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Selection of DRD and DFD cam laws

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THE PREPARATION OF THIS DATA ITEM

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The Mechanisms Committee, which first met in 1975, is continuing the work that previously was the responsibility of the former Gearing and Mechanisms Committee, which first met in November 1964. Like the earlier Committee it has the cooperation of many engineers and scientists in industry, research establishments and universities from whom much assistance and information is being received.

SELECTION OF DRD AND DFD CAM LAWS

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SELECTION OF DRD AND DFD CAM LAWS

1. NOTATION AND UNITS

The units used in this Item are predominantly SI but any coherent system of units may be used. All angles are measured positively in an anticlockwise sense and the cam is considered to rotate in an anticlockwise direction.

		Units
A, a, b, c	coefficients used in Appendix A (see Section 5.2.2 and Equations (A1.1) and (A1.2))	–
E	offset of follower line-of-action from cam centre, O, measured in positive sense to right of cam centre in first quadrant	m
F_E	external force acting on moving follower (positive to increase contact force)	N
$(F_s)_{lim}$	limiting spring force at outer dwell for no-load to ensure force closure	N
f_n	natural frequency of follower assembly	Hz
J	$= \pi/(\pi + 4)$ (see Section 5.1.3)	–
K	$= 2/(\pi + 2)$ (see Section 5.1.2)	–
k_A	acceleration ratio (see Equation (7.18))	–
k_B	blending ratio (see Equation (7.19b))	–
k_f	stiffness of follower assembly	N/m
k_{SC}	compression spring stiffness	N/m
g	acceleration due to gravity	m/s ²
H	total lift of translating follower corresponding to one motion segment	m
m_f	mass of follower assembly	kg
N	cam speed	rev/min
$N1$	component of blended motion blending with inner dwell	–
$N2$	component of blended motion blending with outer dwell	–
n	period ratio (see Equation (4.1))	–

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O	cam centre of rotation and origin of co-ordinate axis system	–
P_{const}	constant load	N
P_{fric}	load due to friction	N
P_{SP}	compression spring pre-load	N
q_{const}, q_I, q_s	torque to drive cam against constant load, inertia force and spring stiffness, respectively	N m
R_a	prime circle radius	m
t_{const}, t_I, t_s	non-dimensional torque factor for constant load, inertia force and spring stiffness, respectively	–
u	normalised cam angle (see Equations (3.2a) and (3.2b))	–
w, w', w'', w'''	normalised follower displacement, velocity, acceleration and jerk, respectively (see Equations (3.3a) to (3.6a) and (3.3b) to (3.6b))	–
X_c, Y_c	axes fixed in, and rotating with, cam with origin at cam centre O	–
X_m, Y_m	axes fixed in machine, with origin at cam centre O	–
y	linear displacement of translating follower	m
y_b	height of translating follower datum above X_m -axis at innermost position of travel	m
y_0	height of translating follower datum above X_m -axis at start of motion segment	m
$\dot{y}$	linear velocity of translating follower	m/s
$\ddot{y}$	linear acceleration of translating follower	m/s ²
$\dddot{y}$	linear jerk of translating follower	m/s ³
Z_A	intermediate calculation parameter (see Equation (A1.4) of ESDU 06002 ^[18] and Section 7.3.2)	–
z_1	force factor (see Equation (7.7))	–
z_2	pre-load factor (see Equation (7.9))	–
ρ	radius of curvature of pitch curve	m
Φ	total angle of cam rotation corresponding to one motion segment	degree
ϕ	intermediate angle of cam rotation from start of motion segment	degree

ϕ_0	angle of cam rotation at start of motion segment	degree
Ψ	total angle of follower swing associated with one motion segment	degree
ψ	angular displacement of swinging follower	rad
ψ_b	angle between X_m -axis and swinging follower datum at innermost position	degree
ψ_m	angle between X_m -axis and swinging follower datum	degree
ψ_0	angle between X_m -axis and swinging follower datum at start of motion segment	degree
$\dot{\psi}$	angular velocity of swinging follower	rad/s
$\ddot{\psi}$	angular acceleration of swinging follower	rad/s ²
$\dddot{\psi}$	angular jerk of swinging follower	rad/s ³
ω_c	angular velocity of cam	rad/s

Subscripts

B	refers to value at Blending Point
$dwel$	refers to dwell of follower
$fall$	refers to fall motion of follower
$Idwell, Odwell$	refer to inner and outer dwell, respectively
lim	limiting value
min	minimum value
$N1$	refers to notional dimension of Component $N1$ blending with inner dwell
$N2$	refers to notional dimension of Component $N2$ blending with outer dwell
q	refers to value at position of maximum separating force
$rise$	refers to rise motion of follower
V	refers to location of maximum follower velocity for single law motions
$V1$	refers to location of maximum follower velocity (Component $N1$)
$V2$	refers to location of maximum follower velocity (Component $N2$)
$(\dots)_1, (\dots)_2, \dots$	relate to Segments 1, 2, ...



The definition of follower motion synchronised with the operation of the machine determines many of the input values for continuing the design of the mechanism. ESDU 03010^[17] explains the application of timing diagrams to define the operation of a proposed design to determine the cam angle for each motion segment of a mechanism. The associated Item, ESDU 01002^[16], provides guidance for progressing the design, *e.g.* for determining the least size of the cam disc compatible with the kinematic constraints imposed by minimum pressure angle and profile curvature.

In many mechanisms force closure to maintain contact between the cam profile and the follower relies upon spring force acting upon the follower. When a rise or fall action consists of DRD or DFD motions separated by an intermediate dwell, the peak acceleration of the motion nearer the cam centre may be limited by the partial deflection of this spring (see Example 3).

