

The stability and response of linear systems

Part VII: examples

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THE PREPARATION OF THIS DATA ITEM

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The Dynamics Committee, which first met in January 1962, has the co-operation of many engineers and scientists in industry, research establishments and universities from whom much assistance and unpublished information is being received.

THE STABILITY AND RESPONSE OF LINEAR SYSTEMS PART VII: EXAMPLES

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THE STABILITY AND RESPONSE OF LINEAR SYSTEMS PART VII: EXAMPLES

1. SYNOPSIS

EXAMPLE A – Design of a control system to position a television camera

The design commences with the general schematic of the system elements required, and concludes with the performance assessment of the final system. The analysis and selection of each element of the control system is considered and the design is tailored to satisfy a performance specification.

Nyquist and Bode plots are used to illustrate features of the control system design; the response of the system in the time domain, to step and sine wave inputs, is used to assess the final design.

EXAMPLE B – Design of an aircraft autopilot height controller

This example considers the design of the simplest type of height controller, one which will direct an aircraft to achieve and maintain a constant altitude.

The aim of this example is to present not only the way in which a height hold system may be constructed and analysed but also give an insight into the way that each of the feedback loops of the autopilot system affects the aircraft characteristic modes.

The design of the system is accomplished with the “two parameter stability diagram” and the use of root locus techniques. The response of the autopilot system in the time domain is found for step pulse gust disturbances, and the non-linear effect of the limited authority of the elevator is examined for a height demand change.

EXAMPLE C – Design of an electromechanical instrument servo

The aim of this example is to present the way in which a data repeater may be designed to meet a given performance specification. It is shown that the system performance can only be partially fulfilled with the use of lead or lag controllers and finally shows how a full realization of the performance requirement of the system can be met with the use of a lag-lead controller.

At each stage of the design of the control system the Nichols chart is used. The final system is assessed in the time domain with the use of the step response, and with the aid of the Bode diagram.

2. REFERENCES

This section gives a selected list of sources of reference supplementary to that presented in this Item

1. ESDU Introduction to notation for aircraft dynamics. Data Item No. 67001, ESDU International, London, January 1967.
2. ESDU Notation for aircraft dynamics. Data Item No. 67002, ESDU International, London, October 1966.
3. ESDU The equations of motion of a rigid aircraft. Data Item No. 67003, ESDU International, London, September 1966.
4. ESDU The response of first- and second-order systems. Data Item No. 69005, ESDU International, London, January 1969.
5. ESDU Solution of ordinary linear differential equations by the Laplace transform method. Data Item No. 69025, ESDU International, London, August 1974.
6. ESDU The stability and response of linear systems. Part I: introductory concepts of stability. Data Item No. 74019, ESDU International, London, August 1974.
7. ESDU The stability and response of linear systems. Part II: methods of displaying stability characteristics. Data Item No. 74020, ESDU International, London, August 1974.
8. ESDU The stability and response of linear systems. Part III: methods of analysis based on frequency response. Data Item No. 74021, ESDU International, London, August 1974.
9. ESDU The stability and response of linear systems. Part IV: specification and measures of system performance. Data Item No. 80002, ESDU International, London, November 1979.
10. ESDU The stability and response of linear systems. Part V: control of dynamic systems. Data Item No. 80018, ESDU International, London, December 1980.
11. ESDU The stability and response of linear systems. Part VI: selection of an assessment method. Data Item No. 81008, ESDU International, London, June 1981.
12. ESDU Numerical methods for the solution of ordinary differential equations: initial value problems. Data Item No. 86011, ESDU International, London, July 1986

EXAMPLE A – DESIGN OF A CONTROL SYSTEM TO POSITION A TELEVISION CAMERA

A1. Introduction

This example considers the design of a control system to position a small television camera. The design commences with the general schematic of the system elements required, and concludes with the performance assessment of the final system. The analysis and selection of each element of the control system is considered and the design is tailored to satisfy a performance specification.

Nyquist and Bode plots are used to illustrate features of the control system design; the response of the system in the time domain, to step and sine wave inputs, is used to assess the final design.

In general two systems would be required to provide full pan and tilt control, but these are identical in nature and so only one degree of freedom is considered. The object of the system would be to position the camera via a signal from a “joystick” situated in a control room.

A simple performance specification can be intuitively chosen in relation to viewer comfort. The system must position the camera quickly with little or no overshoot, but with smooth movement from one position to another. The system must have zero, or at least a small, static position error. Care must be taken in ensuring not too rapid a velocity or acceleration of the camera, which would blur vision to the point of viewer dizziness.

