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Introduction to unsteady aerodynamics

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INTRODUCTION TO UNSTEADY AERODYNAMICS

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INTRODUCTION TO UNSTEADY AERODYNAMICS

1. NOTATION AND UNITS

Some symbols which only appear briefly in the text and are defined where they appear have been omitted from the following Notation.

		<i>SI</i>	<i>British</i>
A	structural inertia matrix in Equation (10.1)	N m s ²	lbf ft s ²
A'_0, A''_0	leading coefficients in Equations (6.27)		
a	speed of sound	m/s	ft/s
B	aerodynamic damping matrix in Equation (10.1)	m ⁴	ft ⁴
B'_1, B''_1	coefficients appearing in Equation (6.26)		
b_{ij}	element of matrix B , see Section 10.2	m ⁴	ft ⁴
C	aerodynamic stiffness matrix in Equation (10.1)	m ³	ft ³
c	aerofoil chord or local chord	m	ft
$\bar{c}$	geometric mean chord	m	ft
c_{ij}	element of matrix C , see Section 10.2	m ³	ft ³
D	structural damping matrix in Equation (10.1)	N m s	lbf ft s
d	reference length	m	ft
E	structural stiffness matrix in Equation (10.1)	N m	lbf ft
F_{ij}	generalized force, see Equations (10.4) and (10.5)	N m	lbf ft
i	$\sqrt{-1}$		
i	integer denoting a particular force, see Section 4.1		
j	integer denoting a particular motion, see Section 4.1		
K	Küssner kernel in Equation (6.7)	m ⁻²	ft ⁻²
K_2	upwash influence function in Equation (8.7)	m ⁻¹	ft ⁻¹
L	lift per unit span, see Equation (5.12)	N/m	lbf/ft
M_L	local Mach number		
m	number of spanwise terms		
l	complex loading function in Equation (6.4)		
M	free-stream Mach number, V/a		
$m_{\dot{\alpha}}$	aerodynamic damping derivative for pitching motion, see Equation (7.4)		
N	number of modes		
n	number of chordwise terms		
O	order, see Equation (5.13)		
P	Theodorsen function in Equation (5.2)		
p	local pressure	N/m ²	N/m ²

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p	integer denoting chordwise position, see Equations (6.16) and (6.17)		
p_∞	free-stream pressure	N/m ²	lbf/ft ²
$-\mathbf{Q}'$	aerodynamic stiffness matrix, $\mathbf{C}/d^3$		
$-\mathbf{Q}''$	aerodynamic damping matrix, $\mathbf{B}/d^4$		
Q_{ij}	complex generalized airforce coefficient, see Equation (6.12)		
Q'_{ij}	in-phase part of Q_{ij}		
Q''_{ij}	in-quadrature (90° out of phase) part of Q_{ij}/v		
$\mathbf{q}$	vector of generalized displacements, see Equation (10.1)		
q_j	complex generalized coordinate in mode j		
$\dot{q}_j$	first derivative of q_j with respect to time	s ⁻¹	s ⁻¹
$\mathcal{R}$	real part of		
Re_c	Reynolds number based on chord		
S	integration over wing planform		
S_1, S_2, S_3	domains of dependence in supersonic flow		
s	semi-span	m	ft
t	time	s	s
V	free-stream velocity	m/s	ft/s
w	upwash velocity	m/s	ft/s
x, z	two-dimensional Cartesian coordinates relative to mean position of aerofoil chord with origin at leading edge; x positive rearwards; z positive upwards (see Sketch 4.1)	m	ft
x, y, z	three-dimensional Cartesian coordinates relative to mean position of wing (see Sketch 3.13)	m	ft
x_h	chordwise location of hinge line for trailing-edge control (see Sketch 4.1)	m	ft
z_c	ordinate of camber and incidence, see Sketch 3.14	m	ft
z_i, z_j	mode shapes, see Equations (4.4) to (4.6)	m	ft
z_t	ordinate of aerofoil thickness, see Sketch 3.14	m	ft
α	incidence	deg	deg
α_0	mean incidence	deg	deg
α_1	amplitude of pitching oscillation	deg	deg
Γ_q	spanwise distribution, see Equations (6.9) and (6.12), $q = 1$ to n		
γ	ratio of specific heat capacities of air, taken as 1.4		
$\bar{\gamma}'_j(\xi)$	in-phase part of chordwise loading in Equation (5.8)		
$\bar{\gamma}''_j(\xi)$	in-quadrature part of chordwise loading in Equation (5.8)		
ΔC_p	complex non-dimensional pressure difference in Equation (3.7)		

Δp	lift per unit area, see Equation (3.8)	N/m ²	lbf/ft ²
δ	flap angle	deg	deg
δ_1	amplitude of flap oscillation	deg	deg
ε	phase angle (in degrees in Equation (6.30))	rad	rad
η	y/s		
η_r	spanwise location, see Equation (6.14), $r = 1$ to m		
Λ	sweepback	deg	deg
μ	$\cos^{-1}(1/M)$ in Section 8		
ν	frequency parameter, $\omega d/V$		
ξ	non-dimensional chordwise location, see Equation (5.9)		
ξ_a	pitching axis location		
ρ	free-stream density	kg/m ³	slug/ft ³
τ	regular interval in time	s	s
Φ	complete velocity potential	m ² /s	ft ² /s
Φ	reduced velocity potential in Sections 5 to 7	m ² /s	ft ² /s
Φ_0	steady part of velocity potential in Equation (3.3)	m ² /s	ft ² /s
ϕ	oscillatory part of velocity potential in Equation (3.3)	m ² /s	ft ² /s
ψ	acceleration potential, see Equation (5.7)	m ² /s ²	ft ² /s ²
ψ_q	chordwise distribution, see Equations (6.9) and (6.10), $q = 1$ to n		
ω	circular frequency	rad/s	rad/s