2.1 Cam Law Selection

This Item deals specifically with the selection of a cam law to suit a complete DRD or DFD motion segment. The equations of cam laws to generate complete DRD and DFD motions are given in the Information Sheets (see Section 5). The cam laws selected cover a wide range of applications and performance. For reasons of clarity, this list is only a representative sample of all possible cam laws, noting that some alternative cam laws have similar performance to those in the Information Sheets. In these equations the angle of cam rotation and the resulting follower displacement are expressed in normalised form. Successive derivatives of the displacement equation yield values for the non-dimensional parameters defining follower velocity, acceleration and jerk.

Tables B1.1 and B1.2 of Appendix B show the critical values of these parameters and enable a direct comparison between alternative cam laws to be made.

Cam laws should be chosen to ensure that the acceleration is a smooth, continuous function, zero at every blend with a dwell. Hence cam laws having finite jerk[†] (continuous or discontinuous) throughout DRD and DFD motions are recommended. Typical follower motions, normalised displacement and non-dimensional velocity, acceleration and jerk, corresponding to two symmetrical single-law motion segments are compared in Sketch 2.3. The sketch provides a comparison between the Modified Sinusoidal cam law, shown for DRD action, and the Triple Harmonic Zero Jerk at Zero Acceleration cam law, shown for DFD action. Note that the sinusoidal law is continuous in displacement, velocity and acceleration, but has a finite discontinuity in jerk at the ends of the motion, whereas the trapezoidal law is continuous up to and including jerk. Notice also the relative magnitudes of the peak acceleration.

The cam laws to generate complete DRD and DFD **finite-jerk motions** given in Section 5.1 of the Information Sheets are:

- Cycloidal (Sine Curve Acceleration),
- Modified Trapezoidal Acceleration,
- Modified Sinusoidal Acceleration,
- Triple Harmonic Modified Trapezoidal Acceleration,
- Triple Harmonic Zero Jerk at Zero Acceleration,
- 3-4-5 Polynomial
- and 4-5-6-7 Polynomial

These motions are also suitable for Component N1 of blended DRFD motions driving the follower from

[†] Jerk is the third derivative of the displacement equation with respect to time.

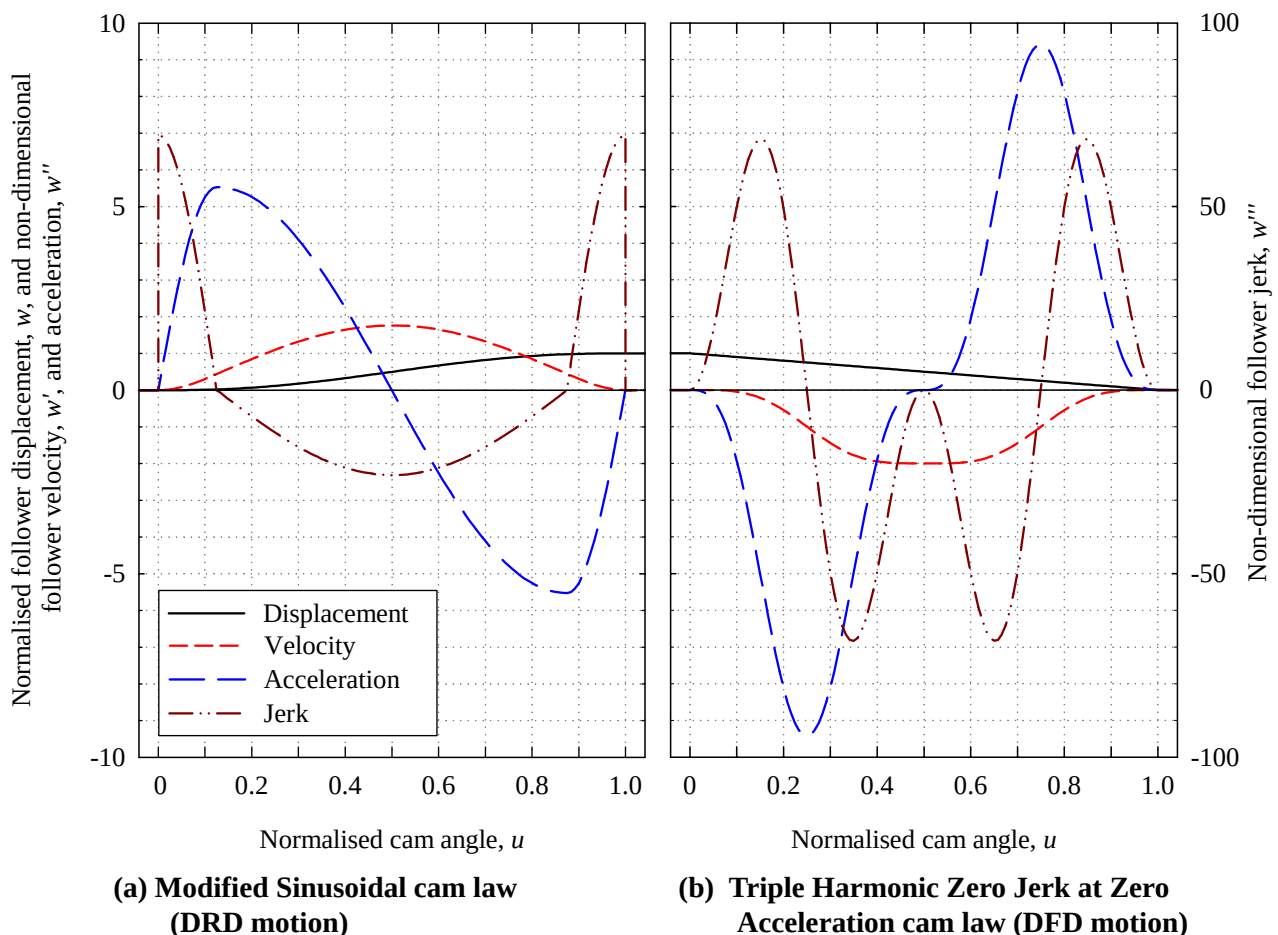
the dwell position to the transition point, where it attains maximum velocity (see ESDU 06002^[18], 06003^[19] and examples in 06004^[20]).

