

APPROXIMATE PARAMETRIC METHOD FOR PROPELLER THRUST ESTIMATION

1. NOTATION AND UNITS

Although the notation for this Item has been presented here in terms of SI and British units, the working Figures and Sketches are presented in **British units only** since this is the most commonly used system of units for propeller performance estimation. However, Table 12.1 of the Item contains all the conversion factors necessary to express the results of the method in SI and other systems of units.

		SI	British
AF	activity factor, per blade, $\frac{10^5}{D^5} \int_{0.2R}^R cr^3 dr$, see Section 4		
c	propeller blade chord at distance r from axis of rotation	m	ft
D	propeller diameter	m	ft
K	correction factor, $T_s/T_{s_{350}}$		
N	propeller rotational speed	rev/min	rev/min
N_{max}	propeller maximum rotational speed	rev/min	rev/min
n	propeller rotational speed (see Section 5)	rev/s	rev/s
n_b	number of blades per propeller		
P	power supplied to propeller	kW	hp
R	propeller radius at tip (= $D/2$)	m	ft
r	radial measurements along blade	m	ft
T	propeller-alone thrust	N	lbf
TAF	total activity factor, $n_b \times AF$		
V	true airspeed	km/h, kn	kn
V_{tip}	propeller tip speed, $\frac{\pi}{60}ND$, πnD	m/s	ft/s
ΔN	difference between propeller maximum rotational speed and rotational speed at speed V , ($N_{max} - N$)	rev/min	rev/min
ρ	density of air	kg/m ³	slug/ft ³
σ	relative density, ρ/ρ_o		

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η propeller efficiency

Subscripts

N_{max} denotes propeller maximum rotational speed

s denotes static conditions

TAF denotes total activity factor

0 denotes value at sea level in International Standard Atmosphere, ISA

350 denotes value for a propeller with $TAF = 350$

2. INTRODUCTION

The propeller thrust estimation method presented in this Item provides an approximate means of determining the “uninstalled” static thrust and the thrust lapse characteristics for a wide variety of constant-speed propellers. The method is based upon data derived from the results of a computer program from Derivation 1 and provides estimates of thrust for propellers operating at airspeeds up to 300 kn, and for which $2 \leq P/\sigma D^2 \leq 16$ hp/ft² ($16 \leq P/\sigma D^2 \leq 128$ kW/m²) and $700 \leq V_{tip} \leq 1000$ ft/s ($200 \leq V_{tip} \leq 300$ m/s). The Item is intended for use in project estimations and is particularly suited to aeroplanes in the light and general aviation categories. The data from Figures 1 to 6 were used in developing the take-off distance estimation method of Item No. 82033.

Addendum A, issued as Item No. 83028, gives a method for estimating the thrust of fixed-pitch propellers utilising the data of Figures 1 to 6 of the present Item.

3. METHOD OF ESTIMATION

Figure 1 presents static thrust to power ratio, $(T/P)_{s_{350}}$, as a function of the disc loading parameter, $P/\sigma D^2$, and tip speed, V_{tip} , for a propeller having a total activity factor, $TAF = 350$. The activity factor per blade, AF , is a function of the chordal distribution of the propeller blade, from root to tip (see Section 4). The values of $(T/P)_s$ for other values of TAF are determined from the correction charts of Figures 2 to 5, for various values of V_{tip} , Figure 6 shows thrust lapse, T/T_s in terms of V and $(T/P)_s$.

The procedure for estimating thrust is as follows:

- (i) Calculate the propeller tip speed

$$V_{tip} = \frac{\pi}{60} ND,$$

and the disc loading parameter,

$$\frac{P}{\sigma D^2}.$$

- (ii) Determine the value of $(T/P)_{s_{350}}$ from Figure 1.

