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Forced convective heat transfer in concentric annuli with turbulent flow

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THE PREPARATION OF THIS DATA ITEM

The work of the permanent professional staff of the Engineering Sciences Data Unit on this particular Item was monitored and guided by the Heat Transfer Steering Group which has the following constitution:

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The Steering Group has benefited from the participation of members from several disciplines. Dr G.F.Hewitt has been appointed to represent the interests of mechanical engineering as the nominee of the Institution of Chemical Engineers.

The work on this Item was carried out in the Fluid Mechanics and Physical Properties Group of the Engineering Sciences Data Unit under the supervision of Mr N.Thompson, Group Head. The member of staff who undertook the technical work involved in the initial assessment of the available information and the construction and subsequent development of the Item was

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The Heat Transfer Steering Group, which first met in February 1966, has the co-operation of many engineers and scientists in industry, research establishments and universities from whom much assistance and information is being received. Particular assistance in the preparation of this Item was received from Dr H.Barrow of University of Liverpool, Dept. of Mechanical Engineering, Mr H.G.Lyall of Central Electricity Generating Board, Berkeley, Dr D.Wilkie, Mr A.C.Rapier and Mr G.Jackson of Heat Transfer and Fluid Flow Group, U.K.A.E.A., Windscale.

FORCED CONVECTIVE HEAT TRANSFER IN CONCENTRIC ANNULI WITH TURBULENT FLOW

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FORCED CONVECTIVE HEAT TRANSFER IN CONCENTRIC ANNULI WITH TURBULENT FLOW

1. NOTATION AND UNITS

		SI Unit
A	flow cross-sectional area	m^2
c_p	specific heat capacity at a constant pressure	$\text{J}/(\text{kg K})$
D_1	outer diameter of inner tube	m
D_2	inner diameter of outer tube	m
D_e	equivalent diameter of annulus, $(D_2 - D_1)$	m
Gr	Grashof number, $\beta g D^3 \rho^2 T_i - T_b / \eta^2$	
f	friction factor	—
f_i	correction factor in Equations (5.4) and (5.5)	—
f_{Di}	factor in Equation (5.1)	—
g	gravitational acceleration	m/s^2
L	tube length	m
$\dot{M}$	mass flow	kg/s
$\dot{m}$	mass flow rate, $\rho_b \bar{u} = \dot{M}/A$	$\text{kg}/(\text{m}^2 \text{ s})$
$\overline{Nu}$	Nusselt number, $\alpha D_e / \lambda_b$	
Pr_b	Prandtl number, $c_{pb} \eta_b / \lambda_b$	
$\dot{q}$	heat flux	W/m^2
$\dot{Q}$	heat transfer rate	W
Re	Reynolds number, $\rho_b \bar{u} D_e / \eta_b = \dot{m} D_e / \eta_b$	
r	radial distance	m
$\overline{St}$	mean Stanton number, $\overline{Nu} / (Re_b Pr_b) = \alpha / (\dot{m} c_{pb})$	
T_b	fluid bulk temperature	K
T_{bo}	value of T_b at $x = 0$	K

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T_i	temperature of wall with heat flux ($i = 1$ or 2 for inner or outer tube wall respectively)	K
ΔT	temperature difference between tube wall and bulk fluid	K
$\bar{u}$	mean fluid velocity	m/s
x	axial distance from commencement of heat transfer or from incremental change in heat flux	m
Y	parameter defined by Equation (5.5)	
α	mean heat transfer coefficient	W/(m ² K)
β	coefficient of volumetric expansion	1/K
η	dynamic viscosity	Ns/m ²
θ_{ij}	influence coefficient, defined in Section 5.2	
λ	thermal conductivity	W/(m K)
ρ	density	kg/m ³

Subscripts

b	evaluated at fluid bulk temperature
c	appropriate to constant fluid properties
i	applies to tube wall considered, $i = 1$ or 2
ij	i ($= 1$ or 2) applies to tube being heated and j ($= 1$ or 2) applies to tube at which Nusselt number is measured
f	appropriate to fully-developed conditions
o	refers to condition at $x = 0$
s	refers to value for plain circular tube
x	applies to station x along annulus
$1, 2$	refers to inner or outer tube walls, respectively

A bar ($\bar{\quad}$) indicates a mean value.

2. PURPOSE AND SCOPE OF THIS DATA ITEM

This Item presents evaluated data relating to forced convective heat transfer in annuli with turbulent flow. The major practical applications for both the presented data, and other data that can be derived from them using the method of superposition*, are in the thermal design of double pipe heat exchangers, aircraft heaters, and cooling channels in process plant. The general calculation procedures for estimating heat transfer and wall temperatures for an annulus with prescribed thermal boundary conditions are summarised in Section 7.

