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Oscillatory aerodynamics of slender bodies

Associated software: ESDUpac A8310

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OSCILLATORY AERODYNAMICS OF SLENDER BODIES

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OSCILLATORY AERODYNAMICS OF SLENDER BODIES

1. NOTATION AND UNITS

		SI	British
$A(x)$	local area of cross-section; $\pi r^2(x)$ for circular cross-section	m^2	ft^2
$a(\xi)$	non-dimensional area of cross-section $A(\xi L)/L^2$		
a_0	speed of sound	m/s	ft/s
$C_p(\xi, \theta)$	pressure coefficient, see Equation (5.7)		
$C'_p(\xi, \theta)$	in-phase part of C_p , see Equation (5.9)		
$C''_p(\xi, \theta)$	in-quadrature (90° out of phase) part of C_p , see Equation (5.10)		
D	maximum diameter of body		
F_{ij}, G_{ij}, H_{ij}	non-dimensional coefficients independent of M and ν in Equations (6.4) to (6.7) and Tables 10.1 to 10.3		
f	overall body fineness ratio, L/D		
f_f	forebody fineness ratio, l_f/D		
i	$\sqrt{-1}$		
L	body overall length	m	ft
l_f	forebody length	m	ft
$\bar{l}(\xi)$	non-dimensional local lift, positive upwards, see Equation (5.2)		
$\bar{l}'(\xi)$	in-phase part of $\bar{l}$, see Equation (5.3)		
$\bar{l}''(\xi)$	in-quadrature (90° out of phase) part of $\bar{l}/\nu$ see Equation (5.4)		
M	free-stream Mach number, V/a_0		
Q_{ij}	complex generalised airforce coefficient, see Equation (6.1)		
Q'_{ij}	in-phase part of Q_{ij} , see Equation (6.2)		
Q''_{ij}	in-quadrature (90° out of phase) part of Q_{ij}/ν , see Equation (6.3)		

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$r(x)$	local radius of axisymmetric body	m	ft
V	free-stream velocity	m/s	ft/s
x, z	two-dimensional Cartesian coordinates relative to mean position of body axis with origin 0, at apex of forebody; x positive rearwards, z positive upwards, see Sketch 3.2	m	ft
$Z(x)$	amplitude of modal deflection, positive upwards	m	ft
$z(\xi)$	non-dimensional amplitude, $Z(\xi L)/L$		
θ	angle between Oxz plane and arbitrary plane through body longitudinal axis, positive clockwise from z position, see Sketch 3.2	degree	degree
$\bar{\lambda}(x)$	oscillatory lift per unit length, see Equation (5.1)	N/m	lbf/ft
ν	frequency parameter, $\omega L/V$		
ξ	non-dimensional longitudinal distance, x/L		
ρ	density of air	kg/m ³	slug/ft ³
ω	circular frequency	rad/s	rad/s

Subscripts

i	integer denoting a particular force, see Section 4
j	integer denoting a particular motion, see Section 4

Dressings

prime (') as in Q' denotes in-phase (real) part

prime (') as in $a'(\xi)$ and $z'(\xi)$ denotes derivative with respect to ξ

double prime (") as in Q'' denotes in-quadrature (imaginary) part

double prime (") as in $z''(\xi)$ denotes second derivative with respect to ξ

2. INTRODUCTION

This Item presents a method for estimating the longitudinal lift distribution and overall aerodynamic forces due to the oscillatory motion of slender bodies of revolution in inviscid airflow and is based on the work of Miles (Derivation 1) and unpublished work at BAe (Warton) (Derivation 3). Such data would typically be used in loading and flutter calculations on missiles and other bodies.

Items No. 81034 (Reference 13) and 82005 (Reference 14) provide similar data for thin, two-dimensional aerofoils in subsonic and supersonic flows, respectively, and Item No. 82020 (Reference 15) provides an introduction to the field of unsteady aerodynamics, including the use of linearized theories in flutter work, which is the basis of Items No. 81034 and 82005.

Appendix C provides details of a program (ESDUpac A8310) embodying the method of this Item, which is based on slender-body theory for unsteady flow (Appendix A). The program supersedes two earlier programs written in Hewlett-Packard BASIC, listings for which were given in the original Appendix C, now completely revised. The current program considerably increases the scope of the earlier ones in that, for example, it now covers every aspect dealt with in the Item and caters for a greater range of body shapes, see Appendix C for details. However, because the new program is now available major changes to the rest of the Item were considered to be unnecessary. Consequently, the data obtained from the original programs have been retained since they remain valid. The data provided relate to a range of bodies having conical, tangent ogive or ellipsoidal forebodies in combination with cylindrical afterbodies, see Section 3. The modes of oscillation considered are heave, pitch about the body mid-point and bending, see Section 4.