A2. Notation and Units

In this notation units are given for both the Imperial System and for SI.

e_b	back emf of motor	V	V
i_a	current in motor armature	A	A
J	inertia	lbf ft s ²	kg m ²
K_A	amplifier gain	V/V	V/V
K_B	tachometer constant	V/rad/s	V/rad/s
K_P	gain constant of transformer	V/rad	V/rad
k_e	voltage constant of motor	V s/rad	V s/rad
k_t	torque constant of motor	lbf ft/A	N m/A
$\mathcal{L}$	Laplace operator	–	–
L_a	inductance of motor armature	H	H
L_m	$20 \log_{10}$	db	db
n	gear ratio	–	–

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P	power	hp	W
R_a	resistance of motor armature	Ω	Ω
s	Laplace variable	1/s	1/s
T	torque	lbf ft	N m
t	time	s	s
V_a	voltage across motor armature	V	V
Ω	angular velocity	rad/s	rad/s
α	angular acceleration	rad/s ²	rad/s ²
ζ	damping ratio	—	—
θ	motor shaft position	rad	rad
ϕ	camera position	rad	rad
ω	frequency	rad/s	rad/s
ω_N	natural frequency	rad/s	rad/s

Suffixes

C	camera, mounting and driven gear
D	demand
M	motor and drive gear
MAX	maximum
T	effective (or total) quantity

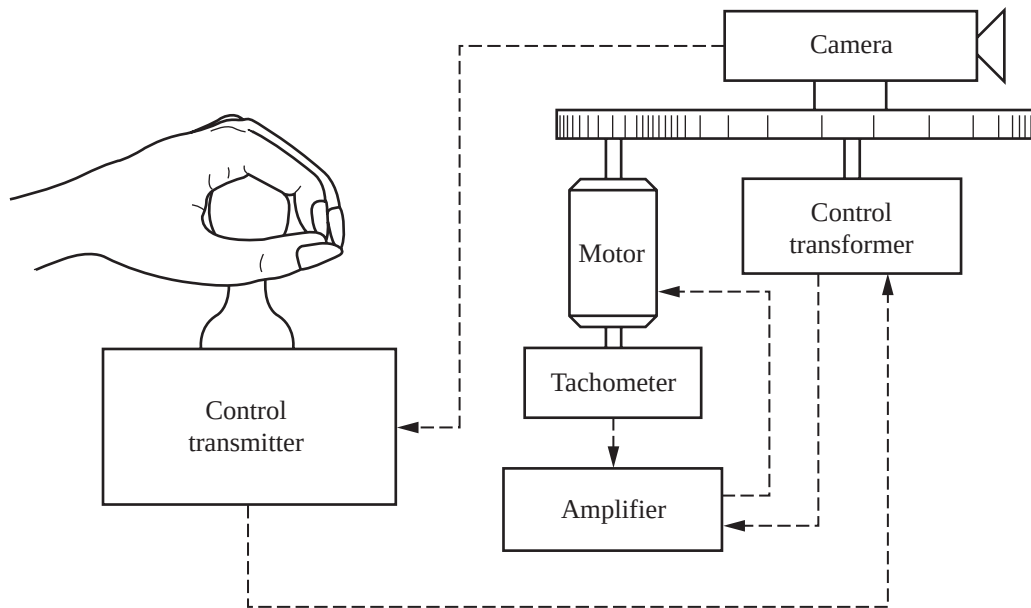
A bar “—” above a quantity indicates the “Laplace” transform of that quantity.

A3. Plant Model

A3.1 General description

The elements of the control system are shown schematically in Sketch [A3.1](#). The input command, or demand signal, is given via a joystick to a control transmitter which computes the error signal. This error signal is the difference between the demand signal and the instantaneous camera position and is fed to a control transformer which converts the error angle in degrees to error voltage, and acts simply as a gain multiplier. The transformer signal is sent to an amplifier and on to the motor which drives the gearing and camera mounting platform. The circuit is completed with the camera position signal being fed back to the control transmitter. An inner loop is also shown with a velocity feedback signal obtained from a tachometer. This inner loop feedback may be required if extra damping is necessary for the control system to meet performance requirements.

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Sketch A3.1 Camera schematic

A3.2 Performance specification

A suitable set of design requirements to achieve a satisfactory system are:

Required damping ratio, ζ	= 0.95
Maximum acceleration, α_{MAX}	= $90^\circ/s^2$ = 1.57 rad/s ²
Maximum velocity, Ω_{MAX}	= $90^\circ/s$ = 15 rpm
Natural frequency of response, ω_N	= 3 Hz = 18.85 rad/s.

The system should position the camera within 1° of the demanded angle.

The inertia of the camera, mounting and gear, J_C , is 176.3 kg m² (130 lbf ft s²) and it is assumed that it has antifriction bearings.

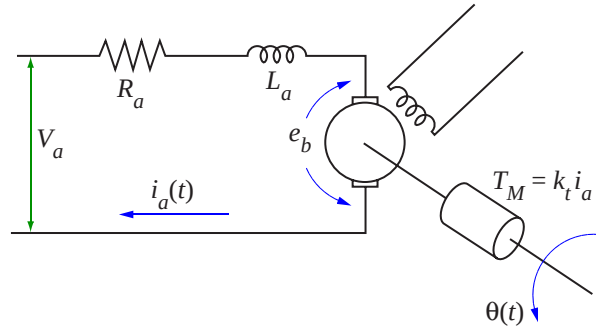
(For information on specification and measures of system performance, see Item No. 80002^[9].)

A3.3 Analysis and selection of servomotor

A DC shunt motor would be best suited for the present application, chosen for its suitability for driving constant loads that require normal starting torque. The torque/speed curves of these motors are usually linear in nature and can be controlled either by variation of the field current or the armature current.

