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Undamped natural vibration of core buildings

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UNDAMPED NATURAL VIBRATION OF CORE BUILDINGS

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UNDAMPED NATURAL VIBRATION OF CORE BUILDINGS

1. NOTATION AND UNITS

Only the SI system of units is quoted, but any consistent system of units may be used.

		SI
D	non-dimensional core rigidity taper factor	
E_r	Young's modulus of elasticity of r th storey	N/m ²
f_j	natural frequency for mode j	Hz
h_r	radius of gyration of floor r	m
I_r	second moment of area of r th storey	m ⁴
K_j	normal-mode generalised stiffness for mode j	N/m
l_r	spacing between r th and $(r-1)$ th storey	m
m_r	lumped mass of r th storey	kg
M_j	normal-mode generalised mass for mode j	kg
n	number of storeys	
r	storey number	
S_r	shear rigidity of r th storey	N
θ_j	non-dimensional generalised mass ratio for mode j , M_j/M_1	
λ	foundation rotational spring stiffness	N m
$\mu_j(\xi_r)$	non-dimensional lateral deflection of the r th floor for mode j , normalised with respect to the lateral deflection of the top storey, so $\mu_j(1) = 1$	
ν_j	natural frequency ratio for mode j , f_j/f_1	
ξ_r	the ratio r/n (for ground level $\xi_0 = 0$ and for top storey $\xi_n = 1$)	
ρ_1	non-dimensional generalised mass for mode 1	
ϕ_1	natural frequency parameter for mode 1	
$\chi_j(\xi_r)$	rotation of the r th floor for mode j , normalised with respect to the lateral deflection of the top storey	m ⁻¹

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Subscripts

j	relates to normal mode number
r	relates to storey number (for ground level $r = 0$ and for top storey $r = n$)

2. NOTES

This Item provides data for the estimation of the natural frequency characteristics for the lateral modes of vibration of core buildings. The way in which these data are used in the case of the calculation of the response of a building under the action of a fluctuating wind load is described in ESDU 87035 and 88019. The Item is intended to complement the work in ESDU 79005, for the undamped natural vibration of shear buildings.

The data provided are applicable to those buildings or other structures which may be modelled as core buildings. A core building may be defined as one which has rigid floor masses attached at the discrete floor levels to a central core which is flexible in both bending and shear. In this Item it is assumed that the reductions in core stiffness due to the axial loads applied by the supported structure are negligible. This stiffness effect would cause a reduction in frequency and may modify the mode shapes, but is generally very small. A typical core building structure is shown in Sketch 2.1. By the suitable choice of bending and shear parameters, the method may allow other structures such as chimneys, lattice towers, or buildings with structural columns to be considered. It is assumed that any soil/substructure interaction may be modelled by a simple rotational spring at the base of the core, permitting rotational displacements but preventing lateral displacement.

An idealised model of the core building is illustrated in Sketch 4.1. A computer program is provided to allow the solution of the model, idealised as described in Section 4.1. Further idealisation of the core building model, as described in Section 4.2, allows a graphical presentation of data relating to the first four natural modes of vibration.

For the torsional vibrations of core buildings, the data are identical to those of shear buildings if warping may be neglected. It is recommended that the shear building ESDU 79005, be used to calculate the torsional mode natural frequency characteristics. Further guidance on warping may be found in Reference 8.



Sketch 2.1 Typical core building structure

3. DATA PROVIDED

The natural frequencies, mode shapes and normal-mode generalised masses of the first four lateral modes of vibration of idealised core buildings with 3* to 100 storeys are provided in graphical form. These data are provided for buildings with uniform storey mass and either uniform or linearly varying stiffness of the central core. Core buildings which cannot be represented by these idealisations should be analysed using the computer program, described in Appendix A.