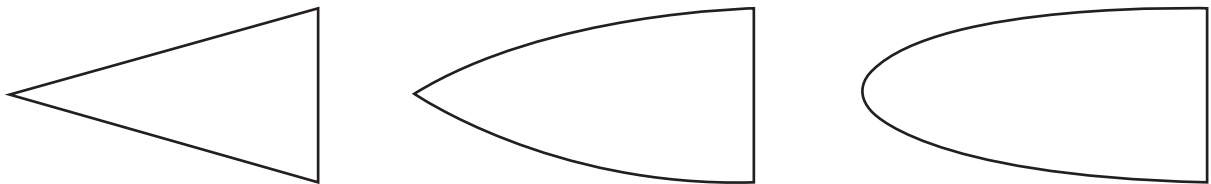
The effects of body geometry and frequency of oscillation on the longitudinal lift distribution (Section 5) are illustrated in graphical form. Tables of non-dimensional coefficients are given, allowing the simple estimation of airforce coefficients (Section 6) for the body shapes and modes of oscillation noted above. The effects of body geometry on airforce coefficients are also presented in graphical form.

The applicability of slender-body theory and the data given in this Item are discussed in Section 7 and a worked example on the use of the Item is given in Section 9.

Currently there are alternative forms in which data for flutter calculations are presented, *i.e.* airforce coefficients and flutter derivatives. Appendix B provides a set of equations to calculate the flutter derivatives from non-dimensional coefficients such as those given in the tables. An example is also given to illustrate the effect of a variation in the location of the pitching axis.

3. GEOMETRY OF BODIES CONSIDERED

This Item presents data for bodies of revolution consisting of conical, tangent ogive and ellipsoidal forebodies (see Sketch 3.1) in combination with cylindrical afterbodies.



(i) Conical (ii) Tangent ogive (iii) Ellipsoidal

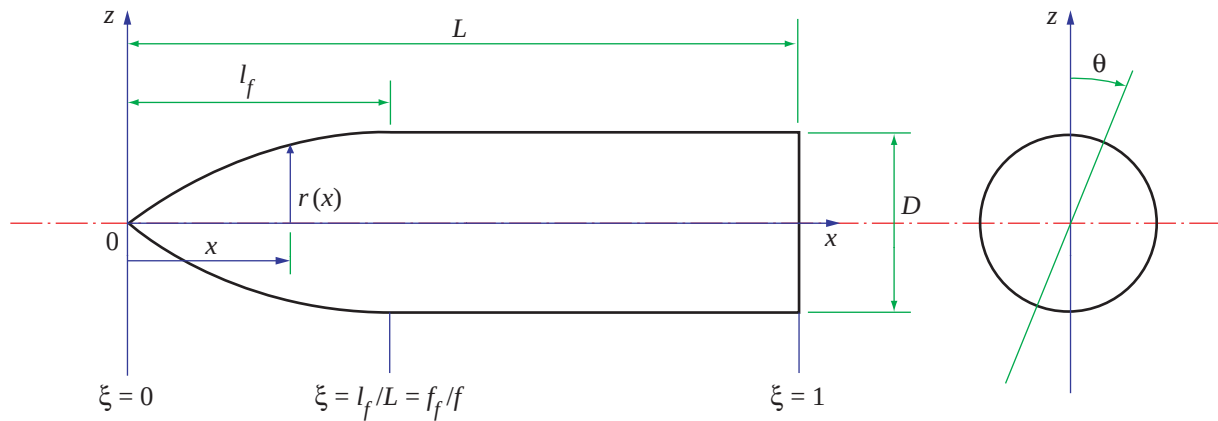
Sketch 3.1 Forebody shapes

The ranges of overall body fineness ratio, f , and forebody fineness ratio, f_f , for which data are given are indicated in Table 3.1.

TABLE 3.1
Body Parameters

$f_f \backslash f$	1	2	3	4	6	8	12	16
0.5	X	X	X	X	X			
1		X	X	X	X	X		
1.5			X	X				
2				X	X	X	X	X
3					X	X	X	X
4						X	X	X

The generating equations of the forebody shapes considered in this Item are defined in Item No. 77028 (Derivation 2). In terms of a non-dimensional area distribution required by the method of this Item the forebody shapes are defined by the following equations.