Suppose that the field excitation of the motor is constant and that a control voltage, V_a , is applied to the armature. See Sketch A3.2.

It is assumed that brush and bearing friction is negligible, which is permissible, because the equation that follows contains a term for the back emf, $e_b(t)$. The torque which accelerates the armature is influenced by the reaction of the back emf of the armature.



Sketch A3.2 Direct-current motor

The equation for the motor is:

$$V_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + e_b(t), \quad (\text{A3.1})$$

where $e_b(t) = k_e \frac{d\theta(t)}{dt}, \quad (\text{A3.2})$

$$T_M(t) = k_t i_a(t) \quad (\text{A3.3})$$

and $T_M(t) = J_M \frac{d^2\theta(t)}{dt^2}. \quad (\text{A3.4})$

The armature inductance of the motor is usually small enough to have negligible effect upon the system operation and so neglecting L_A , Equations (A3.1) to (A3.4) may be rearranged to give

$$J_M \frac{d^2\theta(t)}{dt^2} = k_t \frac{\left\{ V_a(t) - k_e \frac{d\theta(t)}{dt} \right\}}{R_a},$$

or $J_M \frac{d^2\theta(t)}{dt^2} + \frac{k_e k_t}{R_a} \frac{d\theta(t)}{dt} = \frac{k_t V_a(t)}{R_a}. \quad (\text{A3.5})$

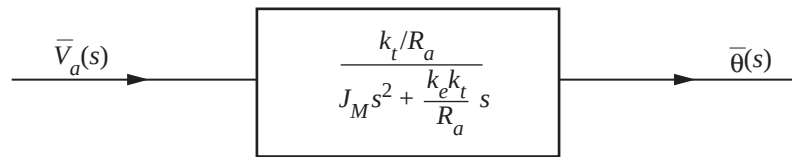
Transforming these equations using the Laplace operator[†], and assuming the motor was at rest at time zero, the motor position transfer function is obtained

$$\frac{\bar{\theta}}{\bar{V}_a}(s) = \frac{k_t/R_a}{\left\{ J_M s^2 + \frac{k_e k_t}{R_a} s \right\}}. \quad (\text{A3.6})$$

Sketch A3.3 shows the block diagram for the motor.

The selection of the motor, which must supply sufficient power for full acceleration at maximum velocity, must now be made.

[†] For reference to the Laplace transform method, see Item No. 69025^[5].



Sketch A3.3 Block diagram for the motor

The maximum torque required is

$$T_{MAX} = J_C \alpha_{MAX} = 176.3 \times 1.57 = 277 \text{ N m (204 lbf ft)}$$

and the maximum velocity

$$\Omega_{MAX} = 90^\circ/\text{s} = 15 \text{ rpm.}$$

The preliminary power requirement, P , for the system is

$$P = \frac{277 \times 15 \times 2\pi}{60} = 0.44 \text{ kW (0.58 hp).}$$

To allow for losses within the system and the power required to accelerate the motor itself, a motor with approximately twice this power capability is sought. Reference to manufacturers specifications could show that a motor with the following characteristics is available.

Power	0.746 kW (1.0 hp)
Rated speed	3250 rpm
Field Winding	Compound shunt
Rated armature volts	115 V
Rated armature current	8.0 A
Torque constant	0.3 N m/A (0.22 lbf ft/A)
Voltage constant	0.30 V s/rad
Rotor Inertia	0.00129 kg m ² (0.00095 lbf ft s ²)

Using these motor characteristics the power requirement is again calculated.

To obtain the maximum power at Ω_{MAX} a gear ratio between the motor and camera shafts of $n = \frac{3250}{15} = 217$ is required.

This gear ratio would normally be achieved with the use of a “gearbox” or “gear train”. A gearbox would introduce additional inertia, damping and friction, and could also introduce non-linear effects such as backlash.

For this example it is assumed that the required gearing is achieved with two gears as shown in Sketch A3.1, and that the inertia of these gears is included in the motor and camera inertias. All other effects, such as damping or friction are assumed to be negligible. The maximum static positioning error requirement of 1° is therefore met implicitly by the system.

With these assumptions it is possible to analyse the system by working in terms of effective conditions at the camera end of the system.

The effective (combined) inertia of the camera, the mounting gears and the motor, at the camera end, is

$$\begin{aligned} J_T &= J_C + n^2 J_M \\ &= 176.3 + (217)^2 \times 0.00129 \\ &= 237 \text{ kg m}^2 \text{ (175 lbf ft s}^2\text{)} \end{aligned}$$

and $T_{MAX} = 237 \times 1.57 = 372 \text{ N m (275 lbf ft)}.$

Hence the power requirement with the addition of the motor inertia is

$$P = \frac{372 \times 2\pi \times 15}{60} = 0.58 \text{ kW (0.78 hp)}.$$

Arbitrarily assuming an overall efficiency of 80%, the power requirement becomes

$$P = \frac{0.58}{0.80} = 0.73 \text{ kW (0.98 hp)},$$

which is just within the rated capability of the proposed motor.

The block diagram of the complete system of motor, gears and camera can now be formed.

