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Pressure loss and heat transfer for single-phase turbulent flow in roughened channels: methods of calculation

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(Continued on inside back cover)

PRESSURE LOSS AND HEAT TRANSFER FOR SINGLE-PHASE TURBULENT FLOW IN ROUGHENED CHANNELS: METHODS OF CALCULATION

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PRESSURE LOSS AND HEAT TRANSFER FOR SINGLE-PHASE TURBULENT FLOW IN ROUGHENED CHANNELS: methods of calculation

1. NOTATION, UNITS AND DEFINITIONS

1.1 Notation and Units

		SI Unit
a_f	total cross-sectional flow area bounded by a nominal surface (see Sketch 1.1)	m^2
a_h	total heat transfer area of nominal surface (see Sketch 1.1)	m^2
A_f	roughness parameter for friction loss, u/u_* at $y = e$	—
A_h	roughness parameter for heat transfer, $(T_w - T)/t_*$ at $y = e$	—
c_p	specific heat capacity at constant pressure	$\text{J}/(\text{kg K})$
D	diameter	m
D_e	equivalent diameter of channel (see Equation (1.1) and Table 4.1)	m
e, e_e	roughness height and equivalent roughness height respectively	m
$\bar{e}$	centre-line-average roughness height (see Sketch 1.3)	m
e_+	roughness Reynolds number, $eu_*/\nu = Re_b(e/D_e)\sqrt{f/2}$	—
F_n, F'_n, F''_n	arbitrarily-defined functions	—
f	Fanning friction factor, $2\tau_w/(\rho\bar{u}^2)$	—
G'	geometry parameter	—
L	length of channel	m
l	roughness element spacing (pitch) (see Sketch 1.3)	m^{-1}
$\dot{M}$	mass flow-rate	kg/s
N_{TU}	number of heat transfer units (see Equation (1.7a))	—
Nu	Nusselt number, $\alpha D_e/\lambda$	—
n	factor by which surface roughening increases heat transfer (see Section 3)	—

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Pr	Prandtl number, $\frac{c_p \mu}{\lambda}$	–
p	pressure	N/m ²
Δp	pressure loss due to friction	N/m ²
$\dot{Q}$	heat transfer rate	W
$\dot{q}$	heat flux	W/m ²
r	radius of curvature of surface (see Sketch 1.4)	m
Re	Reynolds number, $\rho D_e \bar{u} / \mu = D_e \bar{u} / \nu$	–
St	Stanton number, $\alpha / (\rho \bar{u} c_p)$	–
T	temperature	K
t_+	non-dimensionalised temperature, $(T_w - T) / t_*$	–
$\bar{t}_+$	non-dimensionalised mean temperature, $(\overline{T_w - T}) / t_* = 1 / (\bar{St}_w \bar{u}_+)$	–
t_*	friction temperature, $\dot{q} \sqrt{2/f} / (\rho \bar{u} c_p)$	K
u	velocity	m/s
u_*	friction velocity, $\sqrt{\tau_w / \rho} = \sqrt{f \bar{u}^2 / 2} = \bar{u} \sqrt{f/2}$	m/s
u_+	non-dimensionalised velocity, u / u_*	–
$\bar{u}_+$	non-dimensionalised mean velocity, $\bar{u} / u_* = \sqrt{2/f}$	–
$\dot{V}$	volumetric flow rate	m ³ /s
w	roughness element width (see Sketch 1.3)	m
x	axial distance along channel	m
Y	parameter defined by Equation (4.4)	–
Y'	value of Y when $e_+ \geq 50$	–
y	distance normal to channel wall measured from root of roughness element	m
$\bar{y}$	normal distance between channel wall and no-shear surface (see Sketch A1.1)	m
z	radial distance	m
α	heat transfer coefficient	W/(m ² K)

λ	thermal conductivity	W/(mK)
ε	eddy diffusivity	m ² /s
μ	dynamic viscosity	Ns/m ²
ν	kinematic viscosity	m ² /s
ρ	density	kg/m ³
τ	local shear stress	N/m ²
τ_w	shear stress at the wall, $\tau_w = (D_e/4)\Delta p/\Delta x = \frac{1}{2}\rho\bar{u}^2 f = \rho u_*^2$	N/m ²

Subscripts

b	bulk fluid value
f	value for friction loss
h	value for heat transfer
i	wall in annulus at which heat transfer is considered, $i = 1$ for inner wall and $i = 2$ for outer wall
in	inlet conditions
out	outlet conditions
ref	reference conditions
s	smooth surface conditions
w	conditions at wall
x	axial distance along channel
$+$	non-dimensionalised values (see main notation)
0	conditions at $x = 0$

A bar ($\bar{\quad}$) indicates an appropriate mean value.