The cam laws to generate complete DRD and DFD **infinite-jerk motions** given in Section 5.2 of the Information Sheets are:

- Simple Harmonic Motion (SHM),
- Constant-Cosine-Constant Acceleration (CCCA),
- 2-3 Polynomial
- and Parabolic[†] (Constant Acceleration)

SHM, CCCA and the 2-3 Polynomial cam laws have infinite jerk at the start and end of DRD and DFD motions and for this reason they are only suitable for very slow-speed DRD motions. However, in the case of blended DRFD motions, SHM in particular is eminently suitable for the intermediate Component N2 which controls the reversal of follower velocity at maximum displacement (see ESDU 06002^[18], 06003^[19] and examples in 06004^[20]).

The follower displacement, velocity and acceleration are determined by transposing the lift, found from the design specification or the layout drawing, and the segment angle given by the timing diagram (see ESDU 03010^[17]) into the equations for the cam law chosen for the mechanism.

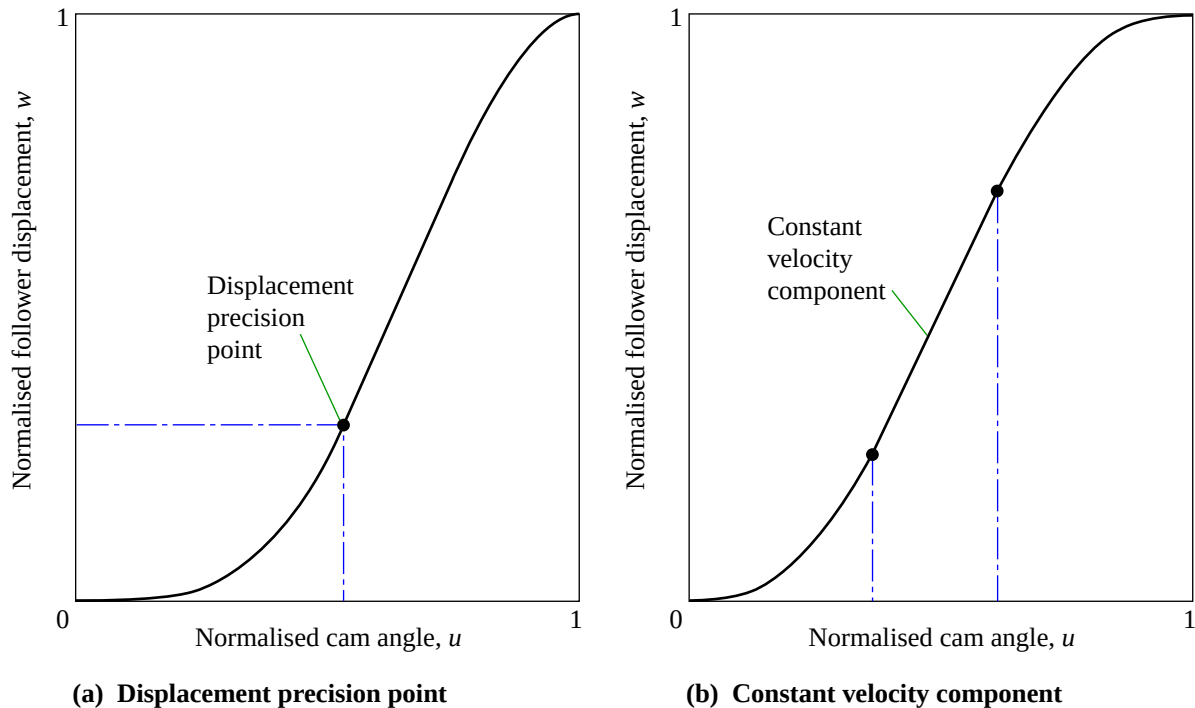


Sketch 2.3 Comparison of two single-law motion segments

[†] Parabolic Motion has infinite jerk at the start, transition point and end of DRD and DFD motions. It is included for information only and is NOT recommended for any application.

2.2 Reasons for Blending

One cam law may not generate the required follower motion for the complete rise or fall segment of a DRFD action. Examples include a displacement curve required to incorporate a specified precision point, Sketch 2.4(a), or a constant velocity component, Sketch 2.4(b). In such cases it can be advantageous to investigate the possibility of forming the segment from blended components of the same or different cam laws.



Sketch 2.4 Special motion requirement

Further reasons for profile blending and potential benefits are summarized as follows.

- It becomes possible to vary the relative magnitudes of the peak follower acceleration and retardation corresponding to a given segment angle to reduce the peak value acting towards the cam and so reduce the spring force required to maintain contact (see Examples 3, 5 and 6 in ESDU 06004^[20]). The consequences of such changes are explained in ESDU 92014^[9] and 93002^[11].
- It becomes possible to vary the follower displacement corresponding to a specific cam angle, for example, to ensure that a specific displacement occurs at the required cam angle (see ESDU 92014^[9]).
- It becomes possible to incorporate an intermediate constant velocity component between the components controlling the follower acceleration and retardation, either to suit a design requirement or to reduce the peak velocity (but increase the acceleration). (See ESDU 93002^[11] and also Examples 2 and 6 in ESDU 06004^[20].)
- The segment angle required for the follower to reach a given lift (for a cam with specified values of reference circle radius and maximum allowable pressure angle) can be minimised by blending (see ESDU 92014^[9]).