In order to relate the motor damping factor $\frac{k_e k_t}{R_a}$ to the load, this damping factor has to be multiplied with the square of the gear ratio. The camera position transfer function becomes

$$\frac{\bar{\phi}}{\bar{V}_a}(s) = \frac{k_t n / R_a}{\left\{ J_T s^2 + \frac{k_t k_e n^2}{R_a} s \right\}}.$$

The amplifier gain is left open as one of the parameters that will be adjusted to meet the design requirements. The sensitivity of the control transformer represents the transducer gain K_P , where

$$K_P = 0.3 \text{ V/}^\circ = 17.2 \text{ V/rad}.$$

The constants can now be calculated.

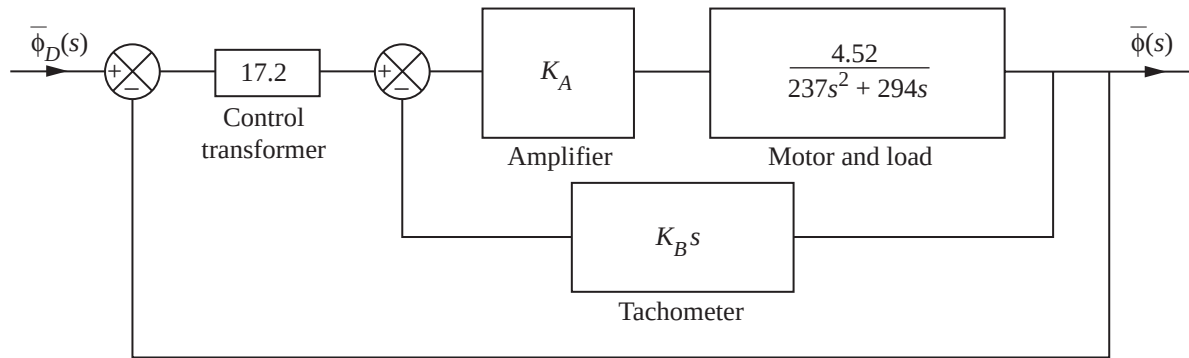
$$\frac{k_t n}{R_a} = \frac{0.3 \times 217}{14.4} = 4.52 \text{ N m/V}.$$

$$J_T = 237 \text{ kg m}^2.$$

$$\frac{k_t k_e n^2}{R_a} = \frac{0.3 \times 0.3 \times (217)^2}{14.4} = 294 \text{ N m s/rad}.$$

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The block diagram for the complete system is given in Sketch A3.4. The diagram shows an inner loop velocity feedback provided by the tachometer attached to the motor, whose transfer function is $K_B s$ V/rad/s. (For reference to introductory concepts of stability including transfer functions and feedback, see ESDU Item No. 74019^[6].)



Sketch A3.4 Block diagram of complete system

A4. Design Requirements

For information relating to control of dynamic systems, see Item No. 80018^[10], and for selection of an assessment method, see Item No. 81008^[11].

A4.1 Without rate feedback

For the control system, given in Sketch A3.4, without the inner loop rate feedback, the open loop transfer function is:-

$$17.2 \times \frac{4.52 K_A}{(237s^2 + 294s)} = \frac{0.327 K_A}{(s^2 + 1.23s)} \quad (\text{A4.1})$$

The closed loop transfer function is:

$$\frac{\bar{\phi}}{\bar{\phi}_D}(s) = \frac{\frac{0.327 K_A}{s^2 + 1.23s}}{\left\{ 1 + \frac{0.327 K_A}{s^2 + 1.23s} \right\}} = \frac{0.327 K_A}{\{s^2 + 1.23s + 0.327 K_A\}}$$

The characteristic equation of this system is

$$s^2 + 1.23s + 0.327 K_A = 0.$$

This second order equation can be equated directly to

$$s^2 + 2\zeta\omega_N s + \omega_N^2 = 0 \quad (\text{A4.2})$$

and hence $0.327 K_A = \omega_N^2$

and $1.23 = 2\zeta\omega_N$.

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The control system specification requires a natural frequency of 18.85 rad/s

and so $K_A = 1087$

and $\zeta = 0.033$,

therefore
$$\frac{\bar{\Phi}}{\bar{\Phi}_D}(s) = \frac{355}{(s^2 + 1.23s + 355)} \quad (\text{A4.3})$$

The system in this form is lightly damped and oscillatory ($\omega_N = 18.85$ rad/s, $\zeta = 0.033$), and will require increased damping so that the damping requirement of $\zeta = 0.95$ for the final system can be met.

A4.2 With rate feedback

The inner loop of the control system, see Sketch A3.4, is that loop involving only the tachometer, the amplifier, and the motor and load.