Subscripts

i	integer denoting a particular force, see Section 4.1
j	integer denoting a particular motion, see Section 4.1
ij	double subscript combining i and j
l	denotes lower surface
le	denotes leading edge
te	denotes trailing edge
u	denotes upper surface

Dressings

prime (') as in $\mathbf{Q}'$ denotes real part

double prime (") as in $\mathbf{Q}''$ denotes imaginary part

2. INTRODUCTION

The information in this Item will introduce the reader to the subject of the theories of oscillatory aerodynamics and their application in aircraft design. Sections 3 and 4 concern the basic principles that underline linearized theory. Sections 5 to 8 are devoted to the various methods of solution for aerofoil sections and wings in simple harmonic motion in subsonic and supersonic flows. Sections 9 and 10 consider the practicalities of semiempirical methods and applications of unsteady aerodynamics to flutter, atmospheric disturbances and active control systems.

From as early as 1920 (Reference 38) the subject of unsteady aerodynamics has been studied with particular effort to determine, mathematically, the pressure distributions on wings oscillating with elastic deformations. The integrated forces are necessary in the investigation of the phenomenon of dynamic aeroelastic instability known as flutter, which occurs above some critical equivalent airspeed. The problems associated with aeroelastic instabilities received growing attention as flight speeds increased, and in recent years flutter considerations have often been a major influence in the design of the primary wing structure. It is therefore important that the unsteady aerodynamic forces be evaluated with some confidence.

For subsonic or supersonic flow linearized theory is known to perform adequately for thin wings. However, even on the basis of linearized theory, the computation of aerodynamic forces on three-dimensional wings, for all the mode shapes, is time consuming. Therefore, in the initial phase of aircraft design, flutter calculations are sometimes made by a “strip theory” analysis in terms of aerodynamic derivatives from two-dimensional theories. ESDU Item Nos 81034 (Reference 40) and 82005 (Reference 41) present some relevant two-dimensional data for subsonic and supersonic speeds respectively. For transonic flow non-linearity in the aerodynamics leads to large errors in the computed flutter speed when, say, $0.8 < M < 1.1$. Another phenomenon that restricts the application of the linearized theories is the occurrence of flow separation. Both types of flow régime are beyond the scope of this Item, but they are discussed briefly in Sections 3.3.1 and 3.3.2.

In contrast to the aeroelastic instability of an aircraft structure, mentioned above, the critical loads that determine structural strength are often those encountered in a gust or atmospheric turbulence. This is particularly true of civil aircraft which have only modest manoeuvring requirements. The calculation of structural loads due to atmospheric disturbances is another important area in which unsteady aerodynamics finds an application. The mathematical models used in such studies usually include a representation of structural deflections, but in less detail than for flutter calculations. The fact that the response of an aircraft to a gust is non-sinusoidal places a different emphasis on the modelling of the unsteady effects.

In recent years, the advent of active control systems designed to alleviate gust loads or to suppress undesirable flutter characteristics has broadened the spectrum of interest in unsteady aerodynamics. As such systems can demand very rapid control movements compared with those normally input by a pilot, the overall performance of an active system will depend to some extent on the rate of growth of the aerodynamic forces.

A vast amount of technical literature has built up over the years, and a comprehensive survey up to the middle fifties is provided in Reference 21. A useful textbook on the subject is Reference 16. This introduction to unsteady aerodynamics does not attempt to review all subsequent developments, but aims to provide a general background, to illustrate current linearized theoretical methods and to discuss their applications.

3. BASIC PRINCIPLES

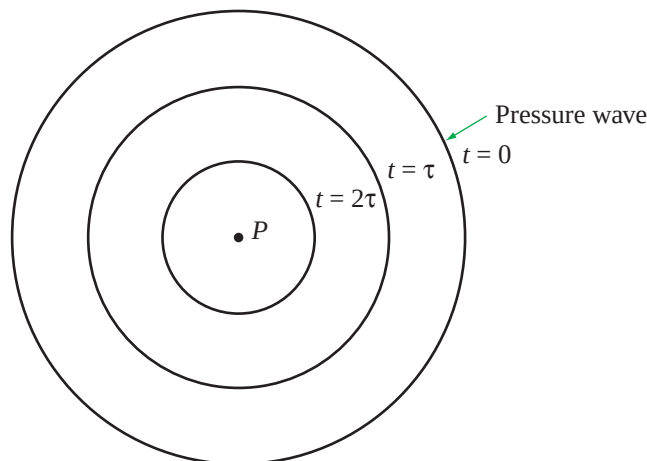
Aircraft wings are complex structures and their external geometry depends on size, taper ratio, leading-edge sweep, aspect ratio, dihedral, twist and aerofoil section, which itself is a function of local thickness and camber. The air in which the wing must operate is both compressible and viscous. Moreover, the wing may move relative to the air within a wide range of speeds and incidences and, in the unsteady case, may oscillate rigidly or in an arbitrary mode of deformation at an arbitrary frequency.

Thus, there is an impressive set of variables upon which the steady or unsteady aerodynamics generated by a wing may depend. Although this introductory Item concerns oscillatory aerodynamics, many of the basic principles will be more readily illustrated by considering steady motion.

In formulating the linearized subsonic and supersonic oscillatory aerodynamics of aerofoils and wings, basic assumptions are made concerning some of the above variables. These assumptions and consequent restrictions on the validity of the theoretical results are discussed where appropriate.

