

FLEXIBILITIES OF AND STRESSES IN THIN UNPRESSURISED PIPE BENDS WITH FLANGED ENDS UNDER IN-PLANE BENDING: INFLUENCE OF BEND ANGLE

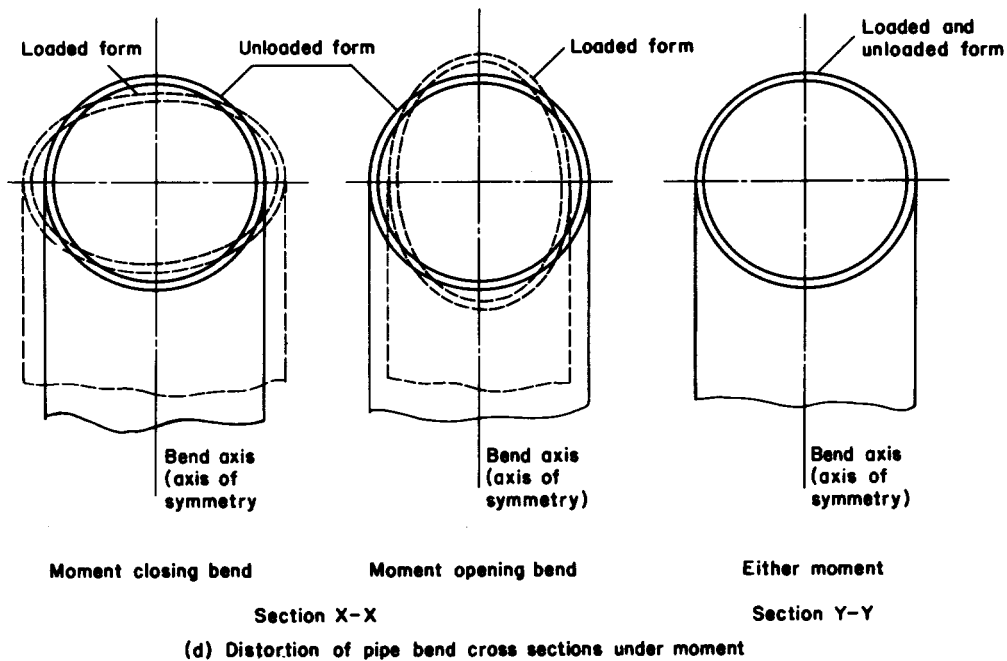
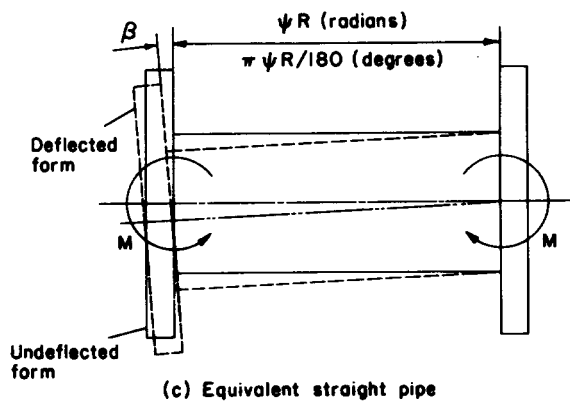
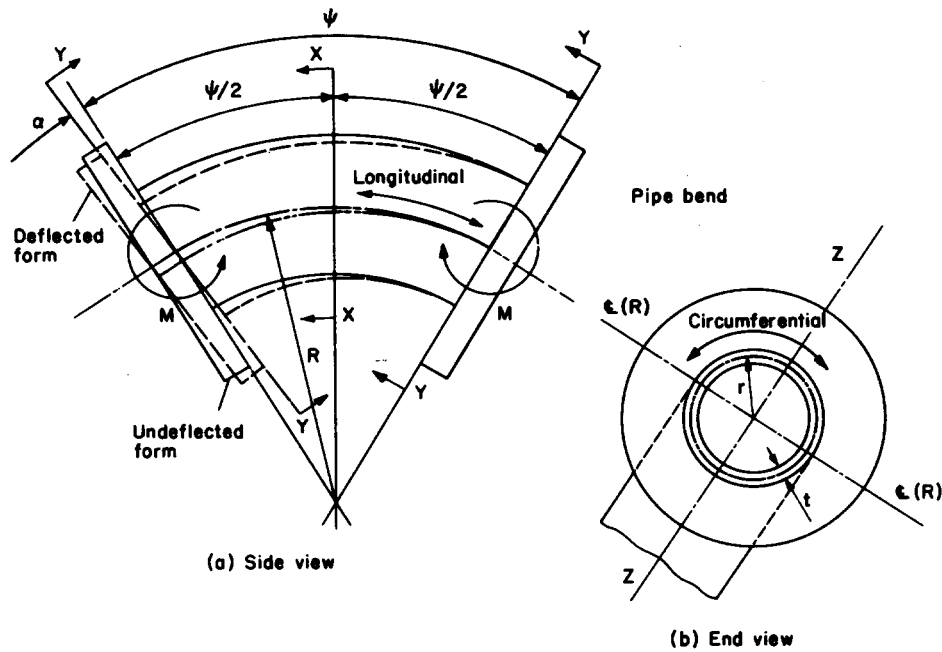
1. NOTATION