Note: $f = L/D$, $f_f = l_f/D$

Sketch 3.2 Body geometry

CONICAL FOREBODY

$$a(\xi) = \frac{\pi}{4f_f^2} \xi^2. \quad (3.1)$$

TANGENT OGIVE FOREBODY

$$a(\xi) = \pi \left\{ 2 \frac{f_f}{f} \xi - \xi^2 + 2 \left(\frac{f_f^2}{f} - \frac{1}{4f} \right)^2 - 2 \left(\frac{f_f^2}{f} - \frac{1}{4f} \right) \left[2 \frac{f_f}{f} \xi - \xi^2 + \left(\frac{f_f^2}{f} - \frac{1}{4f} \right)^2 \right]^{1/2} \right\}. \quad (3.2)$$

ELLIPSOIDAL FOREBODY

$$a(\xi) = \frac{\pi}{4f_f^2} \xi \left(2 \frac{f_f}{f} - \xi \right). \quad (3.3)$$

Similarly, for the cylindrical afterbody,

CYLINDRICAL AFTERBODY

$$a(\xi) = \frac{\pi}{4f^2}. \quad (3.4)$$

4. MODES OF OSCILLATION

The modes of oscillation which were applied to the bodies described in Section 3 are those of heave ($j = 1$), pitch about the body mid-point ($j = 2$) and a typical bending mode ($j = 3$), *i.e.*

$$\text{HEAVE} \quad z_1(\xi) = 1.0 \quad (4.1)$$

$$\text{PITCH} \quad z_2(\xi) = -(\xi - 1/2) \quad (4.2)$$

$$\text{BENDING} \quad z_3(\xi) = 9/2(\xi - 1/2)^2 - 1/8. \quad (4.3)$$

For this set of modes the forces given by $i = 1$ and 2 correspond to lift and nose-up pitching moment respectively. The third mode, $i = 3$, has been included to show a typical bending mode.

5. LONGITUDINAL LIFT DISTRIBUTION

The local lift along a body according to slender-body theory, as derived in Equation (A2.10) of Appendix A, is

$$\bar{\lambda}(x) = -\rho V^2 \left[A(x)Z''(x) + A'(x)Z'(x) - \frac{v^2}{L^2} A(x)Z(x) \right] - i\rho V^2 \frac{v}{L} [2A(x)Z'(x) + A'(x)Z(x)]. \quad (5.1)$$

The non-dimensional local lift acting normal to the longitudinal body axis due to an oscillatory motion, of amplitude $z_j(\xi)$, in the j^{th} mode is

$$\bar{l}_j(\xi) = l_j'(\xi) + i v l_j''(\xi) = \frac{\bar{\lambda}_j(\xi L)}{1/2 \rho V^2 L}, \quad (5.2)$$

where

$$l_j'(\xi) = -2[a(\xi)z_j''(\xi) + a'(\xi)z_j'(\xi)z_j(\xi) - v^2 a(\xi)z_j(\xi)] \quad (5.3)$$

and

$$l_j''(\xi) = -2[a(\xi)z_j'(\xi) + a'(\xi)z_j(\xi)], \quad (5.4)$$

where

$$z_j'(\xi) = \frac{d}{d\xi}[z_j(\xi)] \text{ and } a'(\xi) = \frac{d}{d\xi}[a(\xi)] \quad (5.5)$$

and

$$z_j''(\xi) = \frac{d^2}{d\xi^2}[z_j(\xi)]. \quad (5.6)$$

The relationship between the non-dimensional local lift and the pressure coefficient for a plane at an angle θ to the Oxz plane and passing through the body longitudinal axis (see Sketch 3.2) is

$$C_p(\xi, \theta) = \frac{\bar{l}(\xi)}{2\pi} \left\{ \frac{4\pi}{a(\xi)} + \left[\frac{a'(\xi)}{a(\xi)} \right]^2 \right\}^{1/2} \cos \theta \quad (5.7)$$

where $C_p(\xi, \theta) = C'_p(\xi, \theta) + iC''_p(\xi, \theta).$ (5.8)

From Equations (5.2) and (5.8)

$$C'_p(\xi, \theta) = \frac{l'(\xi)}{2\pi} \left\{ \frac{4\pi}{a(\xi)} + \left[\frac{a'(\xi)}{a(\xi)} \right]^2 \right\}^{1/2} \cos \theta \quad (5.9)$$

and $C''_p(\xi, \theta) = v \frac{l''(\xi)}{2\pi} \left\{ \frac{4\pi}{a(\xi)} + \left[\frac{a'(\xi)}{a(\xi)} \right]^2 \right\}^{1/2} \cos \theta .$ (5.10)