The general applicability of the data in this Item is summarised in Section 3. The definition of the controlling parameters and conditions of heat transfer in an annulus are given in Section 4. The basic correlations used to estimate Stanton or Nusselt numbers for an annulus with particular thermal boundary conditions are described in Section 5. Section 5.1 presents the correlations derived for an annulus with one surface at a constant heat flux and the other surface insulated. Correction factors and methods for dealing with large temperature differences in the radial and in the axial directions are given in Sections 5.1.1 and 5.1.2. For the relatively infrequent case when both surfaces of the annulus are subjected to a heat flux, Section 5.2 presents additional modifications necessary to the correlations in Section 5.1, based on the method of superposition*. Section 5.3 presents a method to be used in conjunction with the basic correlations in order to obtain Nusselt numbers for variable axial heat flux distributions on one or both surfaces of the annulus. Guidance is provided in Section 5.4 on how to obtain an approximate Nusselt number for an annulus with one surface held at constant temperature.

Section 6 provides some qualitative guidance on the influence of effects such as free-convection, eccentricity, developing temperature profile and thermal radiation.

Appendix A details the method of superposition and its use in obtaining data for the case when both surfaces of the annulus are heated with a heat flux that can be varying axially for both, or for only one, of the surfaces. Appendix B (issued in 1982) provides information concerning the derivation of the data presented.

* This method is detailed in Appendix A. The use of the method of superposition is important in extending the applicability of the available experimental data to cover a wider range of thermal boundary conditions.

3. APPLICABILITY OF THE DATA

The data are strictly applicable to:

- (i) Newtonian single-phase fluids,
- (ii) annular passages running full,
- (iii) forced-convective heat transfer with negligible thermal radiation and zero heat generation within the fluid,
- (iv) flow with negligible free-convection effects,
- (v) heating ($T_i/T_b > 1$) and cooling ($T_i/T_b < 1$) of single-phase fluids, *i.e.* not boiling, melting or condensing,
- (vi) clean smooth plain tubes.

The recommended ranges of applicability of the major parameters influencing heat transfer are indicated in the following table which is based on the range of experimental data used to derive the information presented. Most of the experimental data were obtained for air and water.

The recommended correlations, although strictly not derived for parallel plates ($D_2/D_1 = 1$; $D_1/D_2 \rightarrow \infty$) can be applied to this case also with an accuracy of about ± 10 per cent.

<i>Parameter</i>	<i>Range</i>
Re_b	$4 \times 10^3 < Re_b < 3 \times 10^6$
Pr_b	$0.5 < Pr_b < 30$
η_b/η_i (for liquids)	$1 < \eta_b/\eta_i < 2$
T_i/T_b (for gases)	$1 \leq T_i/T_b < 3$
D_2/D_1	$1 < D_2/D_1 < 10$

4. DEFINITION OF THE CONTROLLING PARAMETERS AND CONDITIONS OF HEAT TRANSFER IN AN ANNULUS

4.1 Mean Nusselt Number and Stanton Number

When free-convection effects are negligible (see Section 6.2 for details on free-convection effects) the effective mean Nusselt number at wall i , $\overline{Nu}_i$ (where $i = 1$ or 2 when the inner tube or the outer tube is considered, respectively) is defined as:

$$\overline{Nu}_i = \frac{\overline{\alpha}_i D_e}{\lambda_b} \quad (4.1)$$

where $D_e = (D_2 - D_1)$ and $\overline{\alpha}_i$, the mean heat transfer coefficient, is given by

$$\overline{\alpha}_i = \frac{\dot{q}_i}{(T_i - T_b)}, \quad (4.2)$$

Where $\dot{q}_i$ is the mean flux or the mean heat transfer rate per unit area. Alternatively, the Stanton number, $\overline{St} = \overline{Nu}/(Pr_b Re_b)$, can be used to characterise the heat transfer. A heat balance for the system is obtained by equating the heat transfer to the fluid in the annulus to the heat transfer from the heated surface, *i.e.*

$$\frac{1}{4}\pi(D_2^2 - D_1^2)\rho\bar{u}c_p(T_b - T_{bo}) = \pi D_i L(T_i - T_b)$$

which can be re-arranged to give

$$\overline{St}_i = \frac{\overline{\alpha}}{\rho\bar{u}c_p} = \frac{(T_b - T_{bo}) \times \text{flow cross-sectional area}}{(T_i - T_b) \times \text{heated surface area}}. \quad (4.3)$$

Both $\overline{Nu}_i$ and $\overline{St}_i$ can be expressed as functions of Re_b , Pr_b , T_i/T_b or η_b/η_i , and D_1/D_2 ; the parameter T_i/T_b for gases, or η_b/η_i for liquids, is used to characterise the fluid property variations in the radial direction.