E	modulus of elasticity of pipe material	N/m ²	lbf/in ²
F_c	bend stress factor defined by $F_c = f_c/f_{ref}$ (see Section 2)		
F_{L1}, F_{L2}	bend stress factors defined by $F_L = f_L/f_{ref}$ (see Section 2)		
f_c	maximum circumferential direct stress at centre of bend arc, due to moment M (see Sketch 1.1)	N/m ²	lbf/in ²
f_{L1}, f_{L2}	maximum longitudinal direct stresses at centre of bend arc and at flanged ends, respectively, due to moment M (see Sketch 1.1)	N/m ²	lbf/in ²
f_{ref}	maximum stress in equivalent straight pipe, due to moment M , $f_{ref} = Mr/I$	N/m ²	lbf/in ²
I	second moment of area of pipe ($\pi r^3 t$ for $r/t \geq 10$)	m ⁴	in ⁴
K	bend flexibility factor defined by $K = \alpha/\beta$ or $K = \alpha EI/MR\psi$		
M	applied bending moment in plane of pipe bend (positive when closing bend, negative when opening bend)	N m	lbf in
R	radius of curvature of bend axis of unloaded pipe bend (see Sketch 1.1a)	m	in
r	mid-wall radius of unloaded pipe (see Sketch 1.1b)	m	in
t	pipe wall thickness (see Sketch 1.1b)	m	in
α	change in angle of pipe bend under moment M (see Sketch 1.1a)	rad	degree
β	change in angle of equivalent straight pipe under moment M (see Sketch 1.1c)	rad	degree
λ	pipe factor defined by $\lambda = tR/r^2$		
ψ	angle subtended by ends of unloaded pipe bend (see Sketch 1.1a)	rad	degree

Stress convention: tensile stresses are positive, compressive stresses are negative.

Both SI and British Units are quoted but any coherent system of units may be used.

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Sketch 1.1 (not to scale)

2. NOTES

Curves are given that enable the flexibilities, the maximum circumferential stresses and the maximum longitudinal stresses to be obtained for thin smooth circular, unpressurised pipe bends having fully rigid flanges at each end, when subjected to pure opening or closing bending moments in the plane of the bend. The curves are based on a small deflection elastic thin shell theory. In Figures 1 to 3 the flexibilities of the pipe bends are given in terms of the flexibility factor (K) plotted against the bend angle (ψ) for values of the pipe factor (λ), each figure applying to a particular value of the ratio R/r : 2, 5 and 10, respectively. The maximum direct stress factors F_c , F_{L1} and F_{L2} for a positive, closing moment are given for the same parameters in Figures 4 to 6, 7 to 9 and 10 to 12, respectively. The stresses for a pipe bend subjected to an opening moment will be the same but of opposite sign (e.g. a tensile stress in the figures will become a compressive stress).

