

ACCELERATION FACTORS FOR CLIMB AND DESCENT RATES AT CONSTANT EAS, CAS, M

1. NOTATION AND UNITS*

		SI	British
a	speed of sound*, $(\gamma RT)^{1/2}$, (340.294 m/s, 1116.45 ft/s)	m/s	ft/s
g	acceleration due to gravity	m/s ²	ft/s ²
H	geopotential height	m	ft
H_p	pressure height	m	ft
M	Mach number, $V/a (= V/(a_0 \theta^{1/2}) = V_e/(a_0 \delta^{1/2}))$		
p	static pressure	N/m ²	lbf/ft ²
R	gas constant for air (287.052 87 N m/kg K, 3089.811 4 lbf ft/slug K)	N m/kg K	lbf ft/slug K
T	ambient temperature	K	K
t	time	s	s
V	true airspeed*	m/s	ft/s
V_c	calibrated airspeed* (CAS), see Item No. 69026 (Reference 1)	m/s	ft/s
V_e	equivalent airspeed* (EAS), $V\sigma^{1/2}$	m/s	ft/s
γ	ratio of specific heat capacities of air, taken as 1.4		
δ	relative pressure, p/p_0		
θ	relative temperature, T/T_0		
ρ	density of air	kg/m ³	slug/ft ³
σ	relative density, ρ/ρ_0		
<i>Subscripts</i>			
std	denotes conditions in International Standard Atmosphere, ISA, see Item No. 77022 (Reference 3)		
0	denotes conditions at sea level in ISA		

* The units quoted are coherent sets and this avoids the necessity for introducing numerical constants into otherwise-algebraic equations in the Item. If required, conversion factors to other units can be found in the conversion factors Item in Volume 1 of the Performance Series. However, for purposes of presentation in Table 7.1 and Figures 2a and 2b, limited use is made of the operational unit, the knot (kn).

2. INTRODUCTION

This Item gives algebraic, numerical and graphical forms of “acceleration factors” which provide a simple, direct method of taking into account the acceleration of an aircraft along the flight path in the calculation of instantaneous (or *point*) rates of climb and descent. The particular cases considered are climb or descent at constant equivalent airspeed (EAS), constant calibrated airspeed (CAS) and constant Mach number. Much of the text is written in terms of climbing flight but the expressions and graphical data for acceleration factors are valid for climb and descent.

2.1 Definition of Acceleration Factor

In general climbing and descending flight are accompanied by an acceleration of the aircraft along the flight path. For example, the simplest flight schedules, using pressure-sensing instrumentation, involve flight at constant indicated airspeed or Mach number and, since air density varies with height, the aircraft's true air speed also varies (except in the case of constant Mach number in isothermal conditions).

For still air* or for conditions where wind speed does not vary with height the rate of climb, dH/dt , is calculated from the along-flight-path equation of motion which, for present purposes, can be expressed in the following general forms,

$$\frac{dH}{dt} = \frac{(\text{true airspeed}) \times (\text{component of thrust-minus-drag along airspeed vector})}{\text{aircraft weight}} \times \frac{1}{\left[1 + \frac{V}{g_0} \frac{dV}{dH}\right]} \quad (2.1)$$

$$\text{or} \quad = \frac{\text{specific excess power}}{\left[1 + \frac{V}{g_0} \frac{dV}{dH}\right]} \quad (2.2)$$

From Equations (2.1) and (2.2) it can be seen that dH/dt is inversely proportional to the quantity enclosed in square brackets – *i.e.* the “acceleration factor”. A derivation of this quantity from first principles is given in Section 3.2 of Item No. 78038 (Reference 4) and examples of its use may be found in Section 3.1 of Item No. 70023 (Reference 2) and in Item No. 92019 (Reference 7). The acceleration factor may also be expressed in the forms,

$$\left[1 + \frac{V}{g_0} \frac{dV}{dH}\right] = \left[1 + \frac{1}{2g_0} \frac{d(V^2)}{dH}\right], \quad (2.3)$$

$$= \left[1 + \frac{V_e^2}{2g_0 \sigma} \left(\frac{2}{V_e} \frac{dV_e}{dH} - \frac{1}{\sigma} \frac{d\sigma}{dH}\right)\right], \quad (2.4)$$

$$= \left[1 + \frac{\gamma R T M^2}{2g_0} \left(\frac{2}{M} \frac{dM}{dH} + \frac{1}{T} \frac{dT}{dH}\right)\right]. \quad (2.5)$$

* The effects of wind and wind gradient on rate of climb are considered in Section 5.4 of Item No. 70023 (Reference 2). For a more general formulation of the equations of motion for cases where atmospheric motion must be considered, see Item Nos 78038 and 79018 (References 4 and 5).