- (e) The minimum allowable reference circle radius required for a cam with specified segment angle, follower lift and maximum allowable pressure angle can be reduced by blending (see ESDU 92014^[9] and Example 2 in that Item).
- (f) The maximum allowable follower lift that can be obtained with a specified segment angle, reference circle radius and maximum allowable pressure angle can be increased by blending (see ESDU 92014^[9]).
- (g) Blending provides a means of obtaining a more rapid follower displacement from dwell than possible with a single-law motion (see Example 3 in ESDU 06004^[20]).

In some cases the decision to use a blended profile is determined by the suitability of a particular cam law for that component of the motion. In the absence of further constraints, to minimise vibration the (unique) "notional" dimensions of segment angle and lift for each component should be found by equating the follower velocity and jerk at the blending point. As an alternative, interposing a constant velocity component between components controlling the acceleration and retardation can be beneficial.

Constant velocity sections are sometimes employed in cam profiles for use with translating flat-faced followers because in this arrangement the maximum travel of the contact point on the follower is directly proportional to the peak velocity. Thus the available follower face diameter places a restriction on the peak velocity and so, if it is important to maximise the area under the lift curve, a constant velocity section is useful.

The analyses of blended motion segments are given in ESDU 06002^[18].

2.3 Use of this Data Item

The theoretical background to alternative forms of the mechanism and follower motion, including definitions and terminology, is explained in Section 3. It is strongly recommended that the calculations be performed using the ESDU computer program (see OSCAM in ESDU 00014^[15] and ESDU 06005^[21]), exporting the files defining the motion to other programs to complete the design of the mechanism. ESDU 06005^[21] provides information regarding the use of the computer program including the composition of program input and interpretation of program output.

The boundary conditions defining the general case of complete finite-jerk DRD and DFD motions are given in Table 3.1.

The choice of a cam law is influenced by the interaction between the peak values of follower velocity and acceleration with the segment angle and lift. Hand calculations are convenient for this purpose. A concise form of the equations is obtained by expressing parameters where possible as products of π as given in Appendix D of ESDU 06002^[18]. See also Example 3.

It is emphasized that completion of a dynamically suitable follower motion is no guarantee that the mechanism is a good design. The procedure given in ESDU 01002^[16] should be followed to ensure suitable proportions and adequate strength.

Further analysis of the anticipated kinematic and dynamic performance to assess the short-listed cam laws is readily accomplished with the aid of ESDU computer program OSCAM^[15]. The criteria specific to sliding followers, namely lubrication of the cam/profile/follower interface and the contact stress in that region, can be analysed with the ESDU computer programs W9408^[12] and W9411^[13], respectively.

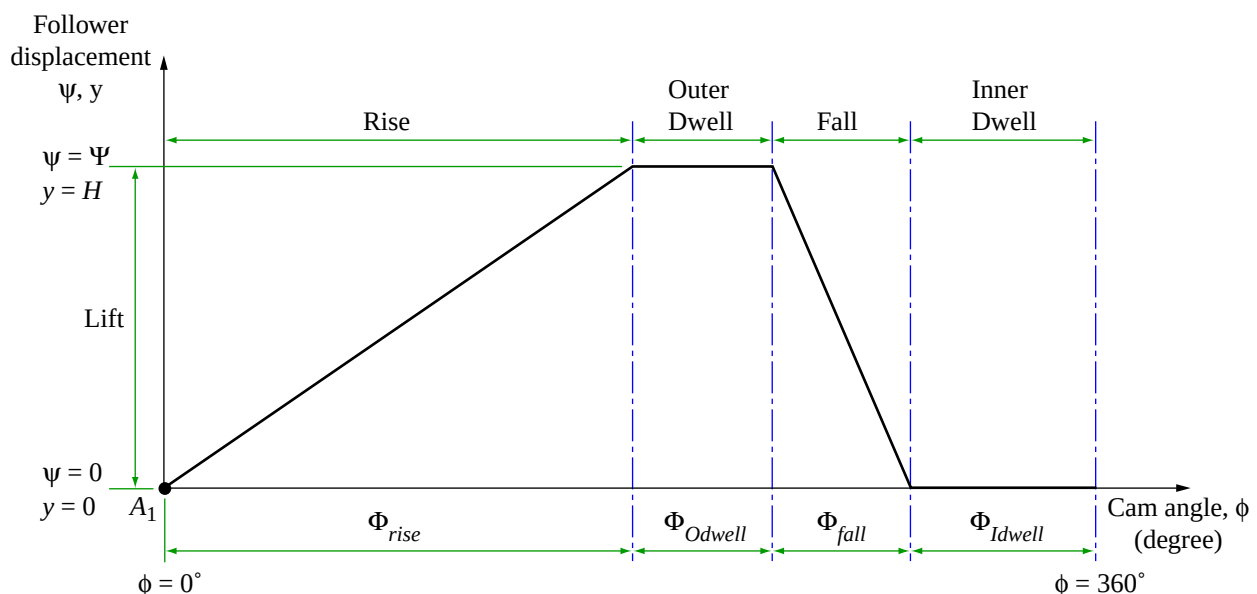
3. THEORETICAL BACKGROUND

3.1 Definitions and Terminology

3.1.1 Motion segments and single-law motion

For the purpose of this Data Item, a motion segment is defined as that part of a cam motion cycle during which the follower moves from one position of zero velocity to the next position of zero velocity. A motion segment may be described as Dwell-Rise-Dwell (DRD) in which the follower moves away from the centre of cam rotation or Dwell-Fall-Dwell (DFD)[†] in which the follower moves towards the centre of cam rotation: the complete sequence is designated DRDF. The most common way to specify follower motion, and hence determine the cam profile, is to use one of a recommended series of equations known as cam laws, which relate the cam angle of rotation to the follower motion. To define a single-law motion segment it is only necessary to specify the cam law, the follower lift and the segment angle.