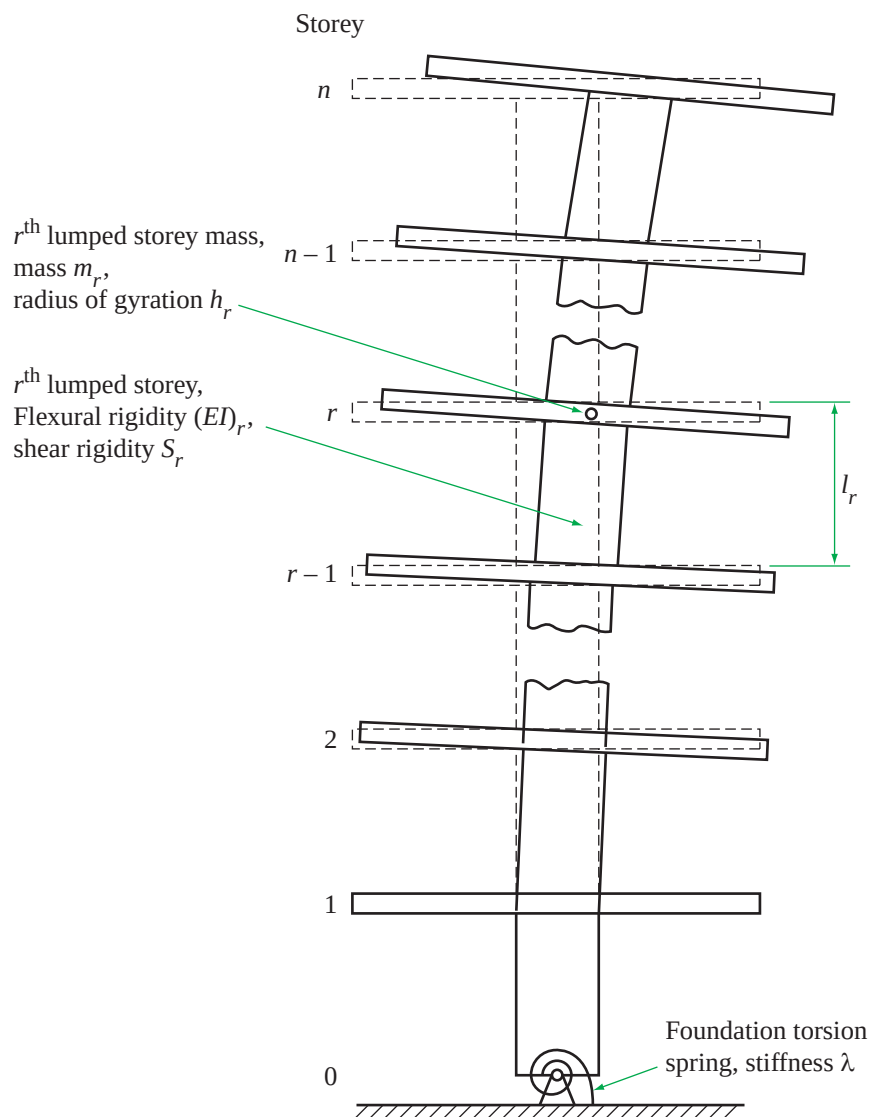
* For the three-storey case, only 3 real solutions exist. Additionally, for certain cases, the range of vibration parameters starts at four or five, for ease of presentation.

4. CORE BUILDING MODELLING AND METHOD OF SOLUTION

4.1 Core Building Model

A core building may be modelled by the lumped parameter system shown in Sketch 4.1. In this model of the core building, the following assumptions are made:

- (i) the foundations may be modelled by a simple rotational spring, with no lateral deflection at the base,
- (ii) the central core is flexible, with finite flexural and shear rigidities, and
- (iii) the masses of the floors are lumped at the floor levels.



Sketch 4.1 n -storey core building model

The lumped floor masses must include the whole of the fixed mass and a proportion of the moveable mass associated with the floor loading (see References 1, 2, 6 and 7). The fixed mass includes the mass of the floor plus the core and walls adjacent to it, together with fittings, partitions and fixed equipment.

