

ELASTIC STRESSES AND DEFLECTIONS OF HELICAL COMPRESSION AND TENSION SPRINGS OF ROUND WIRE

1. NOTATION

a	distance from loading pivot to plane of spring seating (see Sketch 7.1)	m	in
b	damping coefficient (see Section 9)	s^{-1}	s^{-1}
C	end condition factor (see Section 7)		
c	spring index = $D/d = (D_i + d)/d = (D_o - d)/d$		
D	mean diameter of spring = $D_i + d = D_o - d$	m	in
D_i, D_o	inside and outside diameters of spring respectively	m	in
d	diameter of wire	m	in
$\bar{d}$	allowance for end coils at both ends of spring	m	in
E	modulus of elasticity of spring material	N/m ²	lbf/in ²
F	stress increase factor due to cyclic loading (see Section 9)		
f	frequency of applied cyclic loading = $\omega/2\pi$	Hz	cycles/s
f_n	frequency of first natural mode of vibration of spring	Hz	cycles/s
G	shear modulus of spring material	N/m ²	lbf/in ²
L	compressed length of spring (see Sketch 1.1)	m	in
L'_c	solid length of working coils of spring = nd	m	in
L_i	length inside the hooks of closed tension spring	m	in
L_0	free length of compression spring (when $P = 0$)	m	in
N	total number of turns in compression spring = length of wire/ πD		
n	number of effective or working coils in spring (see Section 5)		
n'	number of coils in tension spring between points where end hooks begin (see Sketch 1.2)		
n_0	number of unclosed coils in unloaded compression spring		
P	applied load (see Sketch 1.1)	N	lbf