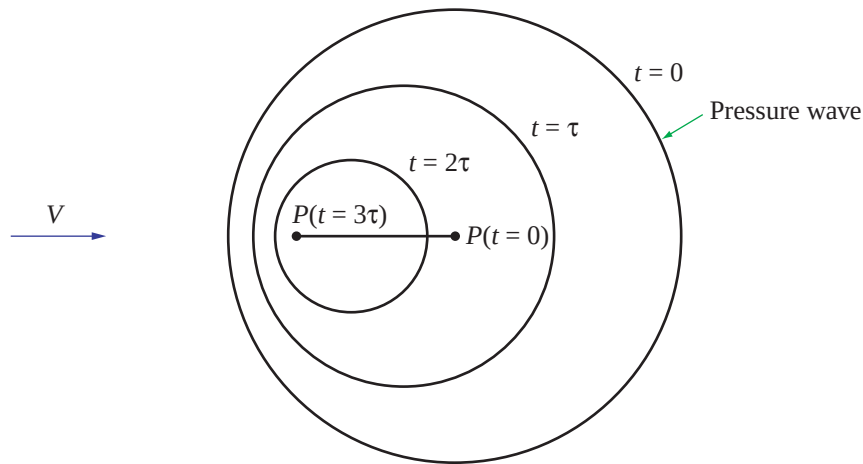
3.1 Basic Flow Characteristics

If a stationary particle, P (Sketch 3.1), in still air is infinitesimally disturbed at regular time intervals, τ , pressure waves will be created which will propagate uniformly in all directions at the speed of sound, a .

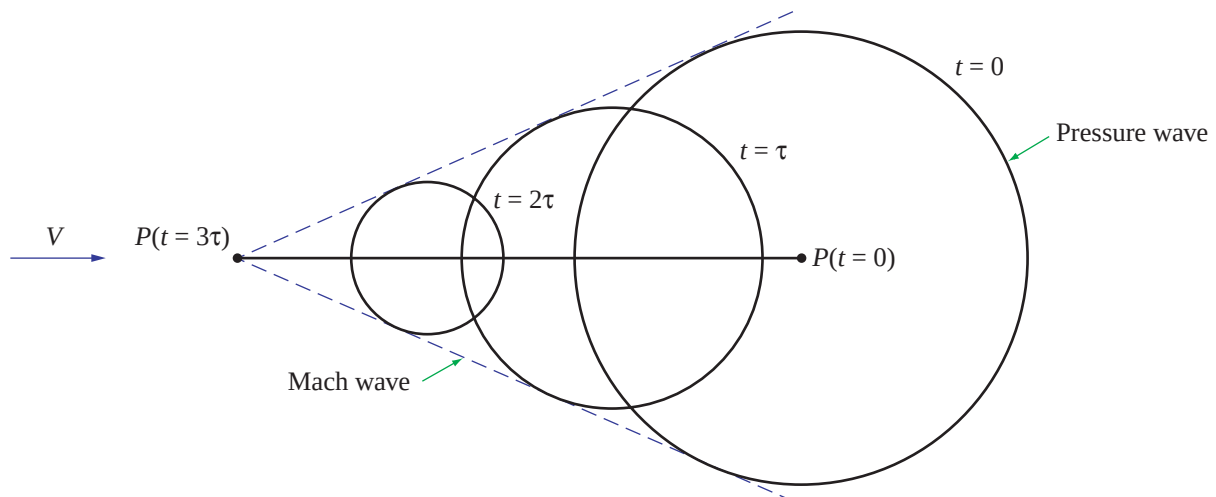


Sketch 3.1 At time 3τ in still air

If the particle now moves at a constant speed V , the pattern of the pressure waves depends on the relative magnitudes of V and a . If $V < a$ (Sketch 3.2) the pressure waves precede the particle, and thus, in subsonic flow, the air ahead of the particle is warned of its approach and will react gradually.



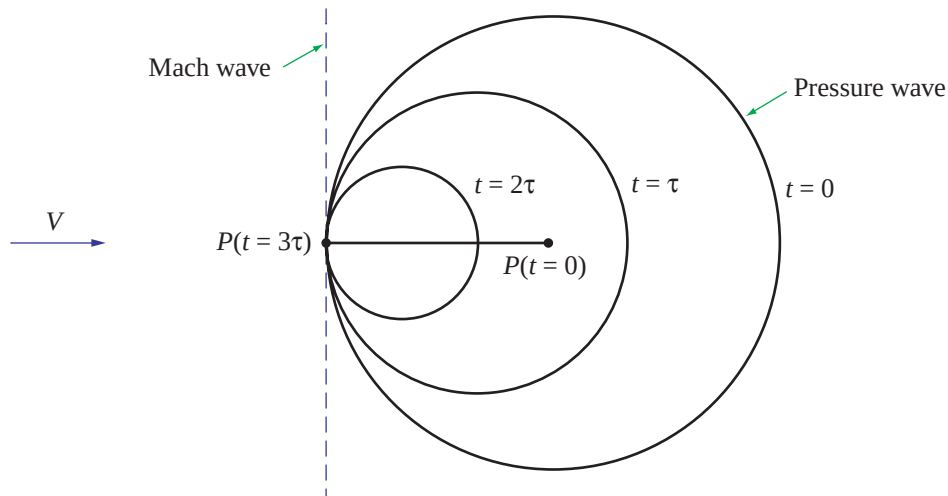
Sketch 3.2 At time 3τ if $V < a$



Sketch 3.3 At time 3τ if $V > a$

If $V > a$ (Sketch 3.3) the particle precedes its emitted pressure waves, which build up along the trailing dotted lines, referred to as Mach waves. Thus in supersonic flow the air has no prior warning of the approaching particle and sudden changes in pressure, velocity, density and temperature are induced by the particle across these Mach waves.