The longitudinal lift distributions for the cases detailed in Sections 3 and 4 have been calculated over a range of frequency parameters. Rather than tabulate the results it was felt better to illustrate some of the more significant variations for the case of the tangent ogive forebody. The results are shown in Figures 1a to 1f which depict the effect of varying the forebody and overall body fineness ratio and frequency parameter on the real and imaginary parts of the loading due to oscillations in heave or pitch. Similar calculations were also carried out for a body of given overall and forebody fineness ratios at a fixed frequency parameter, varying only the forebody shape. The results are shown in Figures 1g and 1h for oscillations in heave and pitch respectively.

For heaving motion, the centres of pressure of both parts of the loading move aft as the forebody fineness ratio f_f increases and f is held constant (Figure 1a). Pitching motion about the body mid-point introduces loadings of opposite sign fore and aft, the change in sign occurring further aft as f_f increases (Figure 1b). The centres of pressure in heaving motion move forward as overall body fineness ratio f increases and f_f is held constant (Figure 1c). For pitching motion, the in-phase loading on the forebody and the in-quadrature loading on the cylindrical afterbody predominate for $f = 2$ (Figure 1d). For heave, the in-phase loading is proportional to v^2 , while the invariance of l_2'' shows that the in-quadrature loading is proportional to v (Figure 1e). In the case of pitch there is a separate contribution to the in-phase loading at $v = 0$ (Figure 1f). Figure 1g shows that for heaving motion the in-phase loading is proportional to area distribution and the in-quadrature loading is proportional to the gradient of cross-sectional area. The effect of forebody shape in pitching motion in Figure 1h is more complicated. The step-discontinuities in the in-quadrature part of the loading for heaving motion, at the nose for the ellipsoidal forebody and at the afterbody junction for the conical forebody, are present in both parts of the loading for pitching motion.

6. GENERALISED AIRFORCE COEFFICIENTS

The non-dimensionalised lift distribution of Equation (5.2) is used to define the generalised airforce coefficients:

$$Q_{ij} = Q'_{ij} + i\nu Q''_{ij} = \int_0^1 \bar{l}_j(\xi) z_i(\xi) d\xi. \quad (6.1)$$

$$\text{Thus } Q'_{ij} = \int_0^1 l'_j(\xi) z_i(\xi) d\xi \quad (6.2)$$

$$\text{and } Q''_{ij} = \int_0^1 l''_j(\xi) z_i(\xi) d\xi. \quad (6.3)$$

From Equations (5.3) and (6.2) the real, or in-phase, part of Equation (6.1) can be expressed as

$$Q'_{ij} = F_{ij} + G_{ij} \nu^2. \quad (6.4)$$

where

$$F_{ij} = -2 \int_0^1 [a'(\xi) z'_j(\xi) + a(\xi) z''_j(\xi)] z_i(\xi) d\xi \quad (6.5)$$

and

$$G_{ij} = 2 \int_0^1 a(\xi) z_j(\xi) z_i(\xi) d\xi \quad (6.6)$$

are coefficients independent of both Mach number and frequency parameter.

Similarly, from Equations (5.4) and (6.3), the in-quadrature coefficient

$$Q''_{ij} = H_{ij} = -2 \int_0^1 [2a(\xi) z'_j(\xi) + a'(\xi) z_j(\xi)] z_i(\xi) d\xi \quad (6.7)$$

is seen to be independent of both Mach number and frequency parameter.

For simple body shapes and simple mode shapes it is possible to evaluate Equations (6.5), (6.6) and (6.7) analytically. However, in order to simplify the procedure, a computer program has been written, see Appendix C. That appendix includes some analytical expressions for F_{ij} , G_{ij} , and H_{ij} as a check on the computed values for a particular forebody shape.

The coefficients F_{ij} , G_{ij} and H_{ij} have been evaluated using the computer program for the range of bodies given in Table 3.1 having the nose shapes defined in Equations (3.1) to (3.3) for the heave, pitch and bending modes of vibration defined in Equations (4.1) to (4.3). The coefficients F_{i1} are zero and, for the modes of vibration considered, there are six other sets of identical coefficients. The results are given in Tables 10.1, 10.2 and 10.3 respectively for bodies with conical, tangent ogive and ellipsoidal forebodies. The generalised airforce coefficients in Equations (6.4) and (6.7) can then be obtained.