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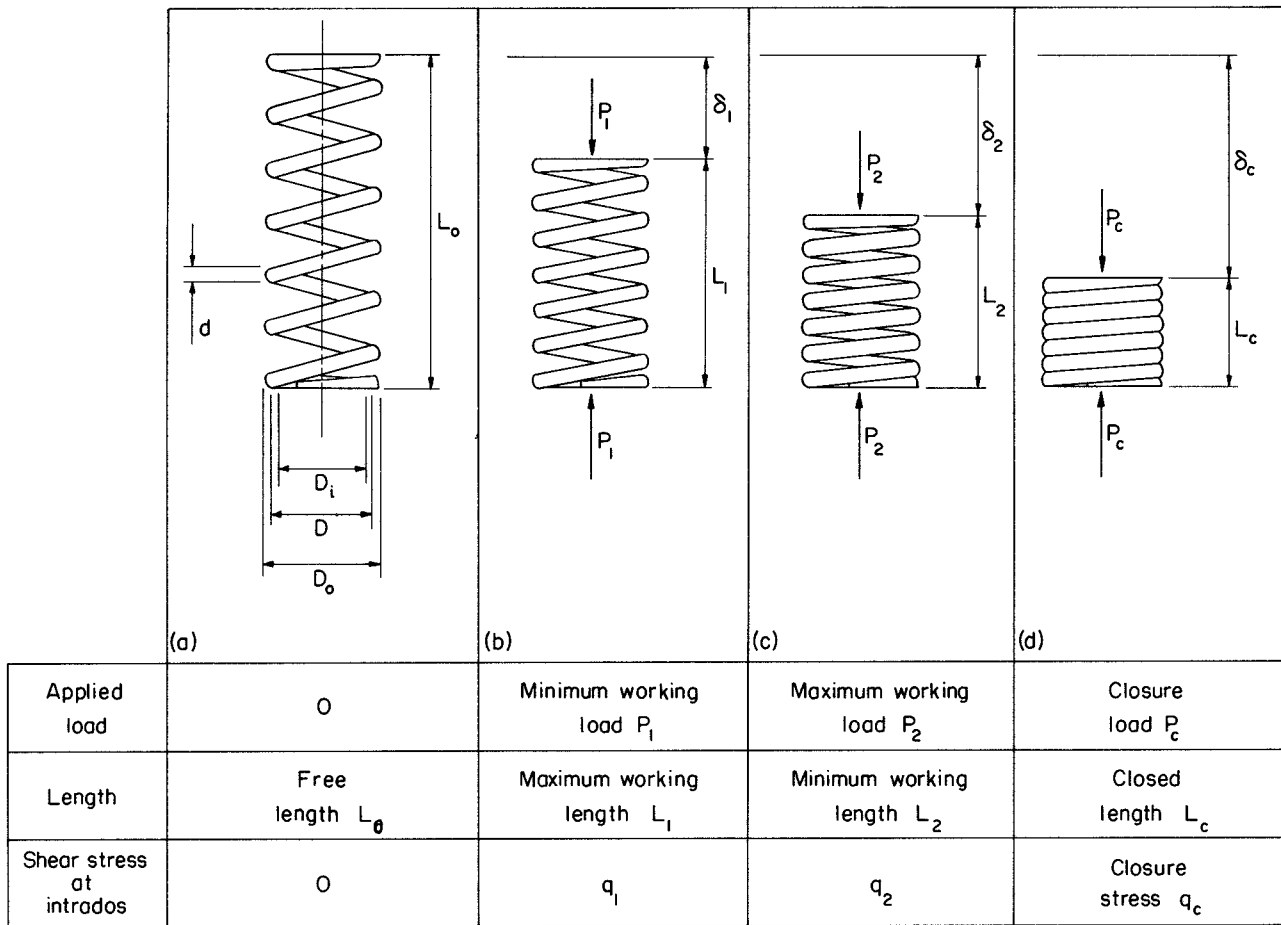
P_i	initial tension induced in tension spring at manufacture, if present	N	lbf
p	unloaded pitch of spring	m	in
q	shear stress at intrados* (including curvature correction factor) (see Sketch 1.1)	N/m ²	lbf/in ²
q_a	value of q under cyclic loading (see Section 9)	N/m ²	lbf/in ²
q_h	shear stress due to deflection equal to amplitude of harmonic in resonance	N/m ²	lbf/in ²
q_r	additional shear stress due to surging (see Sketch 9.1)	N/m ²	lbf/in ²
q_w	total static shear stress including effects of helix warping	N/m ²	lbf/in ²
r_A, r_B	radii of hook bends in tension spring (see Sketch 4.1)	m	in
S	spring rate of stiffness = P/δ	N/m	lbf/in
y	moment arm of load in tension spring hook	m	in
δ	deflection of spring (see Sketch 1.1)	m	in
δ_{crit}	value of δ at which buckling of compression spring occurs	m	in
ν	Poisson's ratio for spring material		
ρ	mass density of spring material	kg/m ³	lb/in ³

* The inner surface of the coils.