The bend flexibility factor (K) is the ratio of the effective overall flexibility of the flanged bend to the flexibility, in simple bending, of an equivalent straight pipe. That is a straight pipe of the same cross section and material properties under the same loading condition having a length equal to ψR where ψ is in radians ($\pi\psi R/180$ where ψ is in degrees), see Sketch 1.1c. The effect of flanges on such an equivalent straight pipe is negligible.

The stress factors F_c , F_{L1} and F_{L2} are ratios of the maximum circumferential stress and the maximum longitudinal stresses in the bend to the maximum (longitudinal) bending stress f_{ref} in an equivalent straight pipe (as previously defined). The maximum combined stresses are not given in the figures; these stresses will occur at different points from those at which the maximum direct stresses occur.

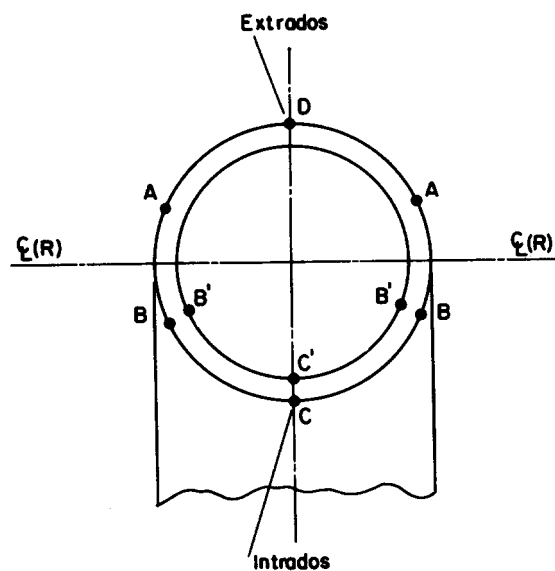
The circumferential and longitudinal stresses vary both around the pipe cross section and along the length of the bend. The stress distribution in any pipe cross section is symmetric about the axis 'Z-Z' of Sketch 1.1b. The stress distribution at any point of the cross section along the length of the bend is symmetric about a radial cross section taken through the pipe at the centre of the bend angle ($\psi/2$), section 'X-X' of Sketch 1.1a. The stresses at any point arise from the combination of two effects; direct longitudinal bending acting on the overall cross section, which induces stresses that vary greatly over the cross section but vary little through the pipe wall thickness, and complementary circumferential distortion of the cross section (as shown in Sketch 1.1d), which induces mainly local bending stresses that change sign (tensile/compression) through the thickness of the pipe wall and vary around the bore.

The maximum direct stress in a particular flanged pipe bend will be one of the three stresses represented by the stress factors F_c , F_{L1} and F_{L2} given in Figures 4 to 12.

The stresses can be a maximum at different positions around the pipe cross section for different values of the parameters used in the figures. Each figure therefore presents the stresses at particular points in the pipe cross section (as shown in Sketch 2.1) over a range of the parameters large enough to include the circumstances when the stress will be a maximum at that point.

Because of the complexity of the analysis the data on which the curves are based are those taken from Derivation 4 supplemented by limited computation by ESDU using the same method. The actual data used for the curves are presented on the figures as an indication of the confidence that may be placed in particular sections of the curves.