4.2 Heat Transfer Rate

Heat transfer is defined as positive when directed into the fluid in the annulus.

The heat transfer rate over the length of the annulus is given by

$$\dot{Q}_i = \overline{\alpha}_i \pi D_i L \Delta T \quad (4.4)$$

where ΔT is the appropriate mean temperature difference, L is the length of the annulus and i ($= 1$ or 2) indicates conditions at either the inner or outer tube wall. (Note that D_i is used to calculate $\dot{Q}_i$, whereas D_e is used in the definition of $\overline{Nu}_i$). Hence, if the temperature of the fluid at the inner and outer tube walls are designated T_1 and T_2 respectively, the corresponding heat transfer rates per unit area or heat fluxes are defined as

$$\dot{q}_1 = \dot{Q}_1/(\pi D_1 L) = \overline{\alpha}_1(\overline{T_1 - T_b}) \quad (4.5)$$

$$\text{and} \quad \dot{q}_2 = \dot{Q}_2/(\pi D_2 L) = \overline{\alpha}_2(\overline{T_2 - T_b}) \quad (4.6)$$

where T_b is the mean fluid bulk temperature (cup-mixing temperature) as defined in Section 4.2.1.

Note that if both the annulus surfaces are subjected to the same heat flux, $\dot{q}_1 = \dot{q}_2$, the heat transfer rate is greater at the outer surface because of the larger surface area of the outer tube, i.e. $\dot{Q}_2 > \dot{Q}_1$, because $D_2 > D_1$.

A measure of the effectiveness of a heat exchanger is the ratio of the heated surface area to the flow cross-sectional area which should be as large as possible. In the present context this ratio is equal to $LD_i/[1/4(D_2 - D_1)(D_2 + D_1)]$ which means that the most effective configuration is a long annulus with $(D_2 - D_1)$ made as small as possible within practical limitations.

4.2.1 Bulk fluid temperature

The local value of the fluid temperature, T , varies according to position (x, r) . The local bulk temperature at any position x is the integrated mean temperature in the radial direction. At any point along the annulus it is given approximately by

$$T_{b,x} = T_{b,o} + \int_{x=0}^x \frac{\dot{Q}_x}{\dot{M}c_p} dx \quad (4.7)$$

where $T_{b,x}$ is the local value of T_b at any length x from the initial reference point, $T_{b,o}$ is the value of T_b at $x = 0$ and the integral term is the sum of the bulk temperature changes in the fluid at length x from the initial reference point; c_p is the appropriate mean specific heat capacity over the temperature range $T_{b,o}$ to $T_{b,x}$.

4.3 Fully-developed Temperature Profile

For a fully-developed velocity and temperature profile arising from a uniform surface heat flux, the condition must be met that

$$\frac{\partial T}{\partial x} = \text{constant for all values of } r \text{ and } x \quad (4.8)$$

$$\text{and } \frac{\partial T}{\partial r} = \text{constant for any given value of } r \text{ and all values of } x. \quad (4.9)$$

Hence, provided that the physical properties of the fluid are constant, the shape of the radial temperature profile and the magnitude of the temperature driving force are invariant with axial position (x) downstream of the entry region.

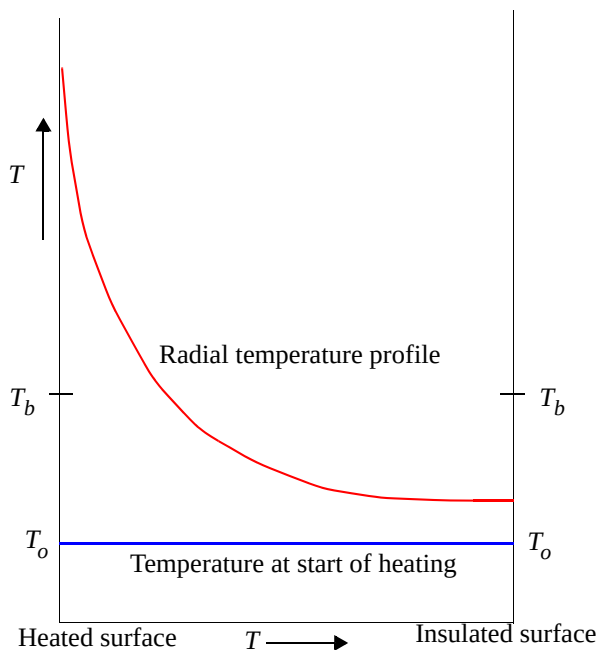
Sketches 5.1 and 5.2 (overleaf) give examples of typical radial and axial temperature profiles expected for a constant heat flux at one wall in an annulus where fully-developed velocity and temperature conditions exist, as would be met in an annulus heated by a constant wattage electrical element wound round one wall.