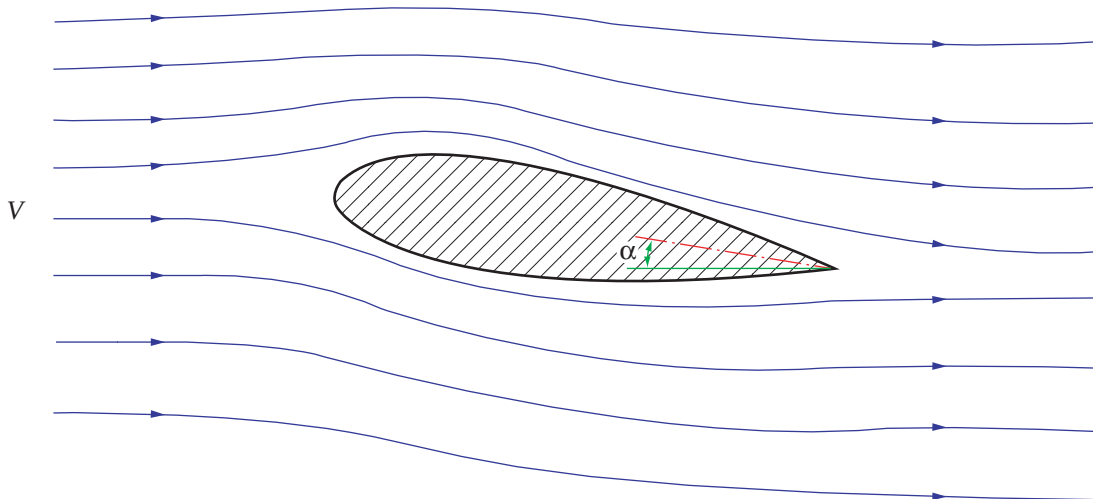
The ratio V/a is called the Mach number, M . In supersonic flow ($M > 1$) the region of disturbance forms a Mach cone, whose apex half-angle, $\sin^{-1}(1/M)$, becomes more acute as Mach number increases. In the special sonic case $M = 1$ (Sketch 3.4) the half-angle is 90° and the Mach wave becomes planar and normal to the direction of travel of the particle.



Sketch 3.4 At time 3τ if $V = a$

3.2 Lift in Two Dimensions

Although the subsonic and supersonic flow characteristics are vastly different, the principles behind the generation of lift have some similarities. Consider, for example, the two-dimensional steady flow past an aerofoil at a small nose-up incidence, α , and at subsonic free-stream velocity, V .

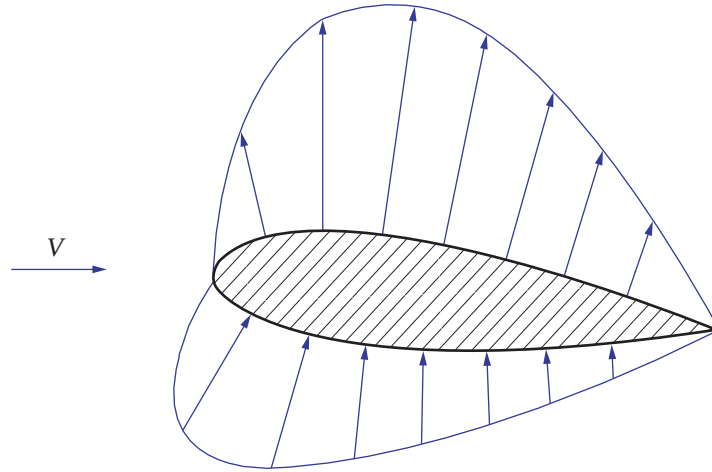


Sketch 3.5 Streamlines in attached flow

The flow of air can be described by the diagram of streamlines (Sketch 3.5), across which there is no mass flow; the closer the streamlines, the faster the local airflow. The effect of α is to speed up the flow over the upper surface and to retard it on the lower surface. The influence on the pressure distribution is described by means of Bernoulli's equation for incompressible flow

$$p + \frac{1}{2}\rho U^2 = \text{constant}, \quad (3.1)$$

where ρ is the air density and p and U are respectively the local values of static pressure and velocity. A decrease in velocity produces an increase in pressure, and vice versa.



Sketch 3.6 Typical pressure distribution

Therefore, for a nose-up incidence (Sketch 3.6), the differential pressure across the aerofoil gives rise to an upward lift force L per unit span, which is identified with a circulation $L/\rho V$.

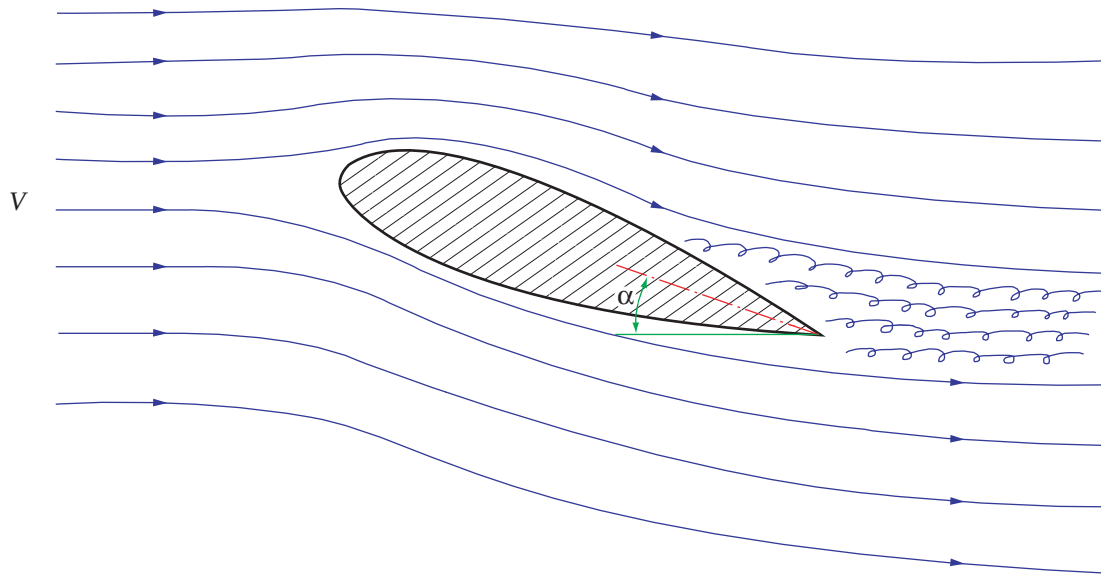
In subsonic flow the circulation is controlled by the Kutta condition of zero differential pressure at the trailing edge. In supersonic flow there is no Kutta condition, and the pressures on the upper and lower surfaces develop independently. When the circulation is steady there is no vorticity in the wake, but vortex shedding occurs in two-dimensional oscillatory flow.