The data in the figures have been obtained using the method and computer program provided in Derivation 4 in which pipe bends, with various end conditions, are analysed by a method based on the principle of minimum total potential energy. The small deflection thin shell analysis assumes that linear elastic relationships hold for the pipe bend and that radial stresses in the pipe wall may be neglected. The data provided relate strictly to pipe bends in which $r/t \geq 10$ but may be used with reasonable accuracy for pipe bends in which $r/t \geq 5$ as in the figures. The flanges are assumed to be rigid, constraining the coincident pipe cross section to rotate and deflect as a flat plane under the action of the moment, allowing no radial, circumferential or longitudinal distortion of its normal cross section. Gross yielding of flanged pipe bends should occur only at higher values of M or α than those considered here and some guidance on the collapse moment in a load controlled situation for unrestrained pipe bends can be obtained from References 7 and 9.



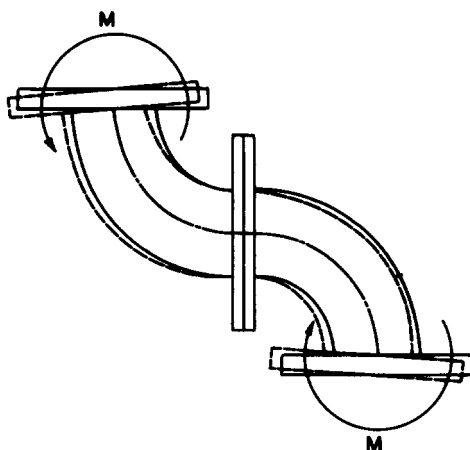
Sketch 2.1

In practice the flanges on the pipe bend will rarely be completely rigid and often may not be immediately adjacent to the ends of the bend. The data provided in this Item represent a lower bound to such cases. In particular the effect of using straight pipes between the bend and the flanges is to decrease the restraining effect of the flanges and increase the flexibility and stress factors to values above those given in this Item. When the lengths of the intermediate pipes are greater than about $6r$, because the restraining effect of the flanges is negligible, the flexibilities and stress factors increase to those of a bend with tangent pipe ends. Detailed guidance on the flexibility and stress factors for bends with adjacent straight pipes will be found in Data Item No. 75014*. Derivation 4 and References 5, 6 and 8 should be consulted for further information on the effects on the flexibilities and stresses in pipe bends arising from combinations of flanges and straight pipes.

The effect of introducing flanges at an intermediate point around a pipe bend is to restrict the ovality of the pipe at that point. Each segment of the bend on either side of the intermediate flange effectively becomes a separate bend with correspondingly reduced flexibility and stress factors. When flanged bends of the opposite direction are attached to each other (for example an S bend flanged at the centre junction, see Sketch 2.2) and are subjected to a constant bending moment they may be considered to act as two separate bends in combination. One bend will tend to open while the other tends to close. If the centre junction of the two segments of an S bend is smooth with no flange the flexibility and stress factors differ very little from those for an S bend with an intermediate flange but in any case are less than those for a smooth S bend

* Data Item No. 75014 "Flexibilities of and stresses in thin pipe bends under in-plane bending: influence of bend angle and tangent pipe ends".

whose ends terminate in straight tangent pipes as described in Data Item No. 75014. The effect of internal pressure is to reduce both the flexibility and stress factors in the bend. Data Item No. 74043* gives detailed guidance on its effect for unrestrained bends; a similar effect can be expected for flanged bends.



Sketch 2.2 Combined bends of opposite direction (S bends)

3. DERIVATION AND REFERENCES

3.1 Derivation

The derivation lists selected sources that have assisted in the preparation of this Item.

1. NATARAJAN, R. Finite element analysis of pipe bends using doubly curved shell elements. Ph.D. Thesis, London University, April 1971.
2. AXELRAD, E. Flexible shell theory and buckling of toroidal shells and tubes. *Ingenieur Archiv*, pp. 95–104, 1978.
3. WATHAM, J.F. In-plane bending of flanged pipe elbows. Metal Struct. Conference, Inst. of Engrs, Perth, Australia, 1978.
4. THOMSON, G. The influence of end constraints on pipe bends. Ph.D. Thesis, University of Strathclyde, 1980.