The straight-line displacement diagram shown in Sketch 3.1 (see ESDU 03010^[17]) represents the simplest form of a complete DRDF action. The sequence begins by the follower rising through the lift, H , from the inner dwell position at the smallest profile radius to reach the outer dwell at maximum profile radius after the cam has rotated through the segment angle, Φ_{rise} . The follower remains stationary during the outer dwell period, corresponding to a cam rotation of Φ_{Odwell} . Then it falls through the same lift, H , returning to dwell at the starting position as the cam rotates through the segment angle, Φ_{fall} . The rise and fall actions can be radically different. With the dwells, they form distinct motion segments each of which is analysed and specified separately.



Sketch 3.1 Diagrammatic displacement of translating follower during one complete cam rotation

The zero datum of the cam angle, ϕ , is located at the start of the rise action, Point A_1 . The displacement of a translating follower, y , or of a swinging follower, ψ , is measured from the inner dwell position, Point A_1 .

[†] "Fall" is designated "Return" in previous editions of this Data Item and some other Items in this Series.

3.1.2 Dimensionless form of displacement, velocity, acceleration and jerk

The cam and follower mechanisms of Sketches 2.1 and 2.2 show two pairs of co-ordinate axes having origins co-incident with the centre of cam rotation. The X_m, Y_m axes are fixed to the machine frame whilst the X_c, Y_c axes are fixed to the cam and rotate with it.

A cam law is the mathematical function defining the normalised follower displacement, w , during a single motion segment in terms of the normalised cam rotation, u . It is conveniently expressed as

$$w = f\{u\} . \quad (3.1)$$

This function is evaluated readily by substituting this form of the equation for the relevant cam law. In this series of Data Items the cam laws are identified by their generic names, *e.g.* "Modified Sine Acceleration" or "4-5-6-7 Polynomial". The equations of motion apply to both DRD and DFD actions. Those for commonly-used DRD cam laws are given as formulae and graphically in Section 5 and for DRFD cam laws in ESDU 06002^[18].

The Notation is given in Section 1. The sign convention and equivalent equations for the various configurations of translating and swinging followers are given in Section 3.2 of ESDU 95001^[14].

Translating followers

For a disc cam driving a translating follower, as shown in Sketch 2.1, the normalised cam angle and normalised follower displacement at an intermediate position during a single-law DRD rise motion segment are given by

$$u_{rise} = \frac{\phi - \phi_0}{\Phi} \quad (3.2a)$$

$$\text{and} \quad w_{rise} = \frac{y - y_0}{H} . \quad (3.3a)$$

Making the usual assumption that the cam rotates at constant angular velocity, the dimensionless velocity, acceleration and jerk parameters for a translating follower are defined respectively as

$$w' = \frac{\dot{y}}{\omega_c H} \left(\frac{\pi \Phi}{180} \right) , \quad (3.4a)$$

$$w'' = \frac{\ddot{y}}{\omega_c^2 H} \left(\frac{\pi \Phi}{180} \right)^2 \quad (3.5a)$$

$$\text{and} \quad w''' = \frac{\dddot{y}}{\omega_c^3 H} \left(\frac{\pi \Phi}{180} \right)^3 . \quad (3.6a)$$

Swinging followers

For a disc cam driving a swinging follower as shown in Sketch 2.2, the normalised cam angle and normalised

follower displacement at an intermediate position during a single-law DRD rise motion segment are given by

$$u_{rise} = \frac{\phi - \phi_0}{\Phi} \quad (3.2b)$$

and

$$w_{rise} = \frac{\psi - \psi_0}{\Psi} \quad (3.3b)$$

Making the usual assumption that the cam rotates at constant angular velocity, the dimensionless velocity, acceleration and jerk parameters for a swinging follower are defined respectively as

$$w' = \frac{\dot{\psi}}{\omega_c \Psi} \left(\frac{\pi \Phi}{180} \right), \quad (3.4b)$$

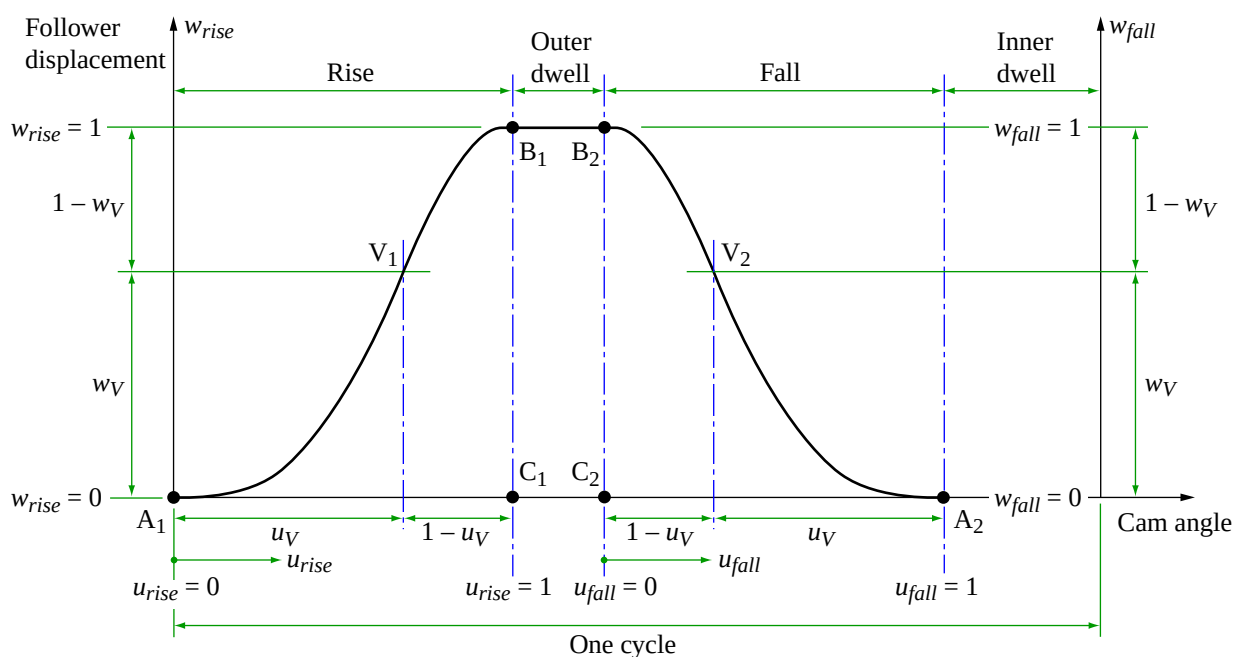
$$w'' = \frac{\ddot{\psi}}{\omega_c^2 \Psi} \left(\frac{\pi \Phi}{180} \right)^2 \quad (3.5b)$$