1.2 Definitions

1.2.1 Equivalent diameter

The equivalent diameter of a flow channel, D_e , is defined as

$$D_e = \frac{4 \times \text{flow cross-sectional area}}{\text{wetted perimeter}} = \frac{4a_f}{(a_h/L)}. \quad (1.1)$$

Rearranging Equation (1.1) gives

$$D_e/4L = a_f/a_h. \quad (1.1a)$$

Values of the equivalent diameter for various channel geometries are given in Table 4.1.

1.2.2 Pressure loss and friction factor

The pressure loss in straight channels resulting from the resistance to flow due to shear stresses induced at the flow channel wall is given by

$$\frac{\Delta p}{\Delta x} = \frac{4}{D_e} (\frac{1}{2} \rho \bar{u}^2) f \quad (1.2)$$

or Equation (1.2) can be written as

$$\frac{\Delta p}{\Delta x} = f (\frac{1}{2} \rho \bar{u}^2) / (D_e/4)$$

where Δx is the length over which the pressure loss, Δp , occurs, and f , the Fanning friction factor, is used to characterise the resistance to flow.

The Fanning friction factor is defined by

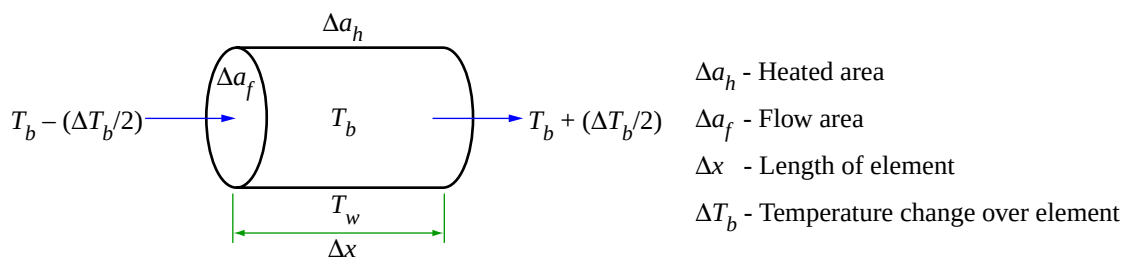
$$f = \tau_w / (\frac{1}{2} \rho \bar{u}^2). \quad (1.3)$$

1.2.3 Mean Stanton number, Nusselt number and N_{TU}

The effective mean Stanton number at the wall at which heat transfer is evaluated is defined as

$$\bar{St}_w = \frac{\bar{\alpha}_w}{\rho \bar{u} c_p} = \frac{\dot{Q}}{(T_w - T_b) a_h \bar{\rho} \bar{u} \bar{c}_p}. \quad (1.4)$$

A heat balance for an element, Δx , of the system (see Sketch 1.1) is obtained by equating the enthalpy change in the fluid to the heat transfer from the surface.



Sketch 1.1 Element over which heat balance is obtained

Hence

$$\Delta a_f \rho \bar{u} c_p \frac{\Delta T_b}{\Delta x} = \bar{\alpha}_w \frac{\Delta a_h}{\Delta x} (T_w - T_b). \quad (1.5)$$

Equation (1.5) can be integrated to give

$$\overline{St}_w = \frac{(T_{b, out} - T_{b, in})a_f}{(\overline{T}_w - \overline{T}_b)a_h} \quad (1.6)$$

where $(\overline{T}_w - \overline{T}_b)$ is the appropriate mean temperature driving force.

Alternatively, the mean Nusselt number $(\overline{Nu} = \overline{St} \times Re_b \times Pr_b)$ may be used to characterise heat transfer.