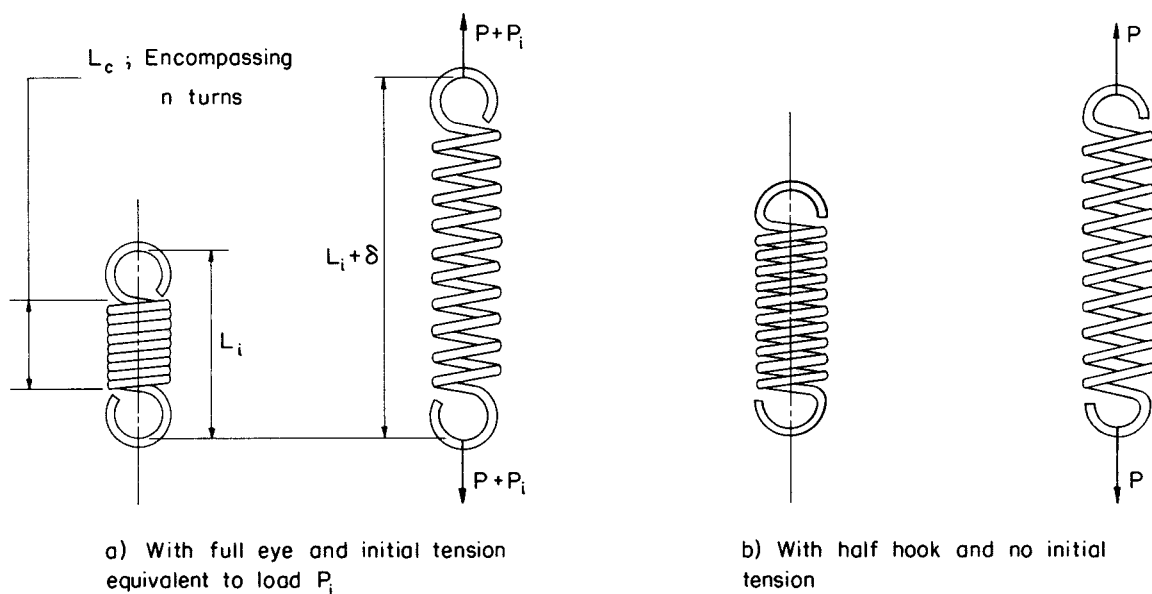
Suffixes

1	relates to spring under minimum working load (see Sketch 1.1b)
2	relates to spring under maximum working load (see Sketch 1.1c)
c	relates to spring when closed solid (see Sketch 1.1d)
t	relates to spring under transverse load (see Section 10)

Both SI and British units are quoted but any coherent system of units may be used. A compression spring and a tension spring are shown in Sketches 1.1 and 1.2.



Sketch 1.1 Compression spring with ends closed and ground under 4 stages of loading



Sketch 1.2 Tension springs under two stages of loading

2. INTRODUCTION

This Item contains design data for helical compression and tension springs of round wire acting within the elastic range of the material. Some design notes to be used in conjunction with this Item are provided in Data Item No. 83003*. The stresses referred to in this Item are additional to any incurred during manufacture by, for example, prestressing (see Data Item No. 83003). The design process presented below indicates the use of each section in the Item, although not all the sections will be applicable to any one spring design. This process may have to be repeated several times before a suitable design is found.

- (1) Choose the type of end coil formation required in the case of a compression spring, or for a tension spring, the type of end eye or hook to be employed. These will influence both the stress and the number of active coils in either spring. Data Item No. 83003 provides notes on the end coils of compression springs.
- (2) Choose a suitable material and a value for the working stress. Guidance on this is given in Data Item No. 83002†.
- (3) D_o or D_i are often fixed by a design problem or can be easily estimated. Section 4 may then be used to determine d and hence c and D .
- (4) Use Section 5 to determine the spring rate, deflection characteristics and number of turns.
- (5) Check that the spring conforms to the limitations of Section 3.
- (6) Check the closed length characteristics using Section 6.
- (7) Check the buckling characteristics using Section 7.
- (8) The coil diameter of compression springs increases slightly when loaded. This increase is generally less than 2 per cent, but may be accurately determined, if necessary, by using Section 8.
- (9) Additional stresses will arise if the spring is subject to the following types of loading,
 - (a) cyclic loading (see Section 9)or
 - (b) transverse loading (see Section 10).
- (10) The phenomenon of helix warping produces a stress increase in compression springs over and above that predicted in Section 4. The increase is, however, slight (being less than 5 per cent for springs for which $N > 11$) and need be considered only in springs designed near to their stress limit or where fatigue is the governing failure mode. See Section 11, for springs with ends closed and ground.

* Data Item No. 83003, "Notes on the design of helical compression and tension springs of round wire".

† Data Item No. 83002, "Allowable stresses for helical compression and tension springs of round wire".

3. NOTES

The data in this Item are subject to the following limitations.

- (a) The helix angle of a spring should not exceed 10° ; therefore, for an unloaded compression spring,

$$L_0 < 0.55n_0D + \bar{d} \quad , \quad (3.1)$$

where values of $\bar{d}$ are given in Table 3.1, or for a loaded tension spring,

$$\delta < n'd(0.55c - 1) \quad . \quad (3.2)$$

- (b) The deflection per coil under load should not be more than about a quarter of the coil diameter, D , for either type of spring.

Most springs will conform to conditions (a) and (b) above but if these limitations are exceeded then significant errors will arise in using the data in this Item. In that case either open coil spring theory should be used (see Reference 10) or the single spring replaced by a nest of 2 or more springs, each conforming to the conditions above. See Data Item No. 82012*.