3.2 References

The references given are a recommended source of information supplementary to that in this Item.

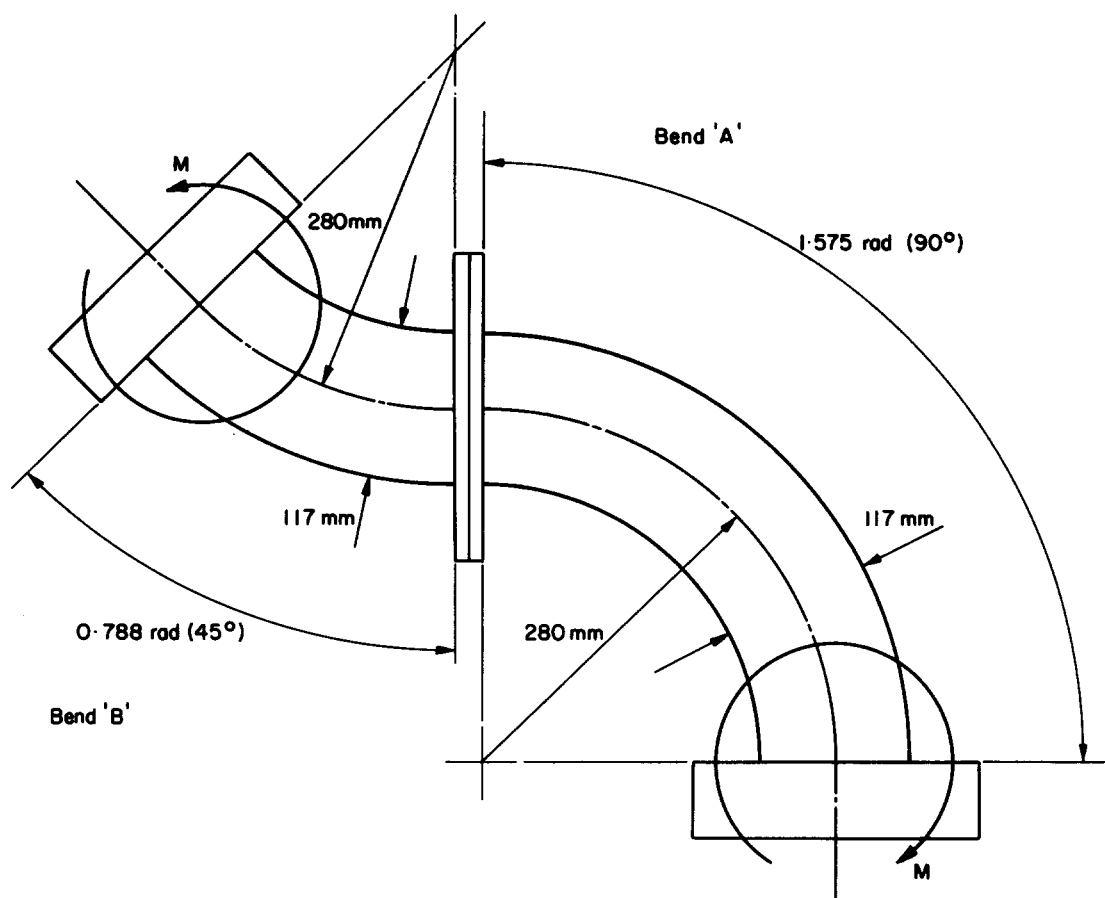
5. ZENO, D.R. Discussion to "Properties of thin-walled curved tubes of short bend radius". *Trans am. Soc. mech. Engrs*, Vol. 73, pp. 77-87, 1951.
6. JACOBS, R.L. SUROSKY, H. Experimental investigation of the effects of end conditions and bend angle on the stress distribution of welding elbows, Phase I. NSRDC Rep 2835, Annapolis, November 1968.

* Data Item No. 74043 "Flexibilities of and stresses in unrestrained pressurised thin pipe bends subjected to in-plane bending".