The building is considered to 'sway' in one plane only, with no torsional rotation of the floor masses or central core. This may not apply if there is no plane of structural symmetry and the elastic and mass centres are not coincident. The building is then analysed as a system with $2n + 1$ degrees of freedom, n lateral and $n + 1$ rotational.

4.2 Method of Solution

A lumped parameter method (Derivation 1) has been used to calculate the vibrational properties of core buildings. The theory for this method is outlined in Appendix B. Limited data available have given good correlation between theoretical and experimental results. The computer program, described in Appendix A, requires the values of the floor masses and radii of gyration, core section flexural and shear rigidities, the storey spacings, and the foundation rotational spring stiffness for the solution. When calculating the values of the flexural and shear rigidities for use in the analysis, it is recommended that attention be paid to Reference 3. For the calculation of the shear rigidity, it may be necessary to consider the use of a shape factor in the analysis. Reference 5 may be found useful if guidance is required on the foundation effects.

To facilitate a graphical presentation, the core building model has been further idealised to include a core rigidity ratio (see Section 4.3), of the core section flexural rigidity to the product of the core section shear rigidity and the square of the storey spacing. This ratio is assumed to be constant throughout the building. In addition, the rotational inertia effects of the floors are neglected. The effect of neglecting the rotational inertia is to give an increase in the frequency and a decrease in the generalised masses and stiffnesses. The mode shapes may also be modified slightly. Using this further idealisation, two types of core building have been considered for the graphical presentation; a uniform core stiffness type building and a uniformly varying core stiffness type building. In both cases, the floor lumped masses and storey spacings are assumed to be uniform. It is convenient to express the core stiffness variations for graphical presentation as variations in the core flexural rigidities.

If it is felt that the further idealisation imposed by the graphical solution is not justified in the case of the analysis of a particular core building, it is recommended that the computer program described in Appendix A be used for the solutions.

4.3 Idealised Core Building

For the graphical solution, it is necessary to use the constant non-dimensional ratio of the flexural rigidity to the product of the shear rigidity and the square of the storey spacing in the analysis. The graphical solution for three values of this non-dimensional ratio are given in the Item, and it is necessary to estimate the required solution by interpolation from these three solutions.

It is then convenient to express the core stiffness variations used for the graphical presentation as variations in the core flexural rigidity only. The graphical data are provided for two cases of idealised core buildings. In the first case a uniform value of flexural rigidity is used and in the second case a uniformly varying value of flexural rigidity is used. In both cases, the floor lumped masses and storey spacing are assumed to be uniform, the floor radii of gyration are assumed to be zero, and the foundations are considered to be completely rigid.

4.3.1 Mass and stiffness characteristics of the idealised building

For the uniform stiffness type building, the lumped storey mass and the storey flexural rigidity characteristics are defined by Equations (4.1) and (4.2) to be

$$m_r = m \quad (r = 1, 2, 3, \dots, n), \quad (4.1)$$

$$(EI)_r = EI \quad (r = 1, 2, 3, \dots, n). \quad (4.2)$$

The uniformly varying stiffness type building has similar lumped mass characteristics, but the storey flexural rigidity characteristics are characterised by Equation (4.3) as follows

$$(EI)_r = (EI)_n + (n-r)\Delta EI. \quad (4.3)$$

The term ΔEI , the constant stepped-change in flexural rigidity per storey, is defined to be positive when the storey flexural rigidity increases from top to bottom and is expressed in the non-dimensional form, D , as

$$D = \frac{\Delta EI}{(EI)_n}. \quad (4.4)$$

Since a building rarely falls exactly into either of the ideal categories considered in this Item, to use the graphical data provided it is necessary to represent the building approximately by one of the idealisations as discussed in Sections 4.3.2 and 4.3.3. It must be emphasised that the accuracy of the data obtained will depend on how closely the idealisation represents the real core building. Results obtained for a building for which the stiffness variations, storey spacing or storey mass variations do not conform closely to the idealised case should be treated with caution, and it is recommended that in such cases the buildings should be analysed with the aid of the computer program.