3.3 Causes of Non-linearity

The primary causes of non-linear aerodynamics in steady and oscillatory flows are boundary-layer separation and shock waves. The implications of separated and transonic flows are considered briefly for two-dimensional subsonic and supersonic free streams.

3.3.1 Subsonic flow

As the incidence is increased, lift is likewise increased in a more or less linear manner and classical theories remain viable. Above a certain incidence attached flow over the upper surface can no longer be maintained and the flow separates leading to a loss of lift. This is known as the stall. Sketch 3.7 shows a “rear separation” having moved forward from the trailing edge as is typical of full-scale subsonic flow. With the advent of separation the classical theories fail and new mathematical modelling is needed.

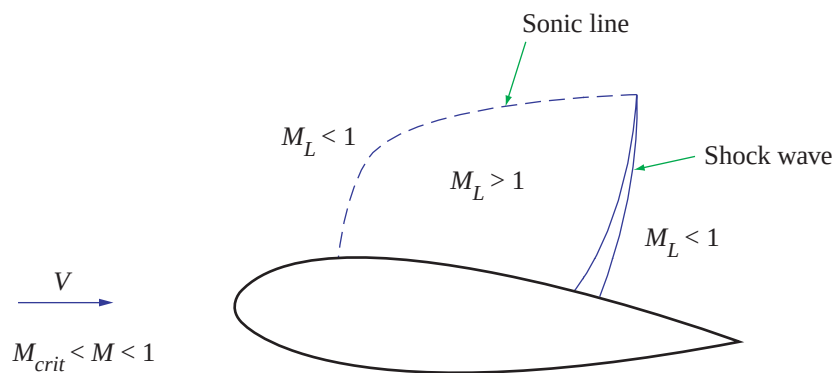


Sketch 3.7 Streamlines in separated flow

The subsonic theories of Section 5 are also invalidated when V is large enough for local supersonic flow to develop. The free-stream velocity V^* at which the local speed of sound is first reached on the aerofoil defines the critical Mach number

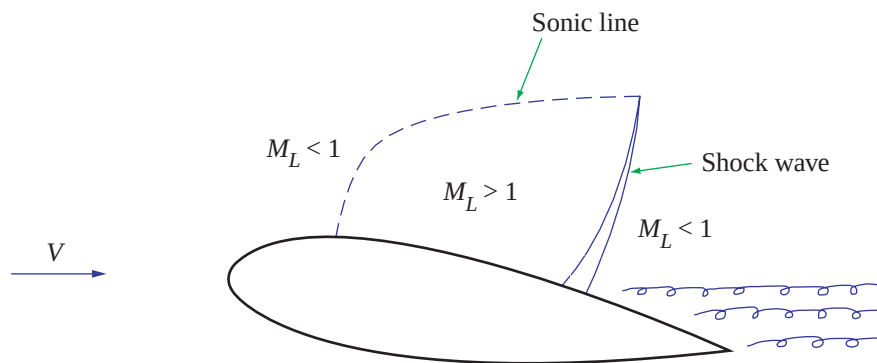
$$M_{crit} = V^*/a. \quad (3.2)$$

For higher velocities there is mixed flow with at least one supersonic region adjacent to the aerofoil and embedded in the otherwise subsonic flow (Sketch 3.8).



Sketch 3.8 Supercritical flow

In this transonic or supercritical flow the supersonic region with local Mach number $M_L > 1$, is terminated by a shock wave which restores the flow to subsonic conditions and is associated with sudden changes in pressure, velocity, density and temperature. Further increase in V results in a larger supersonic region terminated by a stronger shock wave nearer to the trailing edge. Eventually the pressure rise through the shock wave becomes sufficient to cause separation (Sketch 3.9). The manner in which flow separation develops at subsonic and transonic speeds is described in some detail in Reference 26. Flow separation can lead to unsteady phenomena such as buffet and control surface “buzz” (see Section 10.3.2).