The effects of varying the body geometry on the airforce coefficients in heave and pitch (at a fixed frequency parameter where necessary) are shown in Figure 2a for the in-phase part and Figure 2b for the in-quadrature part. These figures are complementary to the lift distributions shown in Figures 1a to 1h. A general feature of the in-phase coefficients in Figure 2a is that the influence of forebody shape diminishes as the overall fineness ratio increases and grows as the forebody fineness ratio increases. Note that for Q'_{21} the trend with varying forebody shape is in opposition to that for the other three coefficients in Figure 2b, but it is observed that Q''_{11} , which represents the damping in heave, is independent of forebody shape and length. The same is true for Q''_{22} , the damping in pitch about the body mid-point, but only for this particular pitching axis.

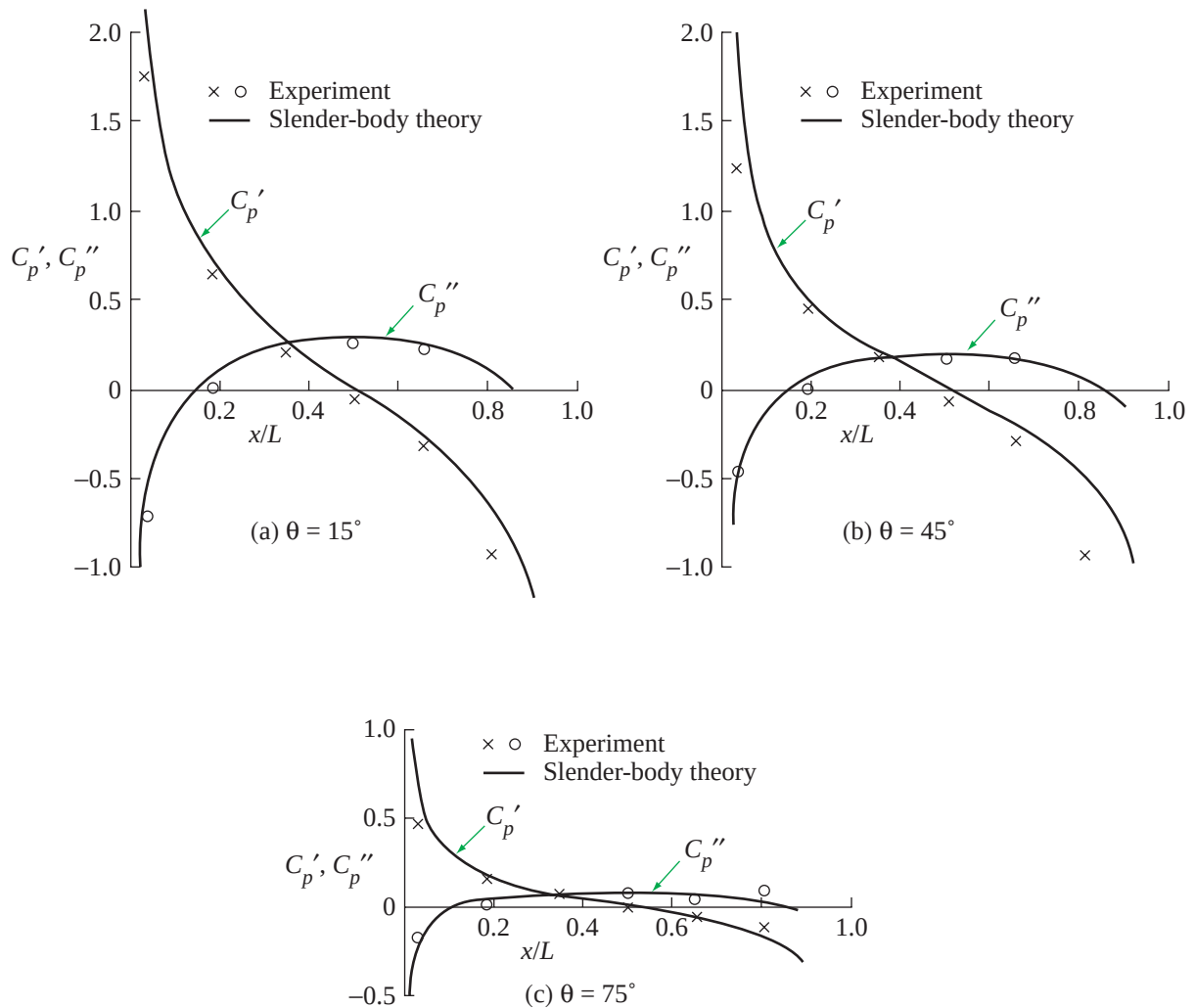
7. APPLICABILITY

The basis of the data presented in this Item is “slender-body” theory. The term “slender” when applied to a body of revolution of length L and maximum diameter D implies that both D/L and (in unsteady flow) MD/L are small. The results for $f < 4$, say, should therefore be used with caution. In practice it is satisfactory to assume that the theory applies to any slender pointed body and that it can be applied satisfactorily to moderately blunt-nosed bodies (see later) up to low supersonic speeds. The amplitude of oscillation must be small compared with the body diameter. The bodies can have any type of symmetrical cross-section (circular, elliptical* etc.), see Appendix A, and the theory can be extended to deal with afterbodies of varying diameter (Reference 9). Conditions for the applicability of slender-body theory to extremes of frequency parameter are given in Section 3.2 of Reference 11, but the restrictions are not thought to be serious in subsonic flow.

Some comparisons have been made between experimental data (Reference 12) for the low-speed pressure distribution on an ellipsoid oscillating at $v = 0.66$ in pitch about the body mid-point and predictions from slender-body theory. The results are shown in Sketch 7.1 for the three longitudinal planes $\theta = 15^\circ, 45^\circ$ and 75° . It can be seen that, with the exception of the rear 25 to 30 per cent of the body where viscous effects would be expected to be significant, the agreement is generally good, with the theory giving a slight overestimate.