4.3.2 Uniform stiffness type building idealisation

For a uniform stiffness type building idealisation the constant lumped mass, storey spacing and storey flexural rigidity for the idealised building can be taken as,

$$m = \frac{1}{n} \sum_{r=1}^n m_r, \quad (4.5)$$

$$l = \frac{1}{n} \sum_{r=1}^n l_r, \quad (4.6)$$

$$EI = \frac{1}{n} \sum_{r=1}^n (EI)_r, \quad (4.7)$$

where the m_r are the lumped storey masses, l_r the storey spacing, and the $(EI)_r$ the storey flexural rigidities of the real building.

4.3.3 Uniformly varying stiffness type building idealisation

Where a real building can be approximated by a uniformly varying stiffness idealisation, the constant lumped mass, storey spacing, storey flexural rigidity, and non-dimensional stepped change in flexural

rigidity for the idealised building can be taken as,

$$m = \frac{1}{n} \sum_{r=1}^n m_r, \quad (4.8)$$

$$l = \frac{1}{n} \sum_{r=1}^n l_r, \quad (4.9)$$

$$EI = \frac{1}{n} \sum_{r=1}^n (EI)_r, \quad (4.10)$$

and
$$\Delta EI = \frac{2}{n(n-1)} \sum_{r=1}^n \{(EI)_r - (EI)_n\}, \quad (4.11)$$

where the m_r are the lumped storey masses, l_r the storey spacings and the $(EI)_r$ the storey flexural rigidities of the real building.

The non-dimensional stiffness taper factor D is given by the equation,

$$D = \frac{\Delta EI}{(EI)_n}. \quad (4.12)$$

5. RANGE OF GRAPHICAL DATA PROVIDED

For the idealised building with constant storey mass, storey spacing and either constant or linearly varying flexural rigidity, graphical data are provided for the first four natural frequencies, corresponding mode shapes and normal-mode generalised masses for buildings from 3* to 100 storeys, for $D = 0$ to 1, and three values of the core rigidity ratio, EI/SI^2 , of 0, 2, and 10. The value $D = 0$ applies to uniform stiffness type buildings, and the core rigidity ratio of 0 applies to pure bending.

5.1 Natural Frequencies

For the fundamental frequency, the frequency parameter, ϕ_1 , is plotted against the number of storeys for a range of values of D in Figure 1, for the three values of the core rigidity ratio considered. The value of fundamental frequency is obtainable from the expression,

$$f_1 = \phi_1 \sqrt{\frac{EI}{mn^4 l^3}}. \quad (5.1)$$

The frequencies of the higher modes are given by

$$f_j = v_j f_1. \quad (5.2)$$

Values of v_2 , v_3 and v_4 are given in Figures 2, 3 and 4 for a range of values of D and the three values of core rigidity ratio considered.

* Lower values of n of four and five respectively have been taken for the third and fourth generalised mass ratio plots.

5.2 Mode Shapes

The normalised mode shapes, $\mu_j(\xi_r)$, for the first four modes are presented in Figures 5, 6, 7 and 8 for a range of values of D and the three values of core rigidity ratio considered. Each mode is normalised with respect to the lateral deflection of the top storey, so that

$$\mu_j(1) = 1. \quad (5.3)$$

The lateral deflection of the base of the building is assumed to be zero, so that

$$\mu_j(0) = 0. \quad (5.4)$$

For each mode, the lumped parameter method gives n discrete values of the lateral deflection $\mu_j(\xi_r)$ for an n -storey building. However, in order to cover the range of storeys considered in this Item, mode shapes are presented as continuous curves of normalised displacements plotted against r/n , where

$$\xi_r = r/n. \quad (5.5)$$

The curves are calculated for a 50-storey building, but provide a reasonable approximation to the mode shapes for other values of n . The error in mode shape will increase as n is reduced and for the lower values of n , (say $n < 10$), the computer program should be used if accurate mode shapes are required.