TABLE 3.1

<i>End coil formation</i>	$\bar{d}$
One coil at each end closed and ground	$1.5d$
End coils unclosed but ground	$-0.5d$
End coils unclosed and unground	d
One coil at each end closed but unground	$3.0d$

4. STATIC SHEAR STRESS

4.1 Compression Springs

The shear stress in a spring may be calculated from Equation (4.1), below.

$$q = \frac{8P}{\pi d^2} \times \frac{c(c + 0.2)}{(c - 1)} \quad . \quad (4.1)$$

Equation (4.1) is presented in graphical form in Figures 1 and 2, which relate the parameters D_o , D_i , d and P/q .

4.2 Tension Springs

For the type of tension spring in which the load is applied through a separate plug connection screwed in

* Data Item No. 82012, "Nested helical compression springs of round wire (To be used in conjunction with Data Item No. 82008)".

to the ends, the maximum stress will occur in the working coils and is given by Equation (4.2)

$$q = \frac{8(P + P_i)}{\pi d^2} \times \frac{c(c + 0.2)}{(c - 1)} . \quad (4.2)$$

Care must be taken not to introduce stress concentrations where the wire leaves the plug.

Most tension springs have their ends turned up to form a hook or eye, in which case the maximum stress occurs in or near these hooks.

At point A in Sketch 4.1, an approximate value of the combined bending and direct stresses in the hook is given by

$$\frac{(P + P_i)y}{(\pi d^3/32)} \left(\frac{2r_A + d}{2r_A} \right) + \frac{(P + P_i)}{\pi d^2/4} , \quad (4.3)$$

where $\frac{2r_A + d}{2r_A}$ is a factor to account for the enhanced stress at the intrados.

For the type of end hook shown in Sketch 1.2 where $y = D/2$, then the combined bending and direct stresses given by Equation (4.3) reduce to

$$\frac{4(P + P_i)}{\pi d^2} \left(\frac{2c}{r_A} (2r_A + d) + 1 \right) . \quad (4.4)$$

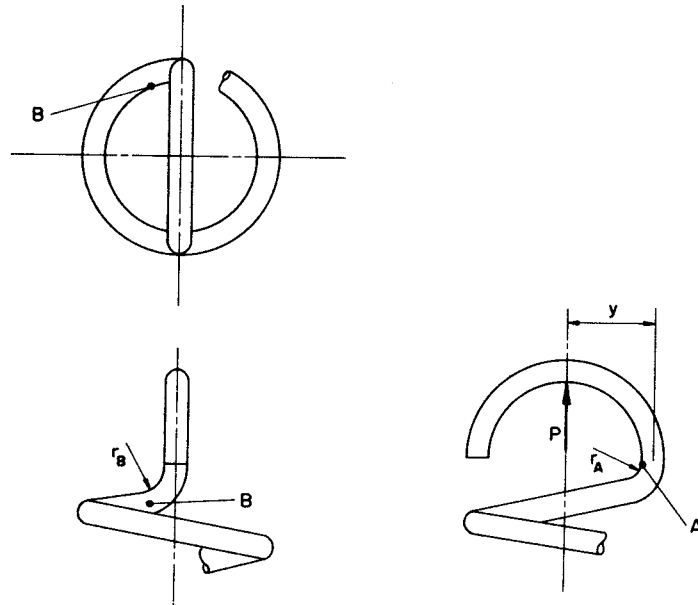
The maximum torsional shear stress will occur at point B (on the intrados) shown in Sketch 4.1 and is given by

$$\frac{(P + P_i)}{\pi d^2 r_B} \times c(8r_B + 3d) . \quad (4.5)$$

Equations (4.2), (4.4) and (4.5) must all be checked to ensure that the allowable stress is not exceeded. Note that Equation (4.3) gives a bending stress. A permissible bending stress may be taken as twice the numerical value of an allowable shear stress. Residual stresses due to the manufacture of the hook must also be accounted for.