Sketch 3.9 Shock-induced separation

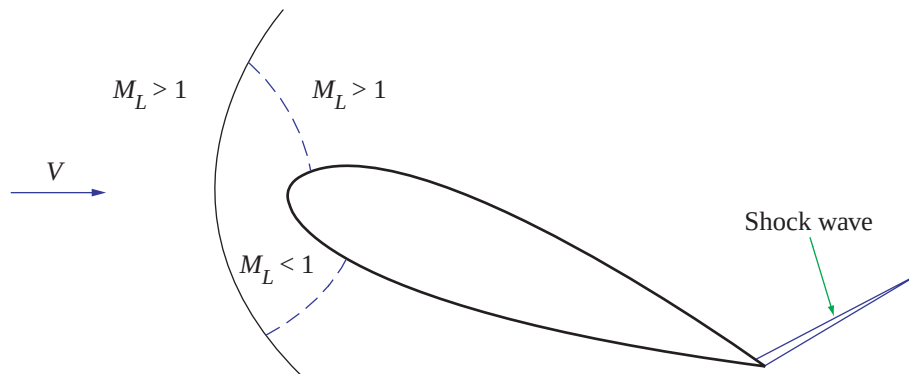
Transonic flows are outside the scope of this Item, but it is helpful to mention some of their characteristics in unsteady flow. The oscillatory pressure distributions depend in a complex manner on aerofoil shape, thickness and mean incidence. There is the likelihood of important amplitude effects, since harmonic oscillations of the aerofoil may cause the shock waves to move non-sinusoidally, or even to detach and move freely upstream!

These non-linear effects are expected to lessen at the higher frequencies which constrain the movement of shock waves. For any given mean flow, linearization for small amplitudes is permissible. References 27 and 30 are recommended for further reading on steady and unsteady transonic flows.

3.3.2 Supersonic flow

It is axiomatic that on a round-nosed aerofoil, however high the stream Mach number, there must always be a stagnation point close to the leading edge. It therefore follows that when the stream is supersonic there is always a local embedded region of subsonic flow (Sketch 3.10). At low supersonic speeds the subsonic region will be quite large, and the classical theories for supersonic flow will not be applicable. With increasing stream Mach number the local subsonic region diminishes in size, and for $M > 1.1$ the theories begin to have practical application. The theories are also subject to an upper limit on Mach number, beyond which air can no longer be regarded as a continuum.

For an aerofoil at incidence or with a non-zero included trailing-edge angle shock waves will emanate from the trailing edge, and in steady motion these redirect the flows above and below the aerofoil. If the angle through which the flow must be turned is sufficiently large, boundary-layer separation will start to occur from the compression corner at the foot of the shock wave (Reference 26). As with subsonic flow, extensive regions of separated flow will invalidate the theories to be considered in Section 7.



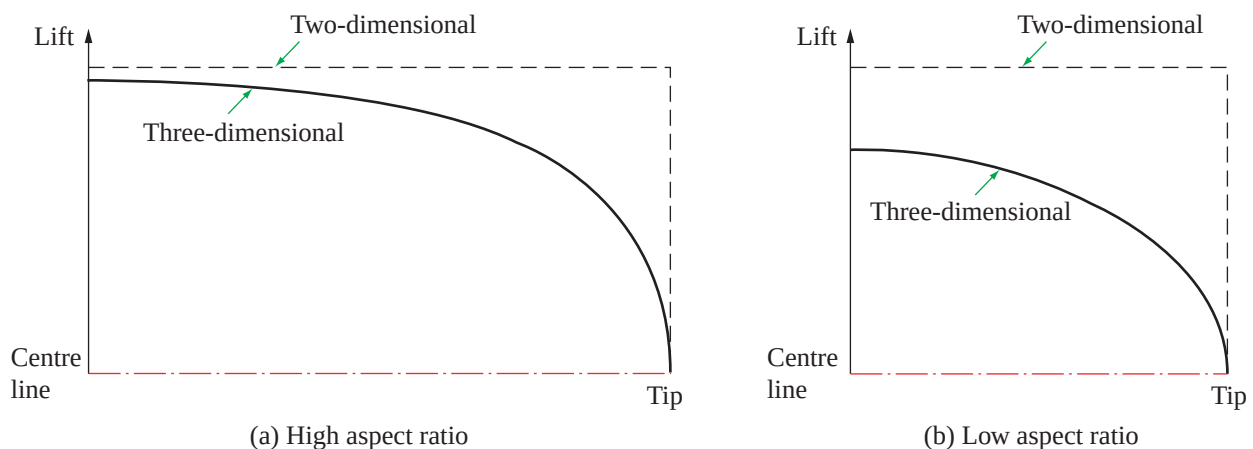
Sketch 3.10 Low supersonic flow

3.4 Three-dimensional Flows

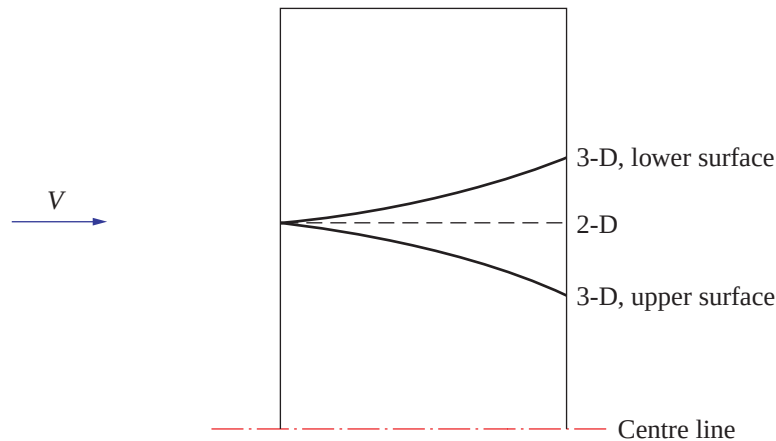
For swept wings of finite span, root and tip effects cause the flow to depart from the idealized concepts of two-dimensional flow. At a streamwise wing tip there can be no pressure difference between the upper and lower surfaces. This is true in both steady and unsteady flows, whether the flight speed is subsonic or supersonic. The spanwise distribution of lift on rectangular wings in steady flow can be compared with the value for a two-dimensional aerofoil (Sketch 3.11).

As aspect ratio is decreased the flow bears less and less relationship to two-dimensional flow and the lift-producing efficiency of the wing is reduced.

