.02709	-0.02349	-0.00964

TABLE 6.4 Generalised Airforce Coefficients of an Oscillating Aerofoil with Trailing-edge Control ($\xi_a = 0.25$, $\xi_h = 0.80$, $M = 0.75$)

$M = 0.75$																		
ν	Q'_{11}	Q''_{11}	Q'_{12}	Q''_{12}	Q'_{13}	Q''_{13}	Q'_{21}	Q''_{21}	Q'_{22}	Q''_{22}	Q'_{23}	Q''_{23}	Q'_{31}	Q''_{31}	Q'_{32}	Q''_{32}	Q'_{33}	Q''_{33}
0.05	-0.0384	-4.2136	-4.2344	13.2514	-2.3116	9.1288	0.0015	-0.0050	-0.0044	-0.9135	-0.4868	-0.2083	-0.00007	-0.01355	-0.01359	0.01126	-0.02706	0.01044
0.10	-0.1001	-3.7483	-3.8040	8.1526	-2.0468	6.1365	0.0056	-0.0135	-0.0116	-0.8622	-0.4920	-0.1754	-0.00013	-0.01232	-0.01239	-0.00331	-0.02638	0.00197
0.20	-0.2120	-3.1236	-3.2489	3.7700	-1.6836	3.4698	0.0193	-0.0314	-0.0254	-0.7972	-0.5036	-0.1285	-0.00001	-0.01081	-0.01085	-0.01523	-0.02559	-0.00508
0.40	-0.3376	-2.5271	-2.7613	0.9117	-1.3144	1.6419	0.0654	-0.0651	-0.0486	-0.7437	-0.5259	-0.0735	0.00126	-0.00964	-0.00916	-0.02280	-0.02513	-0.00936
0.60	-0.3682	-2.2830	-2.6016	-0.0174	-1.1298	1.0068	0.1345	-0.1015	-0.0747	-0.7282	-0.5464	-0.0372	0.00379	-0.00950	-0.00801	-0.02558	-0.02531	-0.01049
0.80	-0.3443	-2.1810	-2.5768	-0.4069	-1.0080	0.7105	0.2237	-0.1457	-0.1130	-0.7233	-0.5631	-0.0067	0.00753	-0.00992	-0.00705	-0.02717	-0.02575	-0.01072
1.00	-0.2951	-2.1459	-2.6194	-0.5782	-0.9065	0.5427	0.3260	-0.1997	-0.1699	-0.7164	-0.5721	0.0205	0.01238	-0.01079	-0.00630	-0.02828	-0.02627	-0.01059
1.20	-0.2418	-2.1397	-2.6906	-0.6407	-0.8095	0.4289	0.4301	-0.2612	-0.2462	-0.7000	-0.5691	0.0430	0.01818	-0.01205	-0.00580	-0.02906	-0.02674	-0.01032
1.40	-0.1939	-2.1390	-2.7589	-0.6463	-0.7156	0.3378	0.5230	-0.3243	-0.3355	-0.6709	-0.5518	0.0584	0.02469	-0.01363	-0.00554	-0.02951	-0.02705	-0.01004
1.60	-0.1458	-2.1315	-2.8008	-0.6273	-0.6321	0.2575	0.5964	-0.3815	-0.4257	-0.6310	-0.5218	0.0652	0.03167	-0.01538	-0.00535	-0.02968	-0.02719	-0.00984
1.80	-0.0806	-2.1159	-2.8061	-0.6056	-0.5686	0.1853	0.6500	-0.4275	-0.5039	-0.5856	-0.4843	0.0635	0.03901	-0.01718	-0.00502	-0.02965	-0.02721	-0.00975
2.00	0.0202	-2.0988	-2.7796	-0.5938	-0.5322	0.1230	0.6909	-0.4607	-0.5619	-0.5410	-0.4457	0.0552	0.04674	-0.01900	-0.00438	-0.02956	-0.02724	-0.00977