The transfer function of the forward path of the inner loop is:

$$\frac{4.52K_A}{237s^2 + 294s} = \frac{0.019K_A}{s^2 + 1.23s}$$

The open loop transfer function of the inner loop is:

$$\frac{(0.019K_A)(K_B s)}{s^2 + 1.23s}$$

and hence the closed loop transfer function of this loop may be written as

$$\frac{\frac{0.019K_A}{s^2 + 1.23s}}{\left\{ 1 + \frac{(0.019K_A)(K_B s)}{s^2 + 1.23s} \right\}} = \frac{0.019K_A}{\{s^2 + (1.23 + 0.019K_A K_B)s\}}$$

This inner loop may be regarded as a block in the forward path of the outer loop and so the open loop transfer function of the outer loop is

$$\frac{(17.2)(0.019K_A)}{\{s^2 + (1.23 + 0.019K_A K_B)s\}} \quad (\text{A4.4})$$

Its closed loop transfer function is then:

$$\frac{\bar{\Phi}}{\bar{\Phi}_D}(s) = \frac{0.327K_A}{\{s^2 + (1.23 + 0.019K_A K_B)s + 0.327K_A\}}$$

The characteristic equation of the above transfer function may again be directly compared with Equation (A4.2). The values of frequency and damping ratio given in the performance requirement necessitate that

$$K_A = 1087$$

and $K_B = 1.67 \text{ V/rad/s.}$

This is the value of the gain required at the load and must be converted to its value at the motor shaft

and so $K_B = \frac{1.67}{217} = 0.0077 \text{ V/rad/s.}$

This is the value that is used for choosing a tachometer suitable for the present application.

The tachometer may not, of course, have a gain exactly as required, but it is a simple job, once the tachometer is chosen, to add another unknown gain into the inner loop and repeat the above process.

This being the case it is assumed that a tachometer satisfying the above gain requirements is available.

It can be seen that the amplifier gain, K_A , was not altered by the introduction of the inner rate loop. This must be true, since K_A occurs in the forward path of the system and is a gain constant relating the magnitudes of input and output of the system.

The transfer function of the closed loop system is

$$\frac{\bar{\Phi}}{\bar{\Phi}_D}(s) = \frac{355}{s^2 + 35.8s + 355} \quad (\text{A4.5})$$

Before the performance of the closed loop system is examined, it is interesting to look at a Nyquist plot of the open loops as given by Equations (A4.1) and (A4.4). (For further information on the Nyquist criterion, see Item No. 74021^[8], Part III in this present Series on the stability and response of linear systems.)

Using the amplifier gain, K_A , of 1087, computed values for the magnitudes and arguments of the two equations as a function of frequency have been obtained and are shown in Table A4.1. Figure A1 shows these two systems plotted as Nyquist diagrams. It can be seen that for both systems the gain margin is infinity, since the curves approach the origin without crossing the ordinate axis.

The phase margins from Figure A1 are

without rate feedback	$180^\circ - 176^\circ = 4^\circ$,
with rate feedback	$180^\circ - 105^\circ = 75^\circ$.

TABLE A4.1

Frequency ω	Without Rate Feedback $\frac{355}{i\omega(i\omega + 1.23)}$		With Rate Feedback $\frac{355}{i\omega(i\omega + 35.8)}$	
	Magnitude	Argument (°)	Magnitude	Argument (°)
1	224	-129	9.9	- 92
5	13.8	-166	2.0	- 98
10	3.52	-173	1.05	-106
15	1.57	-175	0.61	-113
20	0.89	-177	0.43	-119
25	0.57	-177	0.33	-125
30	0.39	-177	0.25	-130
35	0.29	-178	0.20	-134
40	0.22	-178	0.17	-138
45	0.18	-178	0.14	-142
50	0.14	-179	0.12	-144
100	0.035	-179	0.03	-160

A5. System Performance

This section examines the responses of the system, with and without rate feedback, to both a step input and sine wave inputs at various frequencies. The closed loop Bode plots of gain and phase are also given. (For reference to Bode diagrams, see Item No. 74021^[8].)

A5.1 Step input

The response of the closed loop system to a step input is

$$\frac{\phi}{\phi_D}(t) = \mathcal{L}^{-1} \left\{ \frac{\bar{\phi}}{\bar{\phi}_D}(s) \cdot \frac{1}{s} \right\}.$$

Using Equation (A4.3) for system without rate feedback

$$\frac{\phi}{\phi_D}(t) = 1 - 1.0005 \{ e^{-0.62t} \sin(18.8t + 1.534) \}.$$

Using Equation (A4.5) for system with rate feedback

$$\frac{\phi}{\phi_D}(t) = 1 - 3.203 \{ e^{-17.9t} \sin(5.88t + 0.318) \}.$$

Figure A2 shows the responses defined by the above two equations. The curves are plotted on time basis of 1 second and 5 seconds. It is clear from design damping ratio values of the two systems, that of 0.033 and 0.95, that the lightly damped system will be highly oscillatory, and the system with rate feedback will be so highly damped as to be virtually non-oscillatory.

The time taken to reach, and remain within, 10% of the final value is (from Figure A2)

with rate feedback	0.19 s,
without rate feedback	3.8 s.

A5.2 Sine wave inputs

Bode and time history plots are given for the closed loop camera system with and without rate feedback, in Figures A3 to A6.

The Bode plot of Figure A3, for the camera system without rate feedback, shows the highly oscillatory nature of the system response; with a peak in gain occurring at the system natural frequency of 18.85 rad/s. The very rapid phase angle change in this region of frequency clearly shows that the output will be almost 180° out of phase with the input at frequencies a little over 20 rad/s. The system is totally unacceptable near or at this resonant frequency.