Equation (1.6) can be written as

$$\overline{St}_w \frac{a_h}{a_f} = \frac{T_{b, out} - T_{b, in}}{(\overline{T}_w - \overline{T}_b)} = N_{TU} \quad (1.7a)$$

which reduces to

$$\overline{St}_w = N_{TU} / (D_e / 4L) \quad (1.7b)$$

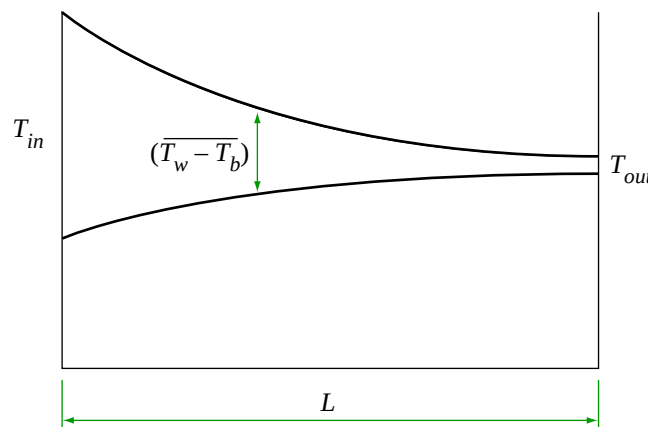
where N_{TU} (number of transfer units) indicates the change in fluid temperature per unit temperature driving force and is a measure of the effectiveness of the heat exchanger as a means of changing the fluid temperatures.

1.2.4 Heat transfer rate

Heat transfer is defined as positive when directed into the fluid. The heat transfer rate over the total length, L , of the channel is given by

$$\dot{Q} = \overline{\alpha}_w a_h (\overline{T}_w - \overline{T}_b) \quad (1.8)$$

where $(\overline{T}_w - \overline{T}_b)$ is the appropriate effective mean temperature difference (see Sketch 1.2) and a_h is the heated area over the length of the channel.



Sketch 1.2 Key temperatures in a co-current heat exchanger

1.2.5 Bulk fluid temperature

The data in this Item apply for fluid properties evaluated at the bulk temperature.

The local value of the fluid temperature, T , is a function of z and y . The local bulk temperature at any axial station x , $T_{b,x}$, is the integrated mean of the local fluid temperature over the channel cross-section. At any station along the channel it is given by

$$T_{b,x} = T_{b,0} + \int_{x=0}^x \frac{4\dot{q}}{\rho \bar{u} c_p D_e} dx \quad (1.9)$$

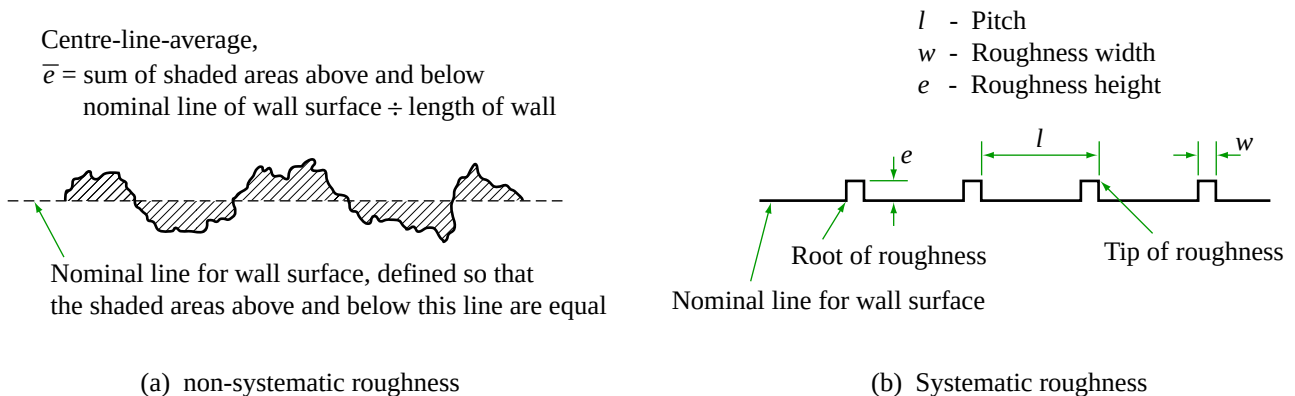
where $T_{b,x}$ is the local value of T_b at a distance x from the initial reference point, $T_{b,0}$ is the value of T_b at $x = 0$ and the integral term is the bulk temperature change in the fluid over distance x from the initial reference point.