The reduction in lift between wing centre-line and tip implies a pressure differential across the span. This induces a cross-flow from high to low pressure areas, so that the three-dimensional streamlines near the surface will be inclined spanwise whereas the two-dimensional streamlines, by definition, are chordwise (Sketch 3.12). The flow in the wake is now complicated by streamwise vorticity. Where the theories are applicable it suffices to assume that the wake remains planar without rolling-up of trailing vortices. In supersonic flow, of course, the wake cannot influence surface pressures on rectangular wings.



Sketch 3.11 Spanwise lift distribution



Sketch 3.12 Surface streamlines

Sweepback of the leading edge of the planform introduces further three-dimensional effects. If the sweepback is high enough, linearized theories can be invalidated by the formation of leading-edge vortices at incidences well below the stall.

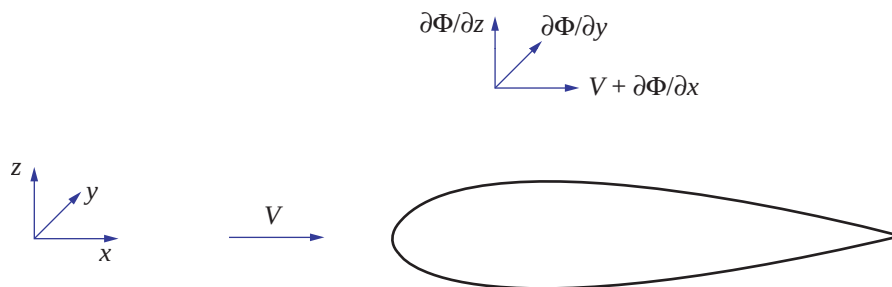
3.5 Linearization

When excited, an aircraft, like any other structure, will respond in a combination of vibration modes, each mode having its own deflection shape and frequency. Structures may be considered to respond linearly and sinusoidally to small disturbances. This then allows the superposition of several simple modal deflections to form a complex vibration mode. If the principle of superposition is to be applied to aeroelastic calculations the unsteady aerodynamics must also be linear and sinusoidal.

Outside the transonic regime it is permissible to regard the unsteady aerodynamics as independent of the aerofoil section and the mean values of incidence and control angle.

To formulate unsteady aerodynamics which are both linear and independent of the steady flow field, the following assumptions have to be made.

- (i) The thickness of the viscous boundary layer is small enough to be ignored (see Section 3.7).
- (ii) Flow outside the vortex sheet representing the wing and wake is irrotational and isentropic. Consequently a potential, $\Phi(x, y, z, t)$, exists whose spatial derivatives describe the perturbation velocities (Sketch 3.13) at any point in space.



Sketch 3.13 Components of velocity

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- (iii) The incidence is small enough to maintain attached flow.
- (iv) Wing motion can be represented as sinusoidal about the plane $z = 0$. Thus the position of the wing surface at any time comprises a symmetrical, non-lifting thick wing, an infinitesimally thin curved plate, and an oscillating flat plate (Sketch 3.14).
- (v) The aerodynamics due to sinusoidal motion are likewise sinusoidal, e.g.,

$$\Phi(x, y, z, t) = \Phi_0(x, y, z) + \mathcal{R}[\phi(x, y, z)e^{i\omega t}] \quad (3.3)$$

where ω is the frequency of oscillation, and Φ_0 and ϕ represent the steady and oscillatory parts of the velocity potential.

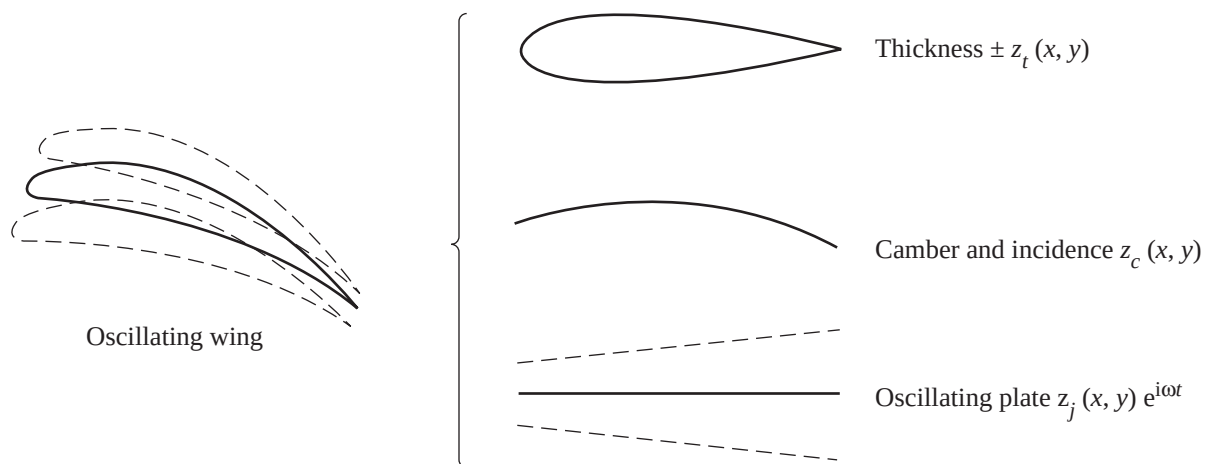
- (vi) Amplitude of oscillation is only considered to first order.
- (vii) The flow is not transonic.