7. BOLT, S.E. Experimental determinations of plastic collapse loads for pipe elbows. Am. Soc. mech. Engrs Paper 71-PVP-37, Pressure Vessel and Piping Conference, San Francisco, May 1971.
8. IMAMASA, J.
URAGAMI, K. Experimental study of flexibility factors and stresses of welding elbows with end effects. Second International Conference on Pressure Vessel Technology, San Antonio, U.S.A., October 1–4, 1973, Proceedings Part 1-30, pp. 417-426, Am. Soc. mech. Engrs, 1973.
9. SPENCE, J.
FINDLAY, G.E. Limit loads for pipe bends under in-plane bending. Second International Conference on Pressure Vessel Technology, San Antonio, U.S.A., October 1–4, 1973, Proceedings Part 1–28, pp. 393-399, Am. Soc. mech. Engrs, 1973.

4. EXAMPLE

It is required to determine the change in angle and the maximum longitudinal and circumferential stresses resulting from the application of a constant in-plane bending moment of 2.5 kN m to a truncated 'S' pipe bend. The bend consists of two unequal bends with flanged ends bolted together in opposition (see Sketch 4.1). The pipe from which the bends are constructed has an outside diameter of 117 mm and a wall thickness of 5.0 mm. The larger bend has a bend radius of 280 mm and a bend angle of 90° (1.575 rad). The smaller, opposite, bend has a bend radius of 280 mm and a bend angle of 45° (0.788 rad). The moment is applied in a direction that tends to close the larger bend and open the smaller. The value of E for the pipe material is $200\,000\text{ MN/m}^2$.



Sketch 4.1

The following quantities required for use with the figures are common to both segments of the bend and are therefore calculated first.

The pipe radius $r = \left(\frac{117-5}{2} = 56 \right) \text{ mm}.$

The second moment of area $I = \pi r^3 t = \pi \times 0.056^3 \times 0.005$
 $= 2.76 \times 10^{-6} \text{ m}^4.$

The reference stress in an equivalent straight pipe is

$$f_{ref} = \left(\frac{2.5 \times 10^3 \times 56 \times 10^{-3}}{2.76 \times 10^{-6}} = 50.7 \right) \text{ MN/m}^2.$$

The parameter $\frac{R}{r} = \left(\frac{280}{56} = 5.0 \right)$

and $\lambda = \frac{5.0 \times 280}{56^2} = 0.446.$

It can be seen from Section 2 that the change in angle of the composite bend will be the sum of the changes in angle of each bend considered separately. The changes in angles of the bends are obtained using the flexibility factors given in Figure 2 for $R/r = 5$.

The flexibility factor for bend A, when $\lambda = 0.446$ and $\psi = 1.575$ rad, obtained by interpolation is $K = 2.0$.

Since $K = \frac{\alpha EI}{MR\psi},$
the change in angle for bend A is therefore:

$$\alpha = \frac{KMR\psi}{EI} = \frac{2.0 \times 2.5 \times 10^3 \times 280 \times 10^{-3} \times 1.575}{200\,000 \times 10^6 \times 2.76 \times 10^{-6}}$$

$$= 0.004 \text{ rad.}$$

Similarly, the flexibility factor for bend B, when $\lambda = 0.446$ and $\psi = 0.788$ rad, is $K = 1.1$. The change in angle for bend B is therefore:

$$\alpha = \frac{1.1 \times 2.5 \times 10^3 \times 280 \times 10^{-3} \times 0.788}{200\,000 \times 10^6 \times 2.76 \times 10^{-6}}$$

$$= 0.001 \text{ rad.}$$

Therefore, the total change in angle of the composite bend is

$$\alpha = 0.004 + 0.001 = 0.005 \text{ rad.}$$

The stress factors F_c , F_{L1} and F_{L2} for bends A and B when $R/r = 5$ and $\lambda = 0.446$ are obtained from