The Bode plot of Figure A4, for the camera system with rate feedback, contrasts with Figure A3 in the gentle decrease of gain and phase exhibited, and the absence of a peak in gain. The phase becomes large only when the gain is greatly reduced. The steady state output signal will follow the input signal closely at low frequencies, with only a small reduction in gain and a small lag in phase angle. As the frequency rises, the gain of the system reduces and the phase angle increases.

The response of the closed loop system to a sine wave input of frequency, ω , is

$$\frac{\phi}{\phi_D}(t) = \mathcal{L}^{-1} \left\{ \frac{\bar{\phi}}{\bar{\phi}_D}(s) \cdot \frac{\omega}{(s^2 + \omega^2)} \right\}.$$

The term $\frac{\bar{\phi}}{\bar{\phi}_D}(s)$ may be written as $\left\{ \frac{c}{(s+a)^2 + b^2} \right\}$

and hence
$$\frac{\phi}{\phi_D}(t) = \frac{c \sin(\omega t + \phi_1) + (c\omega/b)e^{-at} \sin(bt + \phi_2)}{(4a^2\omega^2 + (a^2 + b^2 - \omega^2)^2)^{1/2}},$$

where
$$\phi_1 = \tan^{-1} \left\{ \frac{-2a\omega}{a^2 + b^2 - \omega^2} \right\}$$

and
$$\phi_2 = \tan^{-1} \left\{ \frac{2ab}{a^2 - b^2 + \omega^2} \right\}.$$

Figures A5 and A6 show the time history plots for both systems to sine wave inputs at frequencies of 5, 10, 20 and 50 rad/s. The input signal is bounded by lines at ± 1 . The plots show the first second, or in the case of 5 rad/s the first 2 seconds, of motion which exhibits the response of the system from rest. This transient portion of the plot is followed by the essentially steady state response from the arbitrary time of 10 secs.

For the lightly damped system, given in Figure A5, the transient effect can be seen in the curves at every frequency. A quite noticeable change in amplitude of the output is clearly visible between the transient and steady state portions of the curves. At both 5 and 10 rad/s the output follows the input closely with a very small phase lag, and output amplitude increases with increase of frequency. At the almost resonant frequency of 20 rad/s the output is many times the input and the phase angle has rapidly increased to about 140° . At the higher frequency of 50 rad/s the output is greatly reduced and is almost 180° out of phase with the input signal. At this frequency a number of seconds is required for the system to settle down to a steady oscillation.

For the almost critically damped ($\zeta = 0.95$) system, shown in Figure A6, very little difference is seen between the transient and steady state portions of the curves. The system very quickly settles down to a steady oscillation, even at the higher frequency of 50 rad/s. The Bode plot of Figure A4 shows the steady decrease of gain with frequency and the absence of any peak in gain, and this is borne out in the time history responses. With increasing frequency the amplitude of the output steadily decreases and the phase angle increases.

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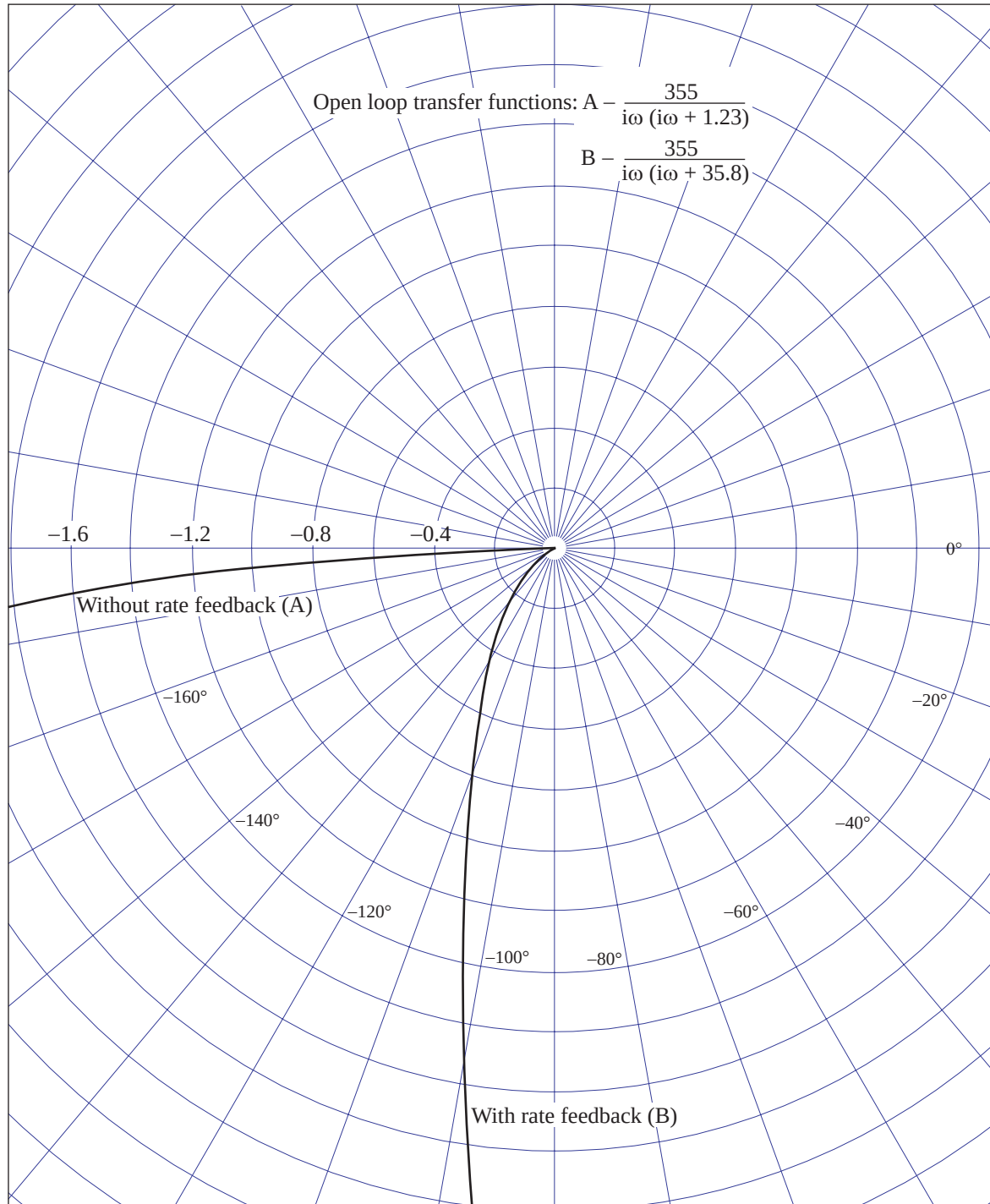