The data presented in this Item can be used to predict loading and unsteady forces on missiles, torpedoes, submarines etc. The data can also be applied to airships and aircraft fuselages ahead of the wing apex. One particular use, as initial or base data for under-fuselage or under-wing store flutter calculations, is illustrated in Section 9. In that example the stiffness and damping matrices are obtained for a missile isolated from its underwing pylon. It is found in practice that the “end plate” effects of the wing and missile allow the pylon to be treated as if in two-dimensional flow, by means of the simple strip theory given in Appendix B of Item No. 81034 (Reference 13), using the data of that Item for subsonic flow or Item No. 82005 (Reference 14) for supersonic flow. The missile-pylon combination could then be modelled by attributing the slender-body contribution from the missile to an additional pylon strip of width equal to half the maximum diameter of the missile.

* However, the theory would not be expected to apply to extremes of ellipticity where one of the axes tends to zero (giving zero cross-sectional area.)



Sketch 7.1 Comparison between slender-body theory and test data (Reference 12) at low speed for pressure distribution around an ellipsoid ($f = 4.67$, $f_f = 2.33$) in pitching oscillation ($v = 0.66$) about body mid-point

8. DERIVATION AND REFERENCES

8.1 Derivation

The Derivation lists selected sources that have assisted in the preparation of this Item.

1. MILES, J.W. On non-steady motion of slender bodies.
Aeronaut. Q. Vol. II, Part III, pp.183-194, November 1950.
2. ESDU Geometrical characteristics of typical forebodies and afterbodies.
ESDU International, Item No. 77028, 1977.
3. BAe Unpublished work at British Aerospace (Warton), 1982.

8.2 References

The References given are recommended sources of information supplementary to that given in this Item.

4. LAMB, H. *Hydrodynamics*.
Cambridge University Press, 1916.
5. MUNK, M.M. The aerodynamic forces on airship hulls.
NACA Rep. 184, 1924.
6. LIGHTHILL, M.J. Supersonic flow past slender bodies.
ARC R & M 2003, 1945.
7. JONES, R.T. Properties of low-aspect-ratio pointed wings at speeds below and above the speed of sound.
NACA Rep. 835, 1946.
8. WARD, G.N. Supersonic flow past slender pointed bodies.
Q.J. Mech. & Appl. Math., Vol. 2, Part I, pp.75-97, March 1949.
9. HOPKINS, E.J. A semiempirical method for calculating the pitching moment of bodies of revolution at low Mach numbers.
NACA RM A51C14 (TIL 2943), 1951.
10. ADAMS, M.C. Slender body theory – Review and extension.
SEARS, W.R. *J. aeronaut. Sci.*, Vol. 20, p.85, 1953.
11. WOODCOCK, D.L. Slender-body theory.
AGARD Manual on Aeroelasticity, Vol. II, Part II, Chapter 7, 1962.
12. GEISSLER, W. Methods of inviscid subsonic flows about aircraft configurations.
In AGARD-FDP-VKI Special Course in Unsteady Aerodynamics, March 1980.
13. ESDU Linearised two-dimensional oscillatory airforce coefficients and load distributions on thin aerofoils in subsonic flow.
ESDU International, Item No. 81034, 1981.