5.3 Normal-Mode Generalised Mass

For a lumped mass building, M_j , the normal-mode generalised mass for mode j , is defined as

$$M_j = \sum_{r=1}^n \left\{ m_r \mu_j^2(\xi_r) + m_r h_r^2 \chi_j^2(\xi_r) \right\}, \quad (5.6)$$

where $\chi_j(\xi_r)$ is the rotation of floor r for mode j when the mode shape is normalised so that $\mu_j(1) = 1$.

In terms of the non-dimensional generalised mass, ρ_1 , for the first mode

$$M_1 = \rho_1 \sum_{r=1}^n m_r, \quad (5.7)$$

where the summation on the right-hand side is the total mass of the building.

Values of ρ_1 are plotted against n for a range of values of D , for the three values of the core rigidity ratio considered, in Figure 9.

The value of the first mode generalised mass of the idealised building is determined from Equation (5.7) and the generalised mass values for the higher modes are obtained from,

$$M_j = \theta_j M_1. \quad (5.8)$$

Values of θ_j are plotted against n for a range of values of D , for the three values of the core rigidity ratio considered, for modes 2, 3 and 4 in Figures 10, 11 and 12 respectively.

5.4 Normal-Mode Generalised Stiffness

The normal-mode generalised stiffness, K_j , for mode j of the idealised building is determined from the corresponding normal-mode generalised mass value using the equation,

$$K_j = (2\pi f_j)^2 M_j. \quad (5.9)$$

6. DERIVATION AND REFERENCES

6.1 Derivation

1. – Private communication with Professor W.H. Wittrick, Department of Civil Engineering, the University of Birmingham.

6.2 References

The references given are recommended sources of information supplementary to that in this Item.

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7. EXAMPLE

It is required to estimate the natural frequencies, mode shapes, normal-mode generalised masses and normal-mode generalised stiffnesses for the first four lateral modes of a sixty-four storey core building ($n = 64$). The mass, flexural rigidity, shear rigidity and storey spacing characteristics are as follows.

r	m_r (kg)	$(EI)_r$ (N m ²)	S_r (N)	l_r (m)	$\frac{(EI)_r}{S_r l_r^2}$
1 – 16	1 380 000	8.757×10^{13}	8.862×10^{11}	3.375	8.7
17 – 24	1 380 000	8.705×10^{13}	8.015×10^{11}	3.375	9.5
25 – 29	1 380 000	5.369×10^{13}	5.649×10^{11}	3.375	8.3
30 – 48	1 380 000	5.347×10^{13}	5.442×10^{11}	3.375	8.6
49 – 64	1 380 000	2.515×10^{13}	3.141×10^{11}	3.375	7.0

For the Example, the rotational inertia effects of the floors are neglected, and perfect fixity at the foundations is assumed.

7.1 Building Idealisation

For this building, the lumped mass is constant, the storey spacing is constant, and it has an approximately uniformly varying flexural rigidity. Therefore, the building may be idealised to the uniformly varying stiffness type. To obtain the required solutions, the following calculation procedure may be followed.

7.2 Idealisation to the Uniformly Varying Stiffness Type

(i) The value of m is given by Equation (4.8) as,

$$\begin{aligned}
 m &= \frac{1}{n} \sum_{r=1}^n m_r \\
 &= \frac{1}{64} \times 88\,320\,000 = 1\,380\,000 \text{ kg} .
 \end{aligned}$$

(ii) The value of l is given by Equation (4.9) as,

$$\begin{aligned}
 m &= \frac{1}{n} \sum_{r=1}^n l_r \\
 &= \frac{1}{64} \times 216 = 3.375 \text{ m} .
 \end{aligned}$$

(iii) The value of EI is given by Equation (4.10) as,

$$\begin{aligned}
 EI &= \frac{1}{n} \sum_{r=1}^n (EI)_r \\
 &= \frac{1}{64} \times (3.784 \times 10^{15}) = 5.913 \times 10^{13} \text{ N m}^2 .
 \end{aligned}$$