and

$$w''' = \frac{\dddot{\psi}}{\omega_c^3 \Psi} \left(\frac{\pi \Phi}{180} \right)^3 \quad (3.6b)$$

3.1.3 Equations for motion segments of DRD and DFD profiles

The curve shown in Sketch 3.2 illustrates the general case of the normalised output displacement with DRD and DFD motions separated by dwells at both the inner and outer limits of follower travel. The Sketch shows the general case in which the transition points (the position of maximum follower velocity), Points V_1 and V_2 , do not occur at the mid-point of either the segment angle or the lift.



Sketch 3.2 Normalised follower displacement during one complete cycle (same single cam law for DRD and DFD actions)

Sketch 3.2 shows the follower displacement during a complete DRDF action. In this case, the rise and fall periods are equal and the same cam law is used for both rise and fall. The cam angle is measured from the start of the rise, Point A₁, where the normalised cam angle of the rise motion segment, u_{rise} , the normalised follower displacement, w_{rise} , the non-dimensional follower velocity, w'_{rise} , and the non-dimensional follower acceleration, w''_{rise} , are all zero[†].

The follower has positive velocity during the rise. The direction of the acceleration, w''_{rise} , reverses from positive to negative at the transition point, V₁, where, in the general case, $u_{rise} = u_V$ and $w_{rise} = w_V$. The rise motion segment is completed when the follower attains the lift, H , at Point B₁, corresponding to a cam rotation through the angle of lift, Φ . At this point the values of both u_{rise} and w_{rise} are unity, whilst w'_{rise} and w''_{rise} are zero[†].

Rise action

The follower displacement, velocity, acceleration and jerk for the rise of a finite jerk single-law DRD motion are given by Equations (3.2a) to (3.6a) and (3.2b) to (3.6b), respectively. The boundary conditions are defined in Table 3.1.

TABLE 3.1 Boundary conditions for general case of single-law finite jerk DRD and DFD cam motions

	u	w	w'	$w''^{\dagger}$	Notes
Rise	0	0	0	0	Profile blends with inner dwell
	$0 \leq u \leq u_V$	$0 \leq w \leq w_V$	+ve	+ve	
	u_V	w_V	Maximum +ve	0	Follower acceleration reverses
	$u_V \leq u \leq 1$	$w_V \leq w \leq 1$	+ve	-ve	
	1	1	0	0	Profile blends with outer dwell
Fall	0	1	0	0	
	$0 \leq u \leq 1 - u_V$	$1 \leq w \leq w_V$	-ve	-ve	
	$1 - u_V$	$1 - w_V$	Maximum -ve	0	Follower acceleration reverses
	$1 - u_V \leq u \leq 1$	$w_V \leq w \leq 0$	-ve	+ve	
	1	0	0	0	Profile blends with inner dwell

Fall action

The fall of the DFD action is analysed separately. Referring to Sketch 3.2, Point C₂ is the datum for the normalised parameters, u_{fall} and w_{fall} . At Point C₂, $u_{fall} = 0$ but $w_{fall} = 1$, with w'_{fall} and w''_{fall} both being zero[†]. The follower has negative velocity during the fall. The peak velocity is attained at the transition point, V₂, where, in the general case, $u_{fall} = (1 - u_V)$ and $w_{fall} = w_V$. At this point the direction of acceleration, w''_{fall} , reverses from negative to positive. At the end of the fall motion segment the normalised cam angle, $u_{fall} = 1$, with w_{fall} , w'_{fall} and w''_{fall} all being zero[†]. The boundary conditions for a complete DRDF motion are defined in Table 3.1.

For compatibility with the position of the datum at Point C₂, Equation (3.1) becomes

$$w_{fall} = f\{1 - u_{fall}\} . \quad (3.7)$$

[†] For infinite jerk cam laws only the velocity is zero at the start and end of DRD and DFD motion segments.

For all the cam laws given in Section 5, the transition point occurs midway through both the lift and the segment angle, *i.e.* $u_V = w_V = 0.5$. In the frequently encountered special case of symmetric cam laws the relationship between the parameters is given in Table 3.2.

TABLE 3.2 Relationship between parameters of symmetric DRD cam laws

u	w	w'	w''	w'''
u_i	w_i	w'_i	w''_i	w'''_i
$1 - u_i$	$1 - w_i$	w'_i	w''_i	w'''_i

4. DYNAMIC PERFORMANCE

Transient oscillations will occur as a result of a discontinuity in the cam law displacement equation or any derivative of that function. The lower the order in which the first discontinuity occurs, the more serious the resulting vibrations (see Koster^[22] or VDI^[24]). For this reason cam laws chosen for DRD and DFD actions should have both zero velocity and zero acceleration of the follower at the blends with the dwell at both start and end of the motion segment. Then the first (finite) discontinuity occurs in the third derivative, the jerk function.