FIGURE A1 POLAR PLOTS OF OPEN LOOP CAMERA SYSTEM WITH AND WITHOUT RATE FEEDBACK

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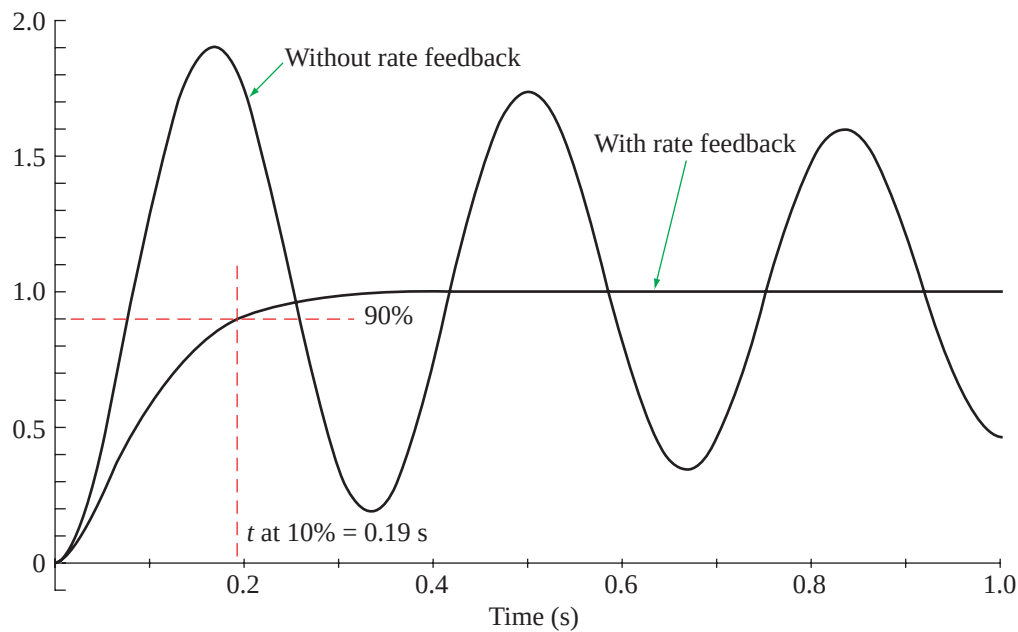
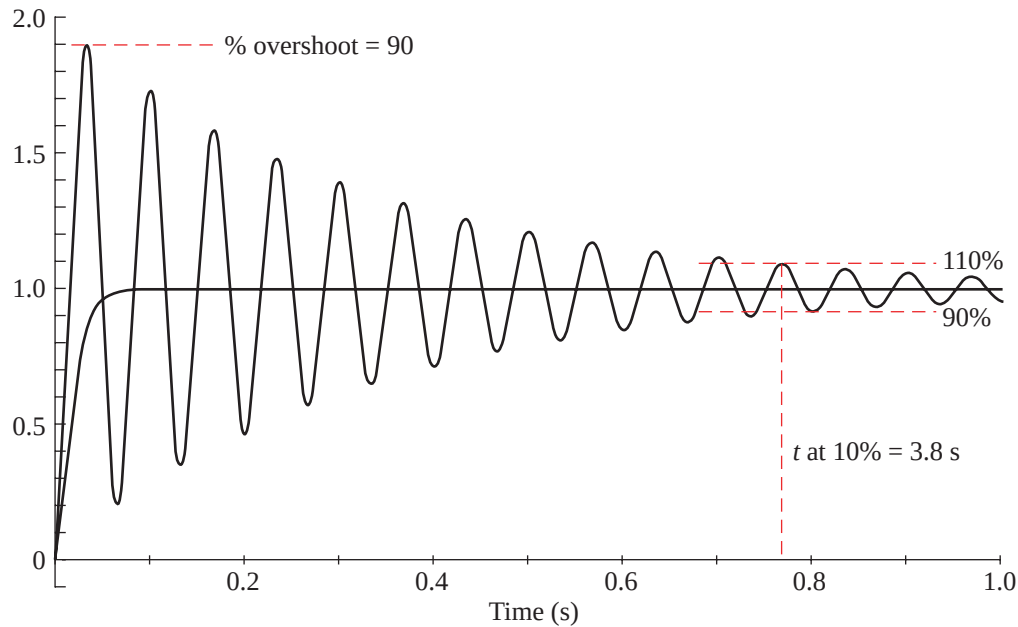