1.2.6 Surface roughness

A measure of the relative roughness in a channel is given by e/D_e . Sketch 1.3 gives examples of rough surfaces and the dimensions necessary to characterise them. In the case of non-systematic roughness it is convenient to use the centre-line-average value, $\bar{e}$, to define the roughness height.

Centre-line-average,

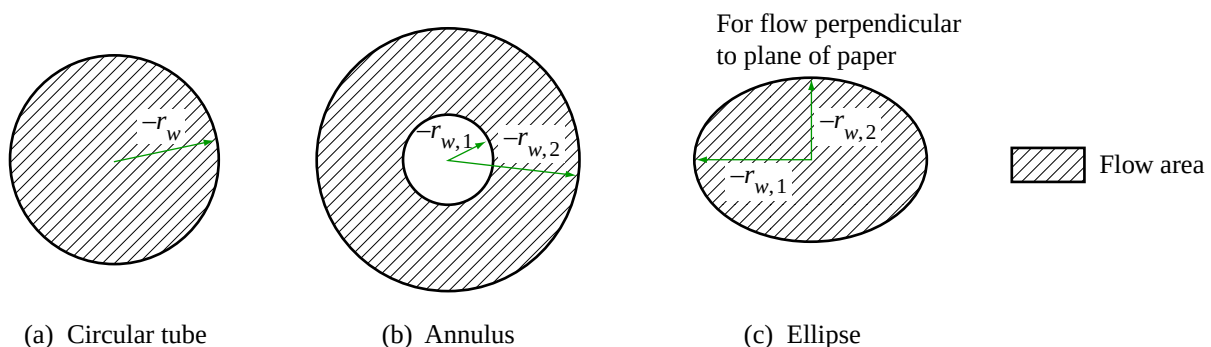
$\bar{e}$ = sum of shaded areas above and below
nominal line of wall surface ÷ length of wall



Sketch 1.3 Examples of roughened surfaces and their characteristic dimensions

1.2.7 Radius of curvature of heat transfer surface

The radius of curvature of the heat transfer surface is defined as positive if the wetted side of the surface is convex. Sketch 1.4 illustrates the radius of curvature of the heat transfer surface in typical channels of conventional geometry.



Sketch 1.4 Radius of curvature for typical channels

2. PURPOSE, SCOPE AND APPLICABILITY

2.1 Purpose and Scope

This Data Item provides a method and data for evaluating the heat transfer and pressure loss for turbulent axial flow in roughened channels. The various types of surface roughness elements for which data are presented are summarised in Table 4.2 and the various channel geometries that can be dealt with directly are described by Table 4.1.

The principal application of this Data Item is to heat exchanger surface design and the potential advantages of surface roughening are explained in Section 3. Methods for calculating the pressure loss and heat transfer are given in Section 4, which contains the following procedures:

- (i) The direct calculation of pressure loss and heat transfer data for roughened surfaces with geometries and roughness shapes specified in Tables 4.1 and 4.2.
- (ii) The estimation of heat transfer data using experimental pressure loss data for any roughness type in the channel geometries described by Table 4.1. This technique is particularly useful because it obviates the need for direct measurement of heat transfer data which are more difficult to obtain experimentally than pressure loss data.
- (iii) The second part of procedure (ii) may be used to extrapolate pressure loss data for a specified roughness height and channel geometry, to obtain heat transfer or pressure loss data for other roughness heights or channel geometries.

The accuracy of the data obtained using these procedures is discussed in Section 5. Calculation procedures and flow charts are also given in Section 5.

A worked example illustrating the use of this Item is given in Appendix B.