With the exceptions of Simple Harmonic Motion (SHM), Constant-Cosine-Constant Acceleration (CCCA), 2-3 Polynomial Motion and Parabolic Motion[†] (constant acceleration), both the velocity and acceleration are zero at each limit of the follower displacement for all the cam laws considered in this Item. Included for completeness, the first three of these motions are unsuitable as single-law motions since they have the undesirable feature of abrupt changes of acceleration, and hence infinite jerk, at the start and finish of the motion segment. However, SHM and CCCA are suitable for the intermediate Component N2 of blended DRFD motions between the transition points through the reversal of follower velocity (see ESDU 06002^[18] and 06003^[19]).

A high rate of acceleration, or a discontinuity in the acceleration curve, *i.e.* excessive jerk anywhere in the motion segment, will be accompanied by abrupt changes of torque in the drive and of forces acting on the cam and following system. These abrupt changes cause fluctuating deflections in the parts concerned. The importance of jerk depends upon the magnitude of these deflections and their relationship to the desired lift curve accuracy. Even with continuous finite jerk, which is zero at the limits of follower travel, there are deflections depending upon the elasticity of the system. As shown in Sketch 2.3, which compares the Modified Sinusoidal Acceleration (see Section 5.1.3) with the Triple Harmonic Zero Jerk at Zero Acceleration (see Section 5.1.5) motions, a cam law with zero jerk at the limits of travel is liable to produce higher maximum accelerations, and therefore higher nominal forces, when the required output displacement is to be reached within the same period.

Rapid changes of forces and of the resulting deflections are likely to cause noise and wear through vibration during operation. Backlash and elasticity in the drive, the cam and following system inevitably lead to the follower deviating from the theoretical lift curve at speed due to the deflections involved. The reduction of noise and avoidance of abrupt changes in the displacement curve are important, especially at high operating speeds (the use of the term *speed* does not refer purely to the cam speed, the periods of the motion segments are also significant). Increasing the stiffness/mass ratio of the following system is potentially superior to reducing jerk and should be considered.

The allowable operating speed is also influenced by the accuracy of cam profile manufacture. The adverse effect of local profile inaccuracies is aggravated as the speed of operation increases but, in some cases, may be compensated by the compliance of the system.

4.1 Period Ratio

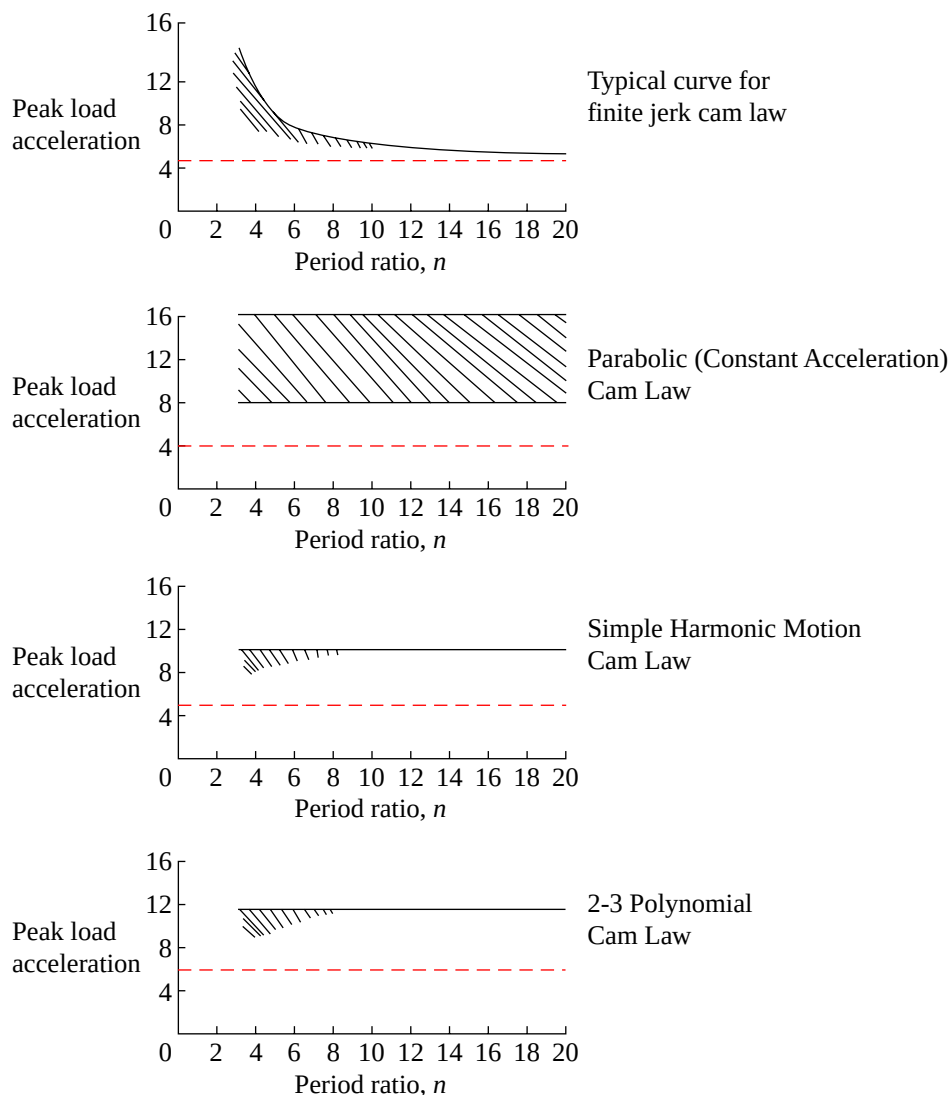
In order to provide an accurate definition of the operating characteristics of the mechanism, including induced vibration of the follower system, the non-dimensional *Period Ratio*, n , is introduced^[4]. The Period Ratio is defined as the ratio of the period of the motion segment to the period of fundamental vibration of the follower system, and is given by

$$n = \frac{\Phi}{360} \frac{\omega_n}{\omega_c} \quad (4.1)$$

[†] The jerk is also infinite at the transition point of Parabolic Motion where the follower attains maximum velocity. Despite the fact that this motion has the least possible peak acceleration, it is NOT recommended.