5. BASIC DATA FOR HEAT TRANSFER BY FORCED CONVECTION IN ANNULI WITH FULLY-DEVELOPED TURBULENT FLOW

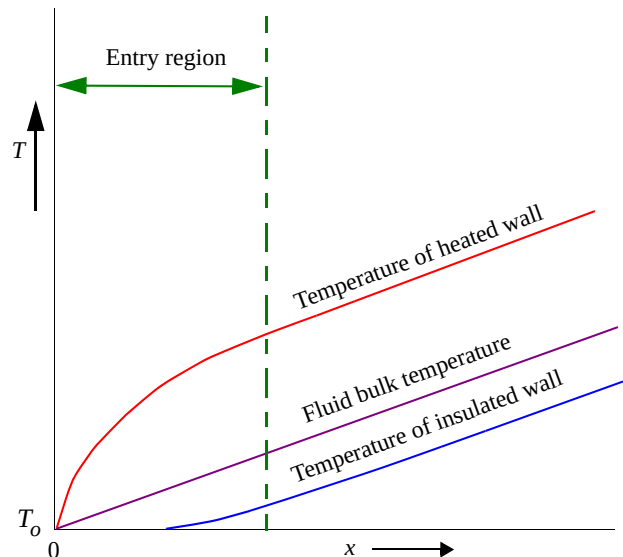
Data and methods for dealing with the following cases are provided in this Section.

- (i) Uniform heat flux at one surface only with the other surface insulated (Section 5.1).
- (ii) Uniform heat flux, of equal or differing magnitudes, at both surfaces (Section 5.2).
- (iii) Varying heat flux at one or both surfaces (Section 5.3).
- (iv) Constant surface temperature at one or both surfaces (Section 5.4).

The derivation of the data presented in Sections 5.1 and 5.2 is described in Appendix B (issued in 1982).



Sketch 5.1 Example of expected radial temperature profile of the fluid at any station x in the annulus for one wall heated



Sketch 5.2 Temperature distributions along the annulus for one wall heated

5.1 Uniform Heat Flux at One Surface Only

The two basic relationships, for uniform heat flux applied to the inner, or outer, tube wall only, are based on the correlation for plain tubes⁷ but modified by a factor f_{Di} to account for the annulus diameter ratio D_1/D_2 . Thus, for an annulus, either

$$\overline{Nu}_i = \overline{Nu}_s \cdot f_{Di} \quad (5.1a)$$

$$\text{or} \quad \overline{St}_i = \overline{St}_s \cdot f_{Di} \quad (5.1b)$$

With constant fluid properties both radially and axially, $\overline{Nu}_s$ or $\overline{St}_s$ for a straight circular tube of diameter D_e is given by Figure 1 which is taken from ESDU 92003⁷. Alternatively, $\overline{Nu}_s$ may be calculated from

Equations (4.1) and (4.2) of ESDU 92003⁷:

$$\overline{Nu}_s = \frac{(f/2)Re_s Pr}{1.07 + 12.7(f/2)^{1/2}(Pr^{2/3} - 1)}, \quad (5.2)$$

where

$$f = \frac{1}{4(1.82 \log_{10} Re - 1.64)^2}. \quad (5.3)$$

The Stanton number expression may be obtained by noting $St = Nu/(RePr)$.

With a uniform heat flux at one wall only ($i = 1$ or 2), adiabatic conditions at the opposite wall, and constant fluid properties radially and axially, the factor f_{Di} is given by Figures 2a or 2b for heat flux at the inner or outer wall respectively.

The factors f_{D1} and f_{D2} have been derived using experimental data from sources marked with an asterisk in Section 8 with an overall uncertainty of ± 15 per cent. In determining the variation of f_{Di} with D_1/D_2 it is assumed that as $D_1/D_2 \rightarrow 0$ (i.e. $D_1 \rightarrow 0$) the value of f_{Di} tends to unity with heat flux at the outer wall; with heat flux at the inner wall this limiting condition is not applicable. For $D_1 \approx D_2$ the configuration approaches that of parallel plates; the value of f_D at $D_1/D_2 = 1.0$ has thus been chosen to correspond with typical data for this situation.