2.2 Cases Considered in this Item

For climb at constant true airspeed, $(dV/dH) = 0$ so that the acceleration factor plays no part in the calculation of dH/dt . In other cases its magnitude depends on the manner in which the aircraft climbs or descends and Sections 3, 4 and 5 give the acceleration factors for three methods of climb or descent as functions of instantaneous speeds and ambient conditions. Section 3 considers flight at constant equivalent airspeed, V_e , which, while not a normal operational procedure, is useful for simple project or engineering calculations and in some circumstances provides a satisfactory approximation to constant calibrated airspeed. Sections 4 and 5 consider the cases of constant calibrated airspeed, V_c , and constant Mach number, M , respectively. Both of these are simple operational procedures that are widely used – many flight profiles having constant V_c and constant M segments; see, for example, Figures A.4 and B.4 of Item No. 90012 (Reference 6).

3. CLIMB OR DESCENT AT CONSTANT EQUIVALENT AIRSPEED, EAS

If the climb or descent is assumed to be carried out at constant equivalent airspeed, $dV_e/dH = 0$ in Equation (2.4) and the acceleration factor is given by,

$$\left[1 + \frac{V}{g_0} \frac{dV}{dH}\right] = \left[1 - \frac{V_e^2}{2g_0\sigma^2} \frac{d\sigma}{dH}\right]. \quad (3.1)$$

From Equation (3.1) and using the additional relationships of the perfect gas law

$$p = \rho RT, \quad (3.2)$$

the hydrostatic equation

$$\frac{dp}{dH} = -\rho g_0 \quad (3.3)$$

and, for the speed of sound, the definitions

$$a^2 = \gamma RT = \frac{\gamma p}{\rho} = a_0^2 \theta \quad (3.4)$$

the acceleration factor for climb at constant equivalent airspeed can be shown to be,

$$\left[1 + \frac{V}{g_0} \frac{dV}{dH}\right] = \left[1 + \frac{\gamma}{2a_0^2} \frac{V_e^2}{\delta} \left(1 + \frac{R}{g_0} \frac{dT}{dH}\right)\right] \quad (3.5)$$

$$\text{or} \quad = \left[1 + \frac{\gamma M^2}{2} \left(1 + \frac{R}{g_0} \frac{dT}{dH}\right)\right]. \quad (3.6)$$

In Equations (3.5) and (3.6) the temperature gradient is expressed in terms of geopotential height but alternative forms may be derived in which the temperature gradient is expressed in terms of pressure height (see Section 7 of Item No. 77022 (Reference 3)) using

$$\frac{dT}{dH} = \frac{dT}{dH_p} \frac{T_{std}}{T} \quad (3.7)$$

to give, for example, from Equation (3.5),

$$\left[1 + \frac{V}{g_0} \frac{dV}{dH} \right] = \left[1 + \frac{\gamma}{2a_0^2} \frac{V_e^2}{\delta} \left(1 + \frac{R}{g_0} \frac{dT}{dH_p} \frac{T_{std}}{T} \right) \right]. \quad (3.8)$$

Table 7.1 gives numerical expressions (Equations (7.1) to (7.12)) for the acceleration factor for climb or descent at constant V_e using the values for dT/dH_p for the International Standard Atmosphere and associated off-standard atmospheres from Table 11.2 of Item No. 77022 (Reference 3).

The relationships between the acceleration factors for climbs at constant V_e and at constant V_c (see Section 4) in ISA and “off-standard” conditions are shown in Figure 1. From Figure 1 it can be seen that for low-speed applications, such as take-off climbs, there is no significant loss of accuracy if climb rates are calculated assuming constant V_e rather than constant V_c .