14. ESDU Linearised two-dimensional oscillatory airforce coefficients and load distributions on thin aerofoils in supersonic flow.
ESDU International, Item No. 82005, 1982.
15. ESDU Introduction to unsteady aerodynamics.
ESDU International, Item No. 82020, 1982.

9. EXAMPLE

As part of the flutter analysis for an underwing store, “base” data in the form of stiffness and damping matrices for the isolated store are required. For this example the store is a missile of fineness ratio 6 with a tangent ogive forebody, of fineness ratio 2, and a cylindrical afterbody. The stiffness and damping matrices, involving lift and pitching moment in heaving and pitching oscillations about the body mid-point, are required for frequency parameters of 0, 0.5 and 1.

From Table 10.2 for $f = 6$ and $f_f = 2$,

F_{11}	G_{11}	F_{12}	G_{12}	F_{21}	G_{21}	F_{22}	G_{22}	H_{11}	H_{12}	H_{21}	H_{22}
0	0.0370	0.0436	-0.0027	0	-0.0027	0.0152	0.0025	-0.0436	0.0588	-0.0152	-0.0109

These coefficients, when used in conjunction with Equations (6.4) and (6.7), give the following stiffness and damping matrices for $v = 0, 0.5$ and 1.

$$\begin{aligned} \text{Stiffness matrix} &= - \begin{bmatrix} Q'_{11} & Q'_{12} \\ Q'_{21} & Q'_{22} \end{bmatrix} & \text{Damping matrix} &= - \begin{bmatrix} Q''_{11} & Q''_{12} \\ Q''_{21} & Q''_{22} \end{bmatrix} \\ \\ v = 0 & \begin{bmatrix} 0 & -0.0436 \\ 0 & -0.0152 \end{bmatrix} & & \begin{bmatrix} 0.0436 & -0.0588 \\ 0.0152 & 0.0109 \end{bmatrix} \\ \\ v = 0.5 & \begin{bmatrix} -0.0093 & -0.0429 \\ 0.0007 & -0.0158 \end{bmatrix} & & \begin{bmatrix} 0.0436 & -0.0588 \\ 0.0152 & 0.0109 \end{bmatrix} \\ \\ v = 1 & \begin{bmatrix} -0.0370 & -0.0409 \\ 0.0027 & -0.0177 \end{bmatrix} & & \begin{bmatrix} 0.0436 & -0.0588 \\ 0.0152 & 0.0109 \end{bmatrix} \end{aligned}$$

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10. TABLES OF COEFFICIENTS F_{ij} , G_{ij} , H_{ij}

TABLE 10.1 Coefficients F_{ij} , G_{ij} , H_{ij} : Conical Forebody with Cylindrical Afterbody