5.1.1 Correction factor for the effect of large temperature differences between wall and bulk fluid

The effect of large temperature differences between wall and the bulk fluid is to produce a radial variation in fluid properties which can be characterised by T_i/T_b for gases and η_b/η_i for liquids. There are insufficient data for annuli from which to derive general correction factors. However, data³⁴ for air suggest that the correction factor, f_i , for annuli is approximately the same as that for plain circular tubes; the use of f_i derived for plain circular tubes tended to overpredict the Nusselt number in annuli with either tube heated by up to about 5 per cent for diameter ratios greater than 5. When both tubes are heated the effects of large temperature differences between each wall and the bulk fluid are considered separately; thus the correction factor f_i must be applied at each wall.

In the absence of specific data on annuli, it is recommended that the correction factor f_i derived for plain circular tubes in ESDU 92003⁷ be used in the present context.

For liquids the correction factor, f_i , can be expressed as

$$f_i = \frac{\overline{Nu}_f}{\overline{Nu}_{f,c}} = \left(\frac{\eta_b}{\eta_i} \right)^n \quad (5.4)$$

where $n = 0.11$ for heating ($T_i/T_b > 1$) and $n = 0.25$ for cooling ($0.1 < T_i/T_b < 1$).

For gases,

$$f_i = \left(\frac{\overline{Nu}_f}{\overline{Nu}_{f,c}} \right) = \left(\frac{T_i}{T_b} \right)^n \quad (5.5)$$

where values of n are given by Table 5.1, taken from ESDU 92003⁷.

TABLE 5.1 Correction Factor for Physical Property Variation in Gases

Fluid	Heating $T_i/T_b > 1$				Cooling $0.1 < T_i/T_b < 1$
	n	Range of T_b K	Range of T_i/T_b	Range of Pressure (bar)	n
Air	-0.40	300 – 1000	1.0 – 2.4	1.0 – 8.0	0
Carbon dioxide	-0.12	300 – 400	1.0 – 2.1	1.0 – 8.0	–
Helium	-0.38	300 – 1200	1.0 – 4.2	1.0 – 8.0	0
Hydrogen	-0.37	300 – 1000	1.0 – 5.0	15 – 500	–
Nitrogen	-0.49	120 – 730	1.0 – 6.7	2.9 – 5.9	0
Steam	+0.18	560 – 750	1.0 – 1.5	21 – 100	–

For all other gases $n = 0$.

5.1.2 Procedure for dealing with the effect of large axial temperature differences

The effect of large axial temperature differences in the wall and bulk fluid is to produce axial variations in the fluid properties. If over the length (a) of the annulus considered $|\bar{\alpha}_{x+a} - \bar{\alpha}_x|/\bar{\alpha}_x > 0.1$ the annulus must be considered in sufficiently small lengths that for each individual length $|\bar{\alpha}_{x+a} - \bar{\alpha}_x|/\bar{\alpha}_x < 0.1$.

5.2 Uniform Heat Flux at Both Surfaces

When the inner tube is heated (or cooled) at a constant heat flux $\dot{q}_1$, and the outer tube wall simultaneously heated (or cooled) at a constant heat flux $\dot{q}_2$, the mean values of Nu_i or St_i at each wall are given by

$$\frac{\bar{Nu}_1}{\bar{Nu}_{11}} = \frac{\bar{St}_1}{\bar{St}_{11}} = \frac{1}{1 - \frac{\dot{q}_2}{\dot{q}_1} \theta_{12}^*} \cdot f_i \text{ (for the inner tube),} \quad (5.6)$$

$$\frac{\bar{Nu}_2}{\bar{Nu}_{22}} = \frac{\bar{St}_2}{\bar{St}_{22}} = \frac{1}{1 - \frac{\dot{q}_1}{\dot{q}_2} \theta_{21}^*} \cdot f_i \text{ (for the outer tube).} \quad (5.7)$$

$\bar{Nu}_{11}$ or $\bar{St}_{11}$ are the reference values (see Section 5.1) for the inner tube wall when that wall alone is heated (or cooled) at a constant heat flux $\dot{q}_1$ and the outer tube wall is well-insulated (adiabatic). Similarly, $\bar{Nu}_{22}$ or $\bar{St}_{22}$ are the reference values for the outer tube wall when that wall alone is heated (or cooled) at a constant heat flux $\dot{q}_2$ with the inner tube wall well-insulated (adiabatic).

To account for the modification to the magnitude of the Nusselt number at one wall, i , due to the other wall, j , being heated or cooled simultaneously, influence coefficients θ_{ij}^* are introduced which are defined in Section A2 of the Appendix. Values of these influence coefficients are given in the Tables in Section 9¹⁶.