In this Item, the term roughness is used to include both naturally occurring and artificially introduced elements. The increase in surface area resulting from roughening is usually small but significant and contributes to the improved heat transfer. This effect is included in the data and is not accounted for separately in the calculation method. (This is in contrast with the heat transfer on finned surfaces where the extended area is a predominant factor and has to be accounted for separately.) The method is based on data for roughness heights less than 4 per cent of the channel equivalent diameter and should not be extrapolated to roughness heights greater than 5 per cent of the channel equivalent.

2.2 Applicability

The data are strictly applicable to:

- (i) channels which can be divided into elements in which uniform axial and circumferential heat flux conditions exist^{*},
- (ii) channels with identical roughness types on all wetted surfaces,
- (iii) most conventional channel geometries (see Table 4.1) except those including narrow regions such as occur with very closely packed tubes or triangular passages,
- (iv) Newtonian single-phase fluids,
- (v) heat transfer with negligible thermal radiation and conduction and zero heat generation within the fluid,
- (vi) forced-convective flow with negligible free-convection effects,
- (vii) clean surfaces and fluids.

3. THE USE OF ROUGHENED SURFACES

3.1 Advantages

The deliberate use of surface roughness in heat exchangers introduces a further design parameter and produces a range of design options from which those most suitable to requirements may be selected. In particular, for a given heat duty, surface roughness may be effectively used to maximise heat transfer surface per unit pressure loss, or to minimise pressure loss in the equipment.

An increase in surface roughness will increase both the heat transfer coefficient (measured by Stanton number) and the friction factor. Experimental data show that, to a first approximation, if roughening the heat transfer surface results in an increase in Stanton number by a factor n , the corresponding increase in friction factor is by a factor n^2 .

The key parameters in heat exchanger design are the wall and bulk fluid temperatures, T_w and T_b , the flow cross-sectional area, a_f , and the heat transfer surface area, a_h . The Stanton number is related to these parameters by

$$\overline{St}_w \frac{a_h}{a_f} = \frac{T_{b, out} - T_{b, in}}{T_w - T_b} = N_{TU}. \quad (3.1)$$

* Most experimental data which have been used to validate the correlations presented have been obtained from measurements on channels with uniform axial and circumferential heat flux conditions but this does not limit the application of the data. The effect of heat flux variation along the length of the tube on the local heat transfer coefficient can be assessed by considering the parameter

$$Z = \frac{1}{\dot{q}} \frac{d\dot{q}}{d(x/D_e)}.$$

If $Z \leq 0.1$, over the entire length x of the heat transfer surface, with the exception of points at the commencement and completion of heat transfer, then the effect of variable heat flux distribution over the length x may be neglected. The correlations presented may then be applied directly to the element considered.

If, as may be the case in nuclear reactors, $Z > 0.1$, the method of superposition described in Reference 8 should be used.

The pumping energy per unit volume flow-rate is equal to the pressure loss across the equipment. Assuming the majority of this loss is due to friction^{*}, the pressure loss in the equipment is given by

$$\Delta p = 4f \frac{L}{D_e} \rho \frac{\bar{u}^2}{2} \quad (3.2)$$

where f is the Fanning friction factor, and $\bar{u}$ the mean fluid velocity.

The pressure loss can also be related to the volumetric flow-rate by Equation (3.3)

$$\Delta p = f \frac{\dot{V}^2 \rho}{2} \frac{a_h}{a_f^3}. \quad (3.3)$$

The effects of an increase in surface roughness such that $St/St_s = n$ and $f/f_s = n^2$ on the key parameters in Equations (3.1) and (3.3) are tabulated in Table 3.1.

Table 3.1 gives the main alternatives for achieving a given performance. These alternatives are obtained by keeping volumetric flow-rate constant and varying one or more of the key parameters in Equations (3.1) and (3.3).