TABLE 6.5 Generalised Airforce Coefficients of an Oscillating Aerofoil with Trailing-edge Control ($\xi_a = 0.25, \xi_h = 0.80, M = 0.80$)

$M = 0.80$																		
ν	Q'_{11}	Q''_{11}	Q'_{12}	Q''_{12}	Q'_{13}	Q''_{13}	Q'_{21}	Q''_{21}	Q'_{22}	Q''_{22}	Q'_{23}	Q''_{23}	Q'_{31}	Q''_{31}	Q'_{32}	Q''_{32}	Q'_{33}	Q''_{33}
0.05	-0.0483	-4.5144	-4.5405	17.0629	-2.4719	11.5682	0.0020	-0.0085	-0.0078	-1.1212	-0.5384	-0.2420	-0.00009	-0.01461	-0.01466	0.01630	-0.02964	0.01393
0.10	-0.1215	-3.9257	-3.9934	10.2044	-2.1317	7.5128	0.0069	-0.0220	-0.0197	-1.0315	-0.5468	-0.1838	-0.00015	-0.01313	-0.01322	-0.00268	-0.02883	0.00285
0.20	-0.2471	-3.1905	-3.3369	4.6321	-1.6937	4.0877	0.0228	-0.0487	-0.0424	-0.9252	-0.5640	-0.1051	0.00002	-0.01149	-0.01153	-0.01719	-0.02800	-0.00570
0.40	-0.3885	-2.5387	-2.8061	1.2560	-1.2685	1.8886	0.0732	-0.0975	-0.0842	-0.8402	-0.5935	-0.0149	0.00148	-0.01047	-0.00992	-0.02574	-0.02778	-0.01024
0.60	-0.4405	-2.2800	-2.6442	0.2339	-1.0478	1.1494	0.1439	-0.1502	-0.1374	-0.8063	-0.6134	0.0460	0.00431	-0.01062	-0.00905	-0.02876	-0.02826	-0.01106
0.80	-0.4508	-2.1610	-2.6124	-0.1643	-0.8883	0.7922	0.2259	-0.2118	-0.2134	-0.7757	-0.6159	0.0932	0.00835	-0.01144	-0.00859	-0.03038	-0.02892	-0.01089
1.00	-0.4439	-2.0948	-2.6212	-0.3258	-0.7536	0.5682	0.3045	-0.2777	-0.3102	-0.7303	-0.5936	0.1243	0.01339	-0.01276	-0.00856	-0.03129	-0.02946	-0.01045
1.20	-0.4226	-2.0439	-2.6214	-0.3913	-0.6449	0.3999	0.3662	-0.3371	-0.4111	-0.6679	-0.5472	0.1345	0.01914	-0.01439	-0.00879	-0.03163	-0.02973	-0.01006
1.40	-0.3726	-1.9985	-2.5940	-0.4283	-0.5753	0.2667	0.4092	-0.3808	-0.4945	-0.5986	-0.4882	0.1243	0.02542	-0.01615	-0.00898	-0.03161	-0.02979	-0.00989
1.60	-0.2783	-1.9654	-2.5487	-0.4667	-0.5513	0.1666	0.4441	-0.4073	-0.5476	-0.5357	-0.4321	0.1005	0.03223	-0.01795	-0.00890	-0.03147	-0.02988	-0.00992
1.80	-0.1394	-1.9548	-2.5095	-0.5120	-0.5666	0.1002	0.4850	-0.4224	-0.5717	-0.4875	-0.3905	0.0719	0.03964	-0.01982	-0.00852	-0.03139	-0.03019	-0.01004
2.00	0.0251	-1.9710	-2.4990	-0.5576	-0.6044	0.0644	0.5405	-0.4342	-0.5773	-0.4556	-0.3680	0.0454	0.04759	-0.02188	-0.00803	-0.03146	-0.03080	-0.01012