- (iii) Select a value of TAF (based upon the criteria described in Section 4 and shown in Sketch 4.1). If $\frac{TAF}{350} \neq 1$, read the value of the correction factor, K , from Figures 2, 3, 4 or 5, as appropriate, then correct the value of $(T/P)_{s_{350}}$ obtained in (ii) to that for the value of TAF chosen,

$$(T/P)_s = (T/P)_{s_{350}} \times K. \quad (3.1)$$

- (iv) The static thrust, T_s , is given by:

$$T_s = (T/P)_s \times P. \quad (3.2)$$

- (v) For the required forward speed and value of $(T/P)_s$ estimated in (iv), determine the value of T/T_s from Figure 6. The thrust at speed V is:

$$T = T/T_s \times T_s. \quad (3.3)$$

4. ACTIVITY FACTOR

The activity factor, AF , is a function of the distribution of chord in a propeller blade from root to tip and is defined here* (as in Derivation 1 and Reference 2) by

$$AF = \frac{10^5}{D^5} \int_{0.2R}^R cr^3 dr \quad (4.1)$$

or
$$AF = \frac{10^5}{16} \int_{0.2}^{1.0} \frac{c}{D} \left(\frac{r}{R}\right)^3 d\left(\frac{r}{R}\right). \quad (4.2)$$

4.1 Effect of Blade Planform

Sketch 4.1 shows a variety of propeller blade planforms of length $0.8R$, and their associated activity factors. For any propeller blade, the torque and hence the power required for rotation is given by,

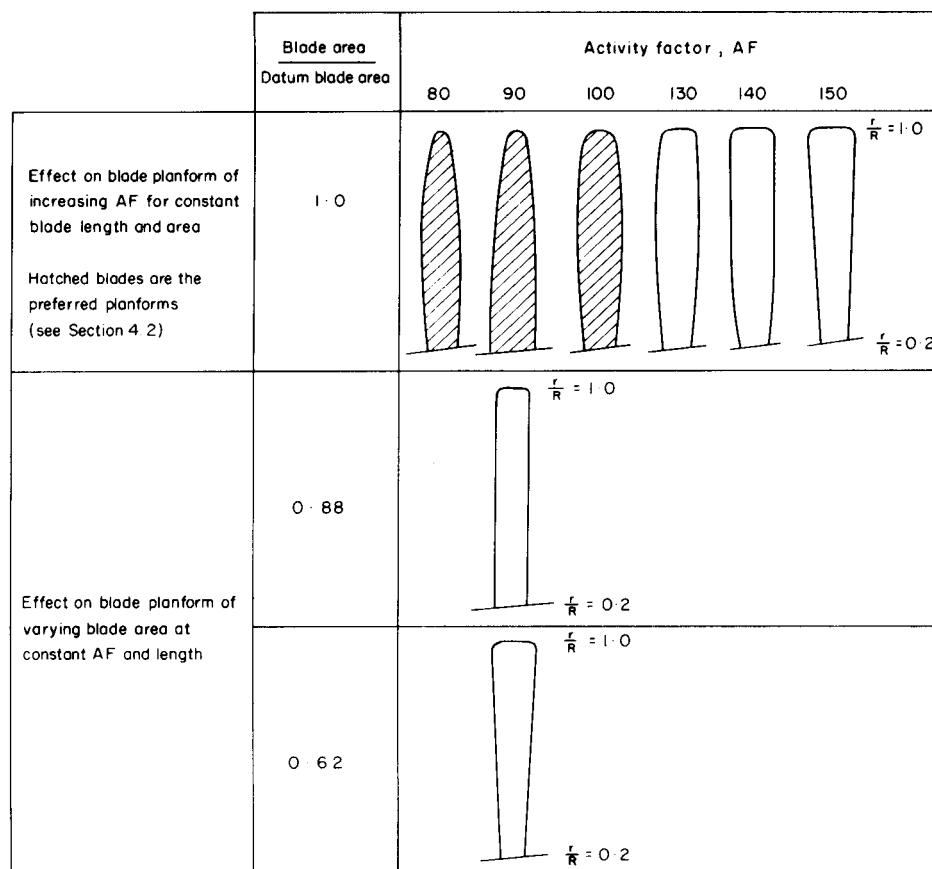
blade torque = $AF \times f$ (section lift to drag ratio, blade angle, rotational speed).