Care must be exercised, particularly on assembly, to ensure that a tension spring is not stretched so far that its free length is altered from the design specification.

Under cyclic loading both the minimum and maximum working stresses must be calculated to obtain the fatigue life.



Sketch 4.1 End hook of tension spring

5. SPRING AND DEFLECTION CHARACTERISTICS

The axial spring rate, S , of both tension and compression springs may be calculated from Equation (5.1). Data on the transverse spring rate, S_t , of compression springs are presented in Section 10.

$$S = \frac{Gd}{8nc^3} . \quad (5.1)$$

Figures 3a and 3b present Equation (5.1) as nomograms. Figure 3a is in SI units and Figure 3b in British units. These figures are based on the number of effective working coils, n . The relationship between N and n is shown in Table 5.1. The value of $N - n$ represents the total number of ineffective coils at both ends of the spring. In each range, the higher number corresponds to compression under a large load and the lower number to compression under a small load.

TABLE 5.1

<i>End coil formation</i>	<i>Total number of ineffective coils in compression spring, (N - n)</i>
One coil at each end closed and ground	1.5–2.0
End coils unclosed but ground	0.5–1.0
End coils unclosed and unground	0–0.5
One coil at each end closed but unground	1.5–2.0

The number of active coils in a tension spring depends on the type of end hook. Values of n are given in Table 5.2.

TABLE 5.2

<i>End hook type</i>	<i>Total number of effective coils in tension spring, n</i>
Half-hook over centre	$n' + 0.2$
Full eye over centre	$n' + 1$

Figure 4 presents the relationship between S/G , c , D and L_c' as a nomogram. The quantity L_c' may be used as an approximation for the solid length of a spring (see Section 6).

6. SOLID LENGTH CHARACTERISTICS

The nominal solid length of a spring is the length at which all coils are touching (see Sketches 1.1d and 1.2a). Some notes on solid length are provided in Data Item No. 83003.

6.1 Compression Springs

Most springs are scragged to introduce favourable residual stresses into the wire. If advantage of this process is not taken then it is necessary to ensure that the spring is not overstressed when compressed solid. At minimum working length clearance should be maintained between the coils. More detailed notes are provided in Data Item No. 83003.

The nominal solid length of a compression spring varies with the type of end coil formation and the total number of turns, N . The relationship between n and N is presented in Table 5.1 and expressions for the nominal solid length are given in Table 6.1.

TABLE 6.1

End coil formation	L_c
One coil at each end closed and ground	$d(N - 0.5)^*$
One coil at each end closed but unground	$d(N + 1)$
End coils unclosed and unground	$d(N + 1)$
End coils unclosed and ground	$d(N - 0.5)^*$

* This expression assumes that the tip of the end coil is ground down to a thickness of $1/4d$; that is, in the case of one coil at each end closed and ground, $3/4$ of the end coil is ground.

6.2 Tension Springs

Tension springs are often coiled with an initial tension, so that the coils are touching when the spring is unloaded. The solid length, excluding the end loops, is given by