4. CLIMB OR DESCENT AT CONSTANT CALIBRATED AIRSPEED, CAS

Calibrated airspeed, V_c , is obtained when the reading of an airspeed indicator has been corrected for instrument, lag and pressure errors. Consequently, flight at or near constant V_c , is one of the simplest methods of scheduling climbs or descents if pressure-sensing instruments are to be used. The relationships between V_c and the pressures recorded by a pitot-static system are given in Item No. 69026 (Reference 1). For the purposes of this Item it is convenient to work in terms of the relationships between V_c and M as follows,*

$$(i) \quad \text{for } \frac{V_c}{a_0} \leq 1.0 \text{ and } M \leq 1.0$$

$$\left\{ \left[1 + \frac{(\gamma - 1)}{2} \frac{V_c^2}{a_0^2} \right]^{\frac{\gamma}{(\gamma - 1)}} - 1 \right\} = \delta \left\{ \left[1 + \frac{(\gamma - 1)}{2} M^2 \right]^{\frac{\gamma}{(\gamma - 1)}} - 1 \right\}, \quad (4.1)$$

$$(ii) \quad \text{for } \frac{V_c}{a_0} \leq 1.0 \text{ and } M > 1.0$$

$$\left\{ \left[1 + \frac{(\gamma - 1)}{2} \frac{V_c^2}{a_0^2} \right]^{\frac{\gamma}{(\gamma - 1)}} - 1 \right\} = \delta \left\{ \left[\frac{(\gamma + 1)}{2} M^2 \right]^{\frac{\gamma}{(\gamma - 1)}} \left[1 + \frac{2\gamma}{(\gamma + 1)} (M^2 - 1) \right]^{\frac{1}{(1 - \gamma)}} - 1 \right\}, \quad (4.2)$$

* A graphical presentation of Equations (4.1) to (4.3) is given in Figure 2 of Item No. 69026 as curves of H_p for fixed values of V_c and M .

(iii) for $\frac{V_c}{a_0} > 1.0$ and $M > 1.0$

$$\left\{ \left[\frac{(\gamma+1)}{2} \frac{V_c^2}{a_0^2} \right]^{\frac{\gamma}{(\gamma-1)}} \left[1 + \frac{2\gamma}{(\gamma+1)} \left(\frac{V_c^2}{a_0^2} - 1 \right) \right]^{\frac{1}{(1-\gamma)}} - 1 \right\} \\ = \delta \left\{ \left[\frac{(\gamma+1)}{2} M^2 \right]^{\frac{\gamma}{(\gamma-1)}} \left[1 + \frac{2\gamma}{(\gamma+1)} (M^2 - 1) \right]^{\frac{1}{(1-\gamma)}} - 1 \right\}. \quad (4.3)$$

Expressions for the acceleration factor for climb at constant V_c are derived by differentiating the expressions in Equations (4.1) to (4.3) with respect to H , setting $dV_c/dH = 0$ and re-arranging the results using Equations (3.2) to (3.4) and (3.7) to give

for $M \leq 1.0$,

$$\left[1 + \frac{V}{g_0} \frac{dV}{dH} \right] = \left[1 + \frac{\left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{\gamma}{\gamma-1}} - 1}{\left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{1}{\gamma-1}}} + \frac{\gamma R M^2}{2 g_0} \frac{dT}{dH_p} \frac{T_{std}}{T} \right], \quad (4.4)$$

and for $M > 1$,

$$\left[1 + \frac{V}{g_0} \frac{dV}{dH} \right] = \left[1 + \frac{\frac{(\gamma-1)}{2} \left\{ 1 - \left(\frac{\gamma+1}{2} M^2 \right)^{\frac{\gamma}{1-\gamma}} \left[1 + \frac{2\gamma}{\gamma+1} (M^2 - 1) \right]^{\frac{1}{\gamma-1}} \right\}}{\frac{1}{M^2} - \frac{2}{(2\gamma M^2 - \gamma + 1)}} + \frac{\gamma R M^2}{2 g_0} \frac{dT}{dH_p} \frac{T_{std}}{T} \right]. \quad (4.5)$$

Table 7.2 gives numerical expressions (Equations (7.13) to (7.18) for the acceleration factor for climb or descent at constant V_c using values of dT/dH_p for the International Standard Atmosphere and associated off-standard atmospheres from Table 11.2 of Item No. 77022 (Reference 3).

For simplicity, Equations (4.4), (4.5) and (7.13) to (7.18) are presented in terms of M , so that to calculate values of the acceleration factor in terms of V_c also requires the use of Equations (4.1) to (4.3). Particular care is then required to identify which of the three possible relationships between V_c and M applies. The following Diagram gives the sequence of “tests” required to identify which equations should be used for given values of V_c and H_p .