For a given blade aerofoil section and operating conditions (such as constant rotational speed, blade lift and drag coefficients *etc.*) it can be shown that the blade torque and hence propeller torque and power required are functions of activity factor only, *i.e.*

$$\text{propeller torque} = \text{constant} \times AF. \quad (4.3)$$

Increasing the diameter of the propeller improves the power absorption characteristics. However, if the propeller diameter cannot be increased, but high power absorption is required, the blade activity factor can be increased by increasing the chord towards the outboard propeller stations. This produces the inverted, trapezoidal shapes shown on the right-hand side of Sketch 4.1. Although the propeller's maximum efficiency is reduced, the use of such blade planforms allows the extra power of larger power plants to be fully utilised to achieve more thrust at take-off.

* Different values for the lower-limit of integration are sometimes used.



Sketch 4.1 Relationship between activity factor and blade planform

4.2 Effect of Blade Aerofoil on Propeller Performance and the Selection of Activity Factor

In Sketch 4.1, the three blade planforms shown in the top left-hand corner are typical of those preferred when propeller cruise efficiency is the primary design parameter. For maximum efficiency, the aerofoil at each blade station must be operated at its maximum lift to drag ratio. Increases in maximum lift to drag ratio by using a more efficient aerofoil usually have a small effect on cruise efficiency. However, the use of aerofoils capable of higher maximum lift coefficients (of greatest significance at take-off) usually allows a reduction in blade chord for the same overall propeller performance.

Hence, for a given level of performance and operating conditions, the use of “new generation” aerofoils (such as those of Derivations 12 and 13) in a propeller allows the TAF to be reduced by as much as 20 per cent when compared to propellers using the earlier generations of aerofoils (Derivations 3 and 10). Therefore, when using this Item to estimate the performance of propellers having new generation aerofoils, the following values of AF should be used to estimate $\frac{TAF}{350}$.

New generation aerofoils, AF	80	90	100	120
Value of AF to be used in Figures 2 to 5	95	110	120	145

5. EXPRESSION OF PROPELLER THRUST CHARACTERISTICS IN TERMS OF C_P , C_T , J AND η

This Item presents propeller performance in terms of $(T/P)_s$, $P/\sigma D^2$, T/T_s and V_{tip} . However, it is a common practice to express the performance in terms of non-dimensional coefficients **using coherent units** as follows:

$$\text{Thrust coefficient, } C_T = \frac{T}{\rho n^2 D^4} \quad (5.1)$$

$$\text{Power coefficient, } C_P = \frac{P}{\rho n^3 D^5} \quad (P \text{ in ft lbf/s or N m/s}) \quad (5.2)$$

$$\text{Advance ratio, } J = \frac{V}{nD} \quad (V \text{ in ft/s or m/s}) \quad (5.3)$$

$$\text{Efficiency, } \eta = \frac{TV}{P} = \frac{C_T}{C_P} J. \quad (5.4)$$

The “advance ratio”, J , is proportional to the ratio of the propeller forward speed to tip speed, and so is a measure of the blade angle of incidence.

The non-dimensional coefficients C_P , C_T and η can be represented in terms of the dependent and independent variables shown in Figures 1 and 6 and outlined in Section 3, **in British units**, as follows:

$$C_P = 7.174\,72 \times 10^6 \left[\left(\frac{P}{\sigma D^2} \right) \left(\frac{1}{V_{tip}^3} \right) \right] \quad (5.5)$$

$$C_T = 2.283\,78 \times 10^6 \left[(T/P)_s T/T_s (P/\sigma D^2) \left(\frac{1}{V_{tip}^2} \right) \right] \quad (5.6)$$

$$\text{and } \eta = 3.068\,7 \times 10^{-3} [(T/P)_s (T/T_s) V]. \quad (5.7)$$

Table 12.1 contains the conversion factors required to express the above equations in alternative systems of units. Additionally, the following relationship between T/P and the coefficient expressions will be found to be useful:

$$\frac{T}{P} = \frac{C_T}{C_P} \times \frac{1727.876}{V_{tip}} \text{ lbf/hp.} \quad (5.8)$$