TABLE 3.1 Effect of Surface Roughening on Heat Exchanger Design Parameters (assuming ρ and $\dot{V}$ are constant)

Option	Description of changes relative to "smooth surface" heat exchanger	Effect on parameters			
		N_{TU}	a_h	a_f	Δp
(i)	Decrease heat transfer surface, a_h , keeping N_{TU} and flow area a_f fixed at the expense of increased Δp .	1	$\frac{1}{n}$	1	n
(ii)	Increase flow area, a_f , to reduce Δp , keeping N_{TU} and heat transfer surface, a_h fixed.	1	1	n	$\frac{1}{n}$
(iii)	Increase N_{TU} keeping flow area, a_f , and heat transfer surface, a_h , fixed at expense of increased Δp .	n	1	1	n^2
(iv)	Decrease heat transfer surface keeping N_{TU} fixed and keep Δp fixed by simultaneously increasing flow area, a_f	1	$\frac{1}{\sqrt{n}}$	$\sqrt{n}$	1

Combinations of the options in Table 3.1 are also possible.

Option (i) is pursued in order to minimise heat transfer surface area, and hence probably minimise capital cost provided the increased pressure loss is not a significant penalty. (For pumping of liquids, increased pressure loss due to surface roughening is not usually significant because $\dot{V}$ is small.) This allows complete flexibility in altering either tube length, or tube perimeter, or number of tubes or any combination of these possibilities provided that required values of a_f and a_h are achieved.

* This pressure loss (calculated in this Data Item) is due to friction for fully-developed turbulent flow over straight surfaces. The additional pressure loss in inlet and outlet regions and bends has to be accounted for separately (see References 5 to 7).

Option (ii) is of particular benefit in cases where pumping costs are significant. The use of option (ii) demonstrates that using a roughened heat transfer surface need not necessarily increase pressure loss in the equipment. (Roughened surfaces were developed in the nuclear energy industry for this very purpose – to reduce gas pumping power in the Advanced Gas-cooled Reactor.) Another important use of option (ii) is that the resulting increased flow-area allows wider spacing between the heat transfer elements in the heat exchanger. This increases the tolerances on tube location and may simplify design and assembly in many applications.

Option (iii) can be used to increase the temperature change in the process fluid. Alternatively it allows the use of a lower driving temperature difference ($T_w - T_b$) which improves the thermodynamic efficiency of the process (less entropy loss) by bringing the wall temperature closer to the bulk fluid temperature either by reducing T_w or increasing T_b . Reduction in the wall temperature when heating the fluid would reduce the risk of degradation in heat-sensitive fluids, and may also permit the use of cheaper construction materials. An increase in wall temperature when cooling may permit more economic or more readily available coolants to be used. (Option (iii) has been used in de-superheaters in order to extract only sensible heat from superheated gas.)

Option (iv) represents the designer's best compromise when faced with a given process duty in terms of fixed N_{TU} and fixed Δp . By using a roughened heat transfer surface in the heat exchanger, the designer is also able to reduce the heat transfer surface area to $1/\sqrt{n}$ of its "smooth surface" value. The resulting exchanger is more compact (this is important if space is limited) and possibly also cheaper. (This option has been used in water chillers to reduce both the volume and weight of the equipment.)

Data given in Figure 3 show that rough surfaces can be designed to give a value of n of up to about 3.5.

3.2 Caveats

Whenever the performance of one side of a heat exchanger is altered, it will affect the other side also. Thus, if the heat transfer surface area or flow cross-sectional area is changed, consideration must be given to the effect on the heat transfer and pressure loss of the other-side-fluid.

In general the beneficial effects of roughening on heat transfer are only significant provided other heat transfer resistances within the heat exchanger are relatively small. The other heat transfer resistances that must be considered include not only those of the other-side-fluid but also those associated with fouling and wall resistance. (This means that clean surfaces and clean fluids are particularly important.)

The thermal bond between the roughness and the surface must be good otherwise the contact resistance will be high.

4. DATA FOR PRESSURE LOSS AND HEAT TRANSFER IN ROUGHENED CHANNELS

4.1 Preamble

Hitherto the effects of a roughened surface have generally been assessed relative to those of a surface covered with sand grains. The characteristic dimension quantifying this roughness is the “equivalent sand-grain roughness height” which is defined as the sand-grain height which will produce the same effect on the pressure drop (measured by friction factor) as the surface being assessed. The equivalent sand-grain roughness has served to quantify naturally rough surfaces very well (see Reference 3). However, when used to quantify surfaces with systematic roughness elements, unrealistic values of the equivalent sand-grain-roughness height (e_e) are obtained ($e_e/D_e > 0.1$). The methods presented in this Item which apply for both pressure drop and heat transfer are based on characteristic dimensions of the roughness being assessed and explicitly take account of

- (i) actual height of roughness element,
- (ii) actual shape of roughness element,
- (iii) channel shape.