**FIGURE A2 SYSTEM RESPONSE TO A STEP INPUT
WITH AND WITHOUT RATE FEEDBACK**

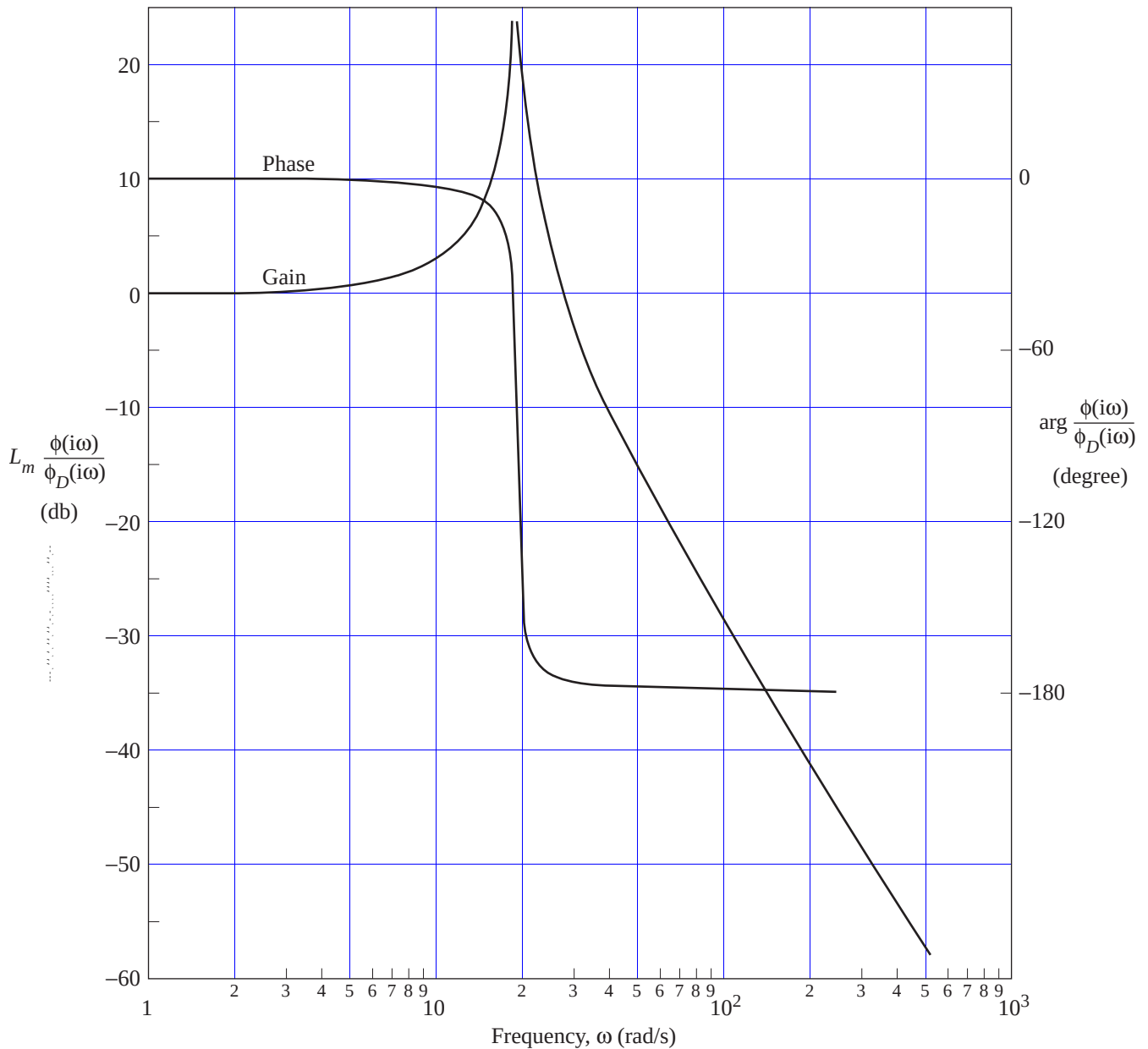


**FIGURE A3 BODE PLOT OF GAIN AND PHASE VERSUS FREQUENCY
WITHOUT RATE FEEDBACK**