6. EFFECTS OF TAF , $P/\sigma D^2$ AND V_{tip} ON PROPELLER PERFORMANCE

Figures 1 to 6 allow the effects of TAF , $P/\sigma D^2$ and V_{tip} on propeller performance to be considered but the influence of these in any particular application depends upon the relative importance of the different flight regimes, for example take-off, climb and cruise. Section 11.2 gives an example of the effect of varying TAF , $P/\sigma D^2$ and V_{tip} on propeller performance, at sea level and for constant power.

Activity factor (or total activity factor) is directly related to power absorption capacity and for some applications, for example where the propeller diameter is restricted for ground clearance reasons, increasing the activity factor is a means to allow a greater power to be absorbed – so increasing the take-off thrust.

However, at the cruise condition the same propeller usually has a lower efficiency when compared to another of smaller TAF (see Sketch 11.3).

Fundamental momentum theory suggests that propeller efficiency increases with diameter. However, for practical propellers where take-off performance is a significant requirement, a reduction in diameter is likely to result in an increase in cruise efficiency. An example of this is shown in Sketch 11.3 for a limited range of TAF 's, rotational speed and power.

Other factors affect the selection of TAF , $P/\sigma D^2$ and V_{tip} . Propeller noise is particularly affected by propellers having high tip speeds, approaching the speed of sound. Increasing the number of blades in the propeller allows a reduction in diameter, for a given power, and alleviates the tip speed problems of the larger unit. However, the increased cost and complexity of multiblade units has to be considered when assessing the relative merits of possible configurations.

7. INSTALLATION EFFECTS

This Item provides estimates of the “free-air”, uninstalled value of propeller thrust. The free-air thrust is that developed by the propeller in the absence of a body. Practically, this is approximated by the “minimum body thrust” concept, where the propeller is mounted at the front of an axisymmetric body having a diameter equal to that of the spinner. The drag of the body and spinner are not accounted for when quoting the free-air value of thrust.

The free-air performance of the propeller is modified when it is fitted to an aeroplane. The installation effects should be considered in two categories:

- (i) The “blockage” effect on propeller thrust due to the presence of a body or other aerodynamic surface.
- (ii) The effect of the slipstream of a tractor propeller or the inflow of a pusher-propeller installation upon the aerodynamic characteristics of the airframe.

For methods of estimating these effects, see Reference 2 or 7.

8. NOTES ON THE CORRELATIONS

The method of estimation presented in this Item is based upon a correlation of data for various propeller types. The effect of propeller geometry is accounted for only at the static conditions by the use of the parameters $P/\sigma D^2$, TAF and V_{tip} and thereafter no further effects of these parameters (upon T/P) are regarded. Thus, for a given propeller, the thrust characteristics throughout the forward speed range are determined by the thrust at static conditions.

The Figures were generated initially using the data of Derivation 1 (itself based on Reference 2 with minor simplifying assumptions) and were subsequently assessed and modified using test or predicted data from Derivations 2 to 15. Analysis of 44 propeller combinations covering a wide range of $P/\sigma D^2$, TAF , V and V_{tip} demonstrated that the method of the Item gave estimates for 90 per cent of the cases considered to within ± 10 per cent of the actual values (see Sketch 8.1). Although the primary source data of Derivation 1 are based upon the performance of propellers using NACA 16-series aerofoils, with design lift coefficients between 0.3 and 0.6, it has been found from the above analysis that the method will give estimates for propellers using other aerofoils, such as Clark Y, RAF 6, NACA 64-series and a limited number of new generation aerofoils (Derivations 12 and 13) to within the stated limits.