$$L_c = d(n' + 1). \quad (6.1)$$

For a tension spring with full loops at each end,

$$L_i \approx L_c + 2D, \quad (6.2)$$

and, with half hooks at each end,

$$L_i \approx L_c + D. \quad (6.3)$$

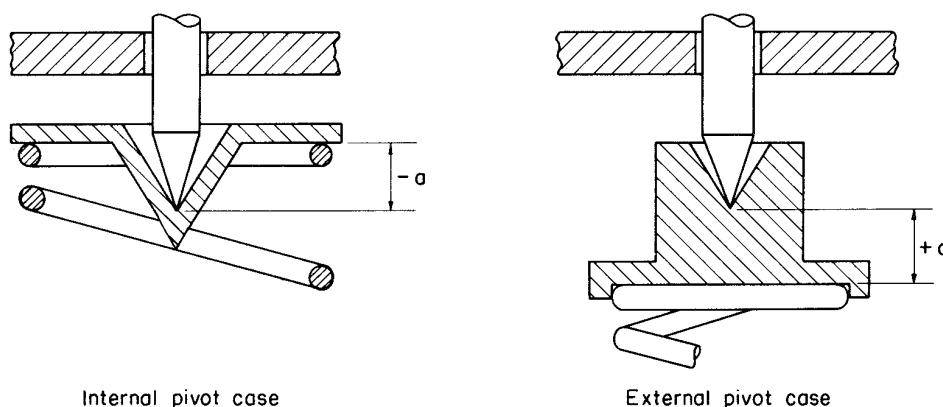
7. BUCKLING OF COMPRESSION SPRINGS

If the length to diameter ratio, L_0/D , is high, buckling may occur during compressive loading. Figure 5 presents curves relating δ_{crit}/L_0 to CD/L_0 and Ca/L_0 for $\nu = 0.3$. Each curve for a given value of Ca/L_0 divides the diagram into two areas; to the left of the curve the spring will buckle, while to the right, it remains stable. The length a is shown in Sketch 7.1 and is zero for non-pivoted springs. Theoretical values of the end condition factor, C , are shown in Sketch 7.2; however, due to effects discussed in Data Item No. 83003, C should be reduced by about 20 per cent. For most spring materials, including steels, Figure 5 yields values of δ_{crit} slightly smaller than would be obtained in practice.

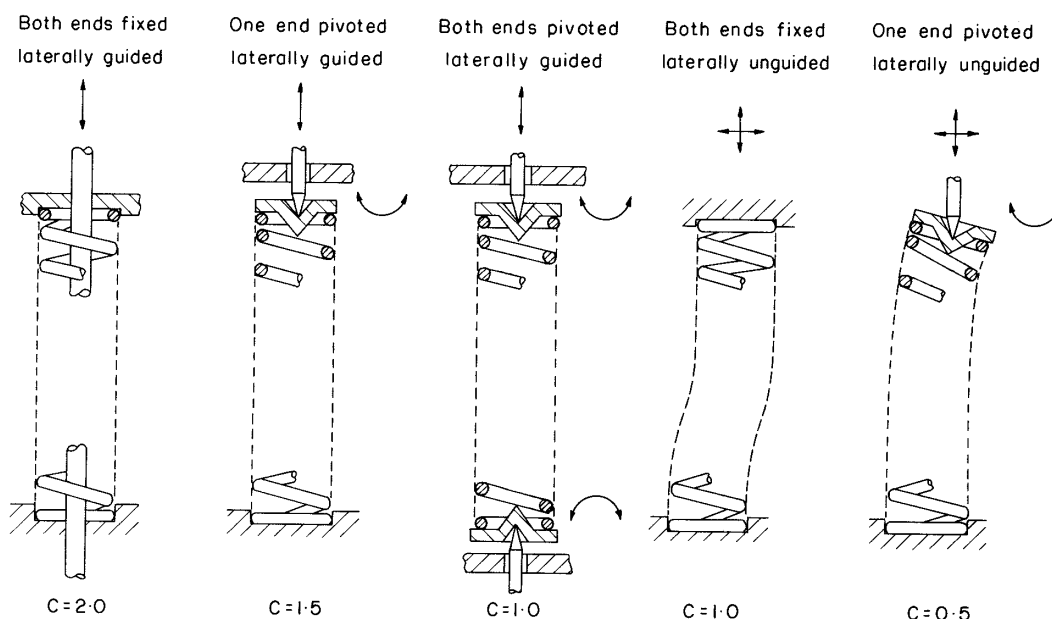
Figure 5 is constructed using Equation (7.1), developed from Derivations 4 and 6.

$$\frac{\delta_{crit}}{L_0} = \left(\frac{1 + \nu}{1 + 2\nu} \right) \left[1 \pm \left[1 - \lambda^2 \pi^2 \left(\frac{1 + 2\nu}{2 + \nu} \right) \left(\frac{CD}{L_0} \right)^2 \right]^{1/2} \right] \quad (7.1)$$

The factor λ depends on Ca/L_0 and δ_{crit}/L_0 and is 1 for non-pivoted seatings. Data on λ may be found in Derivation 6.



Sketch 7.1



Sketch 7.2