TABLE 6.6 Generalised Airforce Coefficients of an Oscillating Aerofoil with Trailing-edge Control ($\xi_a = 0.25, \xi_h = 0.80, M = 0.85$)

$M = 0.85$																		
ν	Q'_{11}	Q''_{11}	Q'_{12}	Q''_{12}	Q'_{13}	Q''_{13}	Q'_{21}	Q''_{21}	Q'_{22}	Q''_{22}	Q'_{23}	Q''_{23}	Q'_{31}	Q''_{31}	Q'_{32}	Q''_{32}	Q'_{33}	Q''_{33}
0.05	-0.0640	-4.9033	-4.9381	23.1740	-2.6743	15.4850	0.0027	-0.0160	-0.0151	-1.4686	-0.6171	-0.2827	-0.00011	-0.01607	-0.01613	0.02378	-0.03344	0.01920
0.10	-0.1524	-4.1228	-4.2080	13.2169	-2.2136	9.5327	0.0090	-0.0391	-0.0364	-1.2945	-0.6314	-0.1676	-0.00017	-0.01425	-0.01435	-0.00250	-0.03245	0.00380
0.20	-0.2935	-3.2415	-3.4158	5.7970	-1.6700	4.9010	0.0275	-0.0811	-0.0753	-1.1066	-0.6575	-0.0228	0.00007	-0.01253	-0.01255	-0.02065	-0.03167	-0.00674
0.40	-0.4524	-2.5226	-2.8289	1.7210	-1.1659	2.1562	0.0800	-0.1545	-0.1524	-0.9551	-0.6895	0.1390	0.00182	-0.01186	-0.01125	-0.03029	-0.03192	-0.01110
0.60	-0.5284	-2.2320	-2.6374	0.5803	-0.8946	1.2379	0.1411	-0.2285	-0.2530	-0.8624	-0.6798	0.2358	0.00498	-0.01255	-0.01105	-0.03335	-0.03279	-0.01102
0.80	-0.5610	-2.0705	-2.5428	0.1372	-0.7136	0.7574	0.1918	-0.2959	-0.3695	-0.7544	-0.6154	0.2720	0.00920	-0.01395	-0.01156	-0.03449	-0.03339	-0.01020
1.00	-0.5488	-1.9618	-2.4601	-0.0925	-0.6173	0.4525	0.2239	-0.3396	-0.4645	-0.6368	-0.5173	0.2444	0.01417	-0.01564	-0.01234	-0.03461	-0.03351	-0.00974
1.20	-0.4833	-1.8971	-2.3909	-0.2555	-0.6030	0.2678	0.2522	-0.3575	-0.5125	-0.5415	-0.4306	0.1794	0.01982	-0.01741	-0.01292	-0.03438	-0.03357	-0.00979
1.40	-0.3790	-1.8785	-2.3646	-0.3811	-0.6353	0.1774	0.2966	-0.3649	-0.5247	-0.4843	-0.3841	0.1132	0.02623	-0.01935	-0.01330	-0.03425	-0.03405	-0.00997
1.60	-0.2715	-1.8937	-2.3902	-0.4631	-0.6634	0.1480	0.3607	-0.3777	-0.5291	-0.4574	-0.3769	0.0671	0.03321	-0.02159	-0.01383	-0.03427	-0.03494	-0.00995
1.80	-0.1875	-1.9174	-2.4410	-0.4998	-0.6500	0.1414	0.4299	-0.4010	-0.5461	-0.4433	-0.3881	0.0450	0.04027	-0.02411	-0.01467	-0.03421	-0.03587	-0.00968
2.00	-0.1166	-1.9296	-2.4749	-0.5072	-0.5958	0.1270	0.4870	-0.4285	-0.5750	-0.4285	-0.3939	0.0384	0.04705	-0.02670	-0.01564	-0.03397	-0.03650	-0.00934