With these assumptions, the general equations for describing the oscillatory part of the flow field reduce to

$$(1 - M^2) \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} - \frac{2i\nu M}{d} \frac{\partial \phi}{\partial x} + \frac{\nu^2 M^2}{d^2} \phi = 0 \quad (3.4)$$

where d is the reference length and the frequency parameter ν is given by

$$\nu = \frac{\omega d}{V}. \quad (3.5)$$



Sketch 3.14 Location of wing surface

Equation (3.4) is linear in ϕ and contains no terms relating to the steady velocity potential, Φ_0 . Thus the wing is regarded as an oscillating flat plate at zero incidence. The oscillatory potential ϕ is subject to the boundary condition that flow is tangential to the oscillating surface, i.e.,

$$\frac{1}{V} \frac{\partial \phi}{\partial z} = \frac{dz_j}{dx} + \frac{ivz_j}{d}, \quad (3.6)$$

which likewise is independent of the mean flow. Thus, apart from the shape and size of the planform the unsteady aerodynamics depend only on

M – free-stream Mach number,

v – frequency parameter,

and z_j – modal deflection shape of the surface.

The non-dimensional pressure difference across the surface is given by

$$\Delta C_p = \frac{2}{V} \left(\frac{\partial}{\partial x} + \frac{iv}{d} \right) (\phi_u - \phi_l), \quad (3.7)$$

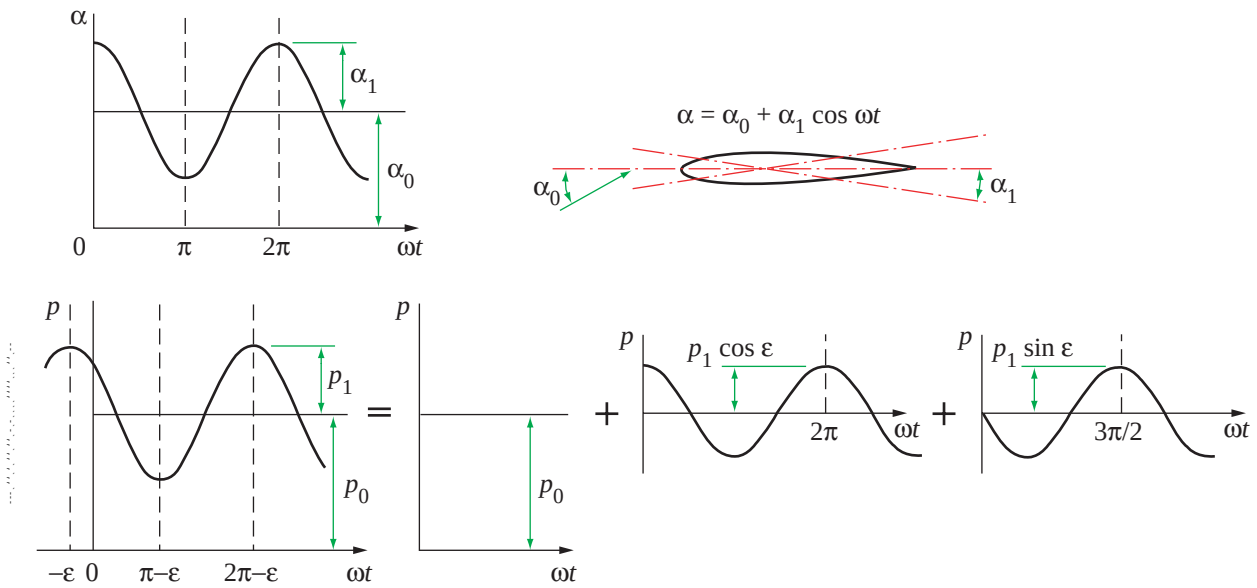
where the subscripts u and l denote the upper and lower surfaces respectively. It may be noted that ΔC_p can be dimensionalized by the kinetic pressure to give the lift per unit area

$$\Delta p = \frac{1}{2} \rho V^2 \Delta C_p. \quad (3.8)$$

Solution of Equation (3.4) with the boundary condition of Equation (3.6) gives the oscillatory aerodynamic response to oscillatory motion at specified frequency parameters. Because of the linearization, the results can be transformed by superposition to generate aerodynamics for a general time-dependent motion.

3.6 Description of Unsteady Pressures

Whereas surface pressures in steady flow are completely described by their magnitude, the unsteady pressure or loading due to some oscillatory aerofoil motion is described by both magnitude and phase angle (Sketch 3.15). In an equivalent representation of the unsteady pressure as a complex number, the real part is in phase with the rotational aerofoil motion, denoted by α , and the imaginary part is in quadrature, i.e., $\pi/2$ or 90° out of phase.



Sketch 3.15 Oscillatory incidence and pressure

When the pressure is described in terms of magnitude and phase angle,

$$p = p_0 + p_1 \cos(\omega t + \epsilon) \quad (3.9)$$

$$= p_0 + (p_1 \cos \epsilon) \cos \omega t - (p_1 \sin \epsilon) \sin \omega t, \quad (3.10)$$

where p_1 is the magnitude and ϵ is the phase angle. When ϵ is positive, as in Sketch 3.15, the oscillatory pressure leads the motion. The pressure is also described as the real part of a complex number

$$p = p_0 + p_1 \Re[e^{i(\omega t + \epsilon)}] \quad (3.11)$$

$$= p_0 + \Re[\bar{p} e^{i\omega t}] \quad (3.12)$$

where the real and imaginary parts of the pressure perturbation $\bar{p}$ are

$$\bar{p}' = p_1 \cos \epsilon \quad (3.13)$$

and
$$\bar{p}'' = p_1 \sin \epsilon. \quad (3.14)$$

Thus the real part is the instantaneous value of $(p - p_0)$ when α is a maximum and the imaginary part is the value as the aerofoil passes through its mean position and $d\alpha/dt$ is a maximum.

The phase angle is expected to be highly dependent on the compressibility of air. A compressible fluid will propagate disturbances at a finite velocity V/M .