High values of Period Ratio indicate stiff, low mass systems operated at moderate or low shaft speeds. Low values of Period Ratio occur in systems with a compliant follower, a large driven mass or high shaft speed which produces a short rise time. As an approximate guide, a low value of n is taken as $n \leq 5$, which represents a very compliant or high speed mechanism. A mechanism operating at moderate speeds with a fairly stiff follower system would be indicated by values of $n \approx 10$. For values of n greater than 20, the acceleration at the driven mass approaches the nominal acceleration defined by the cam law and the dynamic response of the follower system may be neglected when determining accelerations. In the case of the single-law SHM, CCCA, 2-3 Polynomial and Parabolic cam laws, the actual acceleration will always be significantly higher than the nominal value.

All systems will exhibit a degree of elasticity, and in applications where this elasticity is significant, the resulting acceleration at the driven mass will be greater than the nominal acceleration predicted by the cam law formulae. Cam laws which exhibit discontinuities in acceleration (infinite jerk) produce particularly severe vibrations at the driven mass. The "peak load acceleration/deceleration" ratio, *i.e.* the maximum acceleration/deceleration of the driven mass, with an infinite jerk cam law is twice to four times the value predicted by kinematic analysis which neglects flexibility in the follower system. Sketch 4.1 shows the effect of variation of Period Ratio upon the peak load acceleration and deceleration during the motion segment.



Sketch 4.1 Peak load acceleration against period ratio

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In addition to the envelope of a curve typical of all the finite jerk cam laws, separate envelopes for the three infinite jerk laws are given. The nominal acceleration is represented by the horizontal broken lines in Sketch 4.1 which shows how the peak load acceleration approaches the nominal value as the Period Ratio is increased in the case of the finite jerk cam laws but will always be substantially higher than the nominal value for the infinite jerk laws.

It must be noted that the performance of the cam laws discussed in this Item will only be replicated in practice if the manufacturing facilities and techniques available to the user are able to produce the accuracy required (see Reference 23), and if there is no significant backlash in the system.

4.2 Drive Torques

The torque acting on the cam to drive the following system and external load varies throughout the active motion segments. Both the magnitude and the fluctuation of this torque are influenced by the choice of cam law. A low value of peak torque and, especially, a smooth variation of torque during the operation cycle are desirable, but may be subsidiary to other considerations influencing the choice of cam law.

The drive torque can be divided into four components and consist of the following:

1. the torque necessary to overcome the sum of the "constant" forces within the mechanism due to spring pre-load, friction, etc,
 2. the torque required to accelerate the mass of the follower system, lumped at the follower datum,
 3. the torque necessary to overcome the force due to spring stiffness as it is deflected
- and 4. the torque needed to overcome the external load acting on the end effector[†].

The component of torque needed to overcome the external load is unique to the application. The remainder apply equally to no-load operation when there is no contribution from the external load towards the force closure of the mechanism and so is likely to be the worst case for design purposes. The expressions for Components 1, 2 and 3, given as non-dimensional parameters in Table 4.1 (see Derivation 2), apply to the rise of a radial translating roller follower, but provide general guidance for other configurations. To assist the selection of suitable cam laws, curves of these parameters are included in the graphical data given in the Information Sheets of Section 5. As the speed increases, the drive torque needed to accelerate the mass of the following system is likely to be the dominant component of driving torque, hence the importance of it being a smooth, continuous function. (*Speed* does not refer merely to the cam speed, the periods of active motion segments are also significant.)

Inspection of the tables in Appendix B and the graphs included in the Information Sheets in Section 5, shows that the peak negative follower acceleration, w_q'' , occurs at different displacements, w_q , from the inner dwell, depending on the cam law. For a given following system and operating speed, these factors determine the spring stiffness and spring torque.

The curves of non-dimensional torque factors for the components corresponding to no-load operation, *i.e.*

- constant load,
 - force to accelerate lumped mass of following system
- and spring stiffness,

are plotted for every cam law included in the Information Sheets in Section 5.

[†] A general term for the output drive member of any mechanism or actuator.

TABLE 4.1 Non-dimensional torque factors

Component of torque	Expression for component of torque	Non-dimensional torque factor
1. Constant load $P_{const} = m_f g + P_{SP} + P_{frict}$	$q_{const} = \frac{P_{const} H}{\Phi} \left(\frac{180}{\pi} \right) w'$	$t_{const} = \frac{q_{const} \Phi}{P_{const} H} \left(\frac{\pi}{180} \right) = w'$
2. Follower acceleration	$q_I = \frac{m_f \omega_c^2 H^2}{\Phi^3} \left(\frac{180}{\pi} \right)^3 w' w''$	$t_I = \frac{q_I \Phi^3}{m_f \omega_c^2 H^2} \left(\frac{\pi}{180} \right)^3 = w' w''$
3. Spring stiffness	$q_s = \frac{k_{SC} H^2}{\Phi} \left(\frac{180}{\pi} \right) w w'$	$t_s = \frac{q_s \Phi}{k_{SC} H^2} \left(\frac{\pi}{180} \right) = w w'$