Equations have been developed (see Appendix A and Reference 1) giving pressure drop and heat transfer (measured by f and St respectively) for fully-developed turbulent flow over roughened surfaces. These are:

$$\sqrt{\frac{2}{f}} = \bar{u}_+ = 2.39 \ln \frac{D_e}{e} + A_f - G', \quad (4.1)$$

$$\frac{1}{St_w \sqrt{2/f}} = \bar{t}_+ = 2.39 \ln \frac{D_e}{e} + A_h - G'. \quad (4.2)$$

The term D_e/e takes into account the actual roughness height. The terms A_f and A_h are known as roughness parameters* and account for the effect of roughness shape on pressure drop and heat transfer respectively. The term G' is known as the geometry parameter† and accounts for the channel cross-sectional shape.

The use of roughness and geometry parameters simplifies the presentation of the data considerably and obviates the numerous carpet plots that would otherwise have to be produced to show the effect of varying each of the parameters in Equations (4.1) and (4.2) on friction factor and Stanton number.

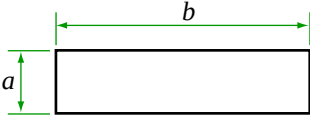
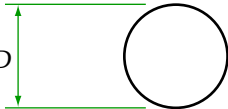
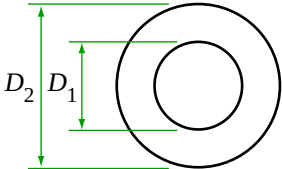
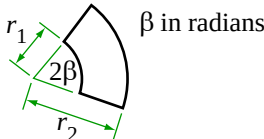
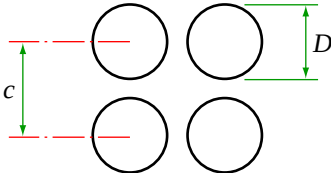
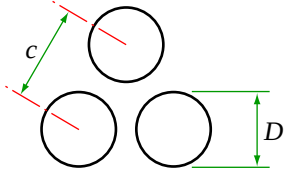
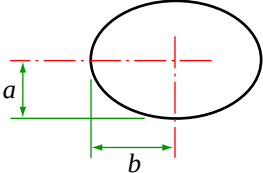
* A_f and A_h represent the non-dimensionalised velocity and temperature characteristics respectively at approximately the roughness height. These characteristics can be regarded as the boundary conditions of flow near the rough surface. Roughness parameters are, in general, independent of the channel cross-section if $e/D_e \leq 0.05$.

† Values of the geometry parameter have been obtained by integration of the velocity and temperature profiles in various channel cross-sectional shapes and are a measure of the overall effect of channel shape on momentum and heat transfer.

4.2 Equivalent Diameter

Table 4.1 provides expressions giving the equivalent diameter for channel geometries to which the methods and data presented are applicable.

TABLE 4.1 D_e for Various Channel Cross-sections

Channel geometry		D_e
Rectangular channel		$2ab/(a+b)$
Circular tube		D
Annulus (both surfaces similarly roughened)		$D_2 - D_1$
Section of an annulus $\beta > 0.2$		$\frac{2\beta(r_2^2 - r_1^2)}{\beta(r_2 + r_1) + 2(r_2 - r_1)}$
Tube bundle, square pitch $c > 1.25D$		$\frac{4c^2}{\pi D} - D$
Tube bundle, equilateral triangle pitch $c > 1.25D$		$2\frac{\sqrt{3}c^2}{\pi D} - D$
Elliptical tube $b < 1.2a$		$\frac{2\sqrt{2}ab}{\sqrt{a^2 + b^2}}$

4.3 Pressure Loss Data

As derived in Appendix A and discussed in Section 4.1, the friction factor in a channel with surface roughness elements of height e is given by

$$\sqrt{\frac{2}{f}} = 2.39 \ln\left(\frac{D_e}{e}\right) + A_f - G' \quad (4.3)$$