$$F_{11} = F_{21} = F_{31} = 0$$

f	f_f	F_{12} $-H_{11}$ $-H_{33}$	F_{13}	F_{22} $-H_{21}$	F_{23}	F_{32} $-H_{31}$	F_{33}	G_{11}	G_{12} G_{21}	G_{13} G_{31}	G_{22}	G_{23} G_{32}	G_{33}	H_{12}	H_{13}	H_{22}	H_{23}	H_{32}
1	0.5	1.5708	-7.0686	0.2618	2.0617	0.0982	-1.2370	1.0472	-0.1636	0.1931	0.0720	-0.0826	0.1436	1.8326	-3.0434	-0.3927	1.2403	0.3305
2	0.5	0.3927	-1.7671	0.1309	0.6443	0.1534	-0.0110	0.3272	-0.0266	0.0567	0.0217	-0.0177	0.0393	0.5236	-0.6320	-0.0982	0.3348	0.0579
2	1.0	0.3927	-1.7671	0.0654	0.5154	0.0245	-0.3093	0.2618	-0.0409	0.0483	0.0180	-0.0207	0.0359	0.4581	-0.7609	-0.0982	0.3101	0.0826
3	0.5	0.1745	-0.7854	0.0679	0.3163	0.0982	0.0894	0.1551	-0.0085	0.0292	0.0108	-0.0064	0.0193	0.2424	-0.2509	-0.0436	0.1553	0.0193
3	1.0	0.1745	-0.7854	0.0485	0.2618	0.0436	-0.0698	0.1357	-0.0145	0.0228	0.0088	-0.0086	0.0166	0.2230	-0.3054	-0.0436	0.1440	0.0305
3	1.5	0.1745	-0.7854	0.0291	0.2291	0.0109	-0.1374	0.1164	-0.0182	0.0215	0.0080	-0.0092	0.0160	0.2036	-0.3382	-0.0436	0.1378	0.0367
4	0.5	0.0982	-0.4418	0.0409	0.1875	0.0648	0.0842	0.0900	-0.0037	0.0180	0.0065	-0.0030	0.0118	0.1391	-0.1315	-0.0245	0.0896	0.0086
4	1.0	0.0982	-0.4418	0.0327	0.1611	0.0383	-0.0028	0.0818	-0.0066	0.0142	0.0054	-0.0044	0.0098	0.1309	-0.1580	-0.0245	0.0837	0.0145
4	1.5	0.0982	-0.4418	0.0245	0.1415	0.0188	-0.0525	0.0736	-0.0088	0.0124	0.0048	-0.0050	0.0092	0.1227	-0.1776	-0.0245	0.0799	0.0183
4	2.0	0.0982	-0.4418	0.0164	0.1289	0.0061	-0.0773	0.0654	-0.0102	0.0121	0.0045	-0.0052	0.0090	0.1145	-0.1902	-0.0245	0.0775	0.0207
6	0.5	0.0436	-0.1963	0.0194	0.0879	0.0334	0.0550	0.0412	-0.0011	0.0088	0.0031	-0.0010	0.0058	0.0630	-0.0539	-0.0109	0.0409	0.0027
6	1.0	0.0436	-0.1963	0.0170	0.0791	0.0245	0.0224	0.0388	-0.0021	0.0073	0.0027	-0.0016	0.0048	0.0606	-0.0627	-0.0109	0.0388	0.0048
6	2.0	0.0436	-0.1963	0.0121	0.0654	0.0109	-0.0175	0.0339	-0.0036	0.0057	0.0022	-0.0022	0.0042	0.0558	-0.0764	-0.0109	0.0360	0.0076
6	3.0	0.0436	-0.1963	0.0073	0.0573	0.0027	-0.0344	0.0291	-0.0045	0.0054	0.0020	-0.0023	0.0040	0.0509	-0.0845	-0.0109	0.0345	0.0092
8	1.0	0.0245	-0.1104	0.0102	0.0469	0.0162	0.0211	0.0225	-0.0009	0.0045	0.0016	-0.0008	0.0029	0.0348	-0.0329	-0.0061	0.0224	0.0022
8	2.0	0.0245	-0.1104	0.0082	0.0403	0.0096	-0.0007	0.0205	-0.0017	0.0035	0.0014	-0.0011	0.0025	0.0327	-0.0395	-0.0061	0.0209	0.0036
8	3.0	0.0245	-0.1104	0.0061	0.0354	0.0047	-0.0131	0.0184	-0.0022	0.0031	0.0012	-0.0012	0.0023	0.0307	-0.0444	-0.0061	0.0200	0.0046
8	4.0	0.0245	-0.1104	0.0041	0.0322	0.0015	-0.0193	0.0164	-0.0026	0.0030	0.0011	-0.0013	0.0022	0.0286	-0.0476	-0.0061	0.0194	0.0052
12	2.0	0.0109	-0.0491	0.0042	0.0198	0.0061	0.0056	0.0097	-0.0005	0.0018	0.0007	-0.0004	0.0012	0.0152	-0.0157	-0.0027	0.0097	0.0012
12	3.0	0.0109	-0.0491	0.0036	0.0179	0.0043	-0.0003	0.0091	-0.0007	0.0016	0.0006	-0.0005	0.0011	0.0145	-0.0176	-0.0027	0.0093	0.0016
12	4.0	0.0109	-0.0491	0.0030	0.0164	0.0027	-0.0044	0.0085	-0.0009	0.0014	0.0006	-0.0005	0.0010	0.0139	-0.0191	-0.0027	0.0090	0.0019
16	2.0	0.0061	-0.0276	0.0026	0.0117	0.0041	0.0053	0.0056	-0.0002	0.0011	0.0004	-0.0002	0.0007	0.0087	-0.0082	-0.0015	0.0056	0.0005
16	3.0	0.0061	-0.0276	0.0023	0.0108	0.0032	0.0022	0.0054	-0.0003	0.0010	0.0004	-0.0002	0.0007	0.0084	-0.0091	-0.0015	0.0054	0.0007
16	4.0	0.0061	-0.0276	0.0020	0.0101	0.0024	-0.0002	0.0051	-0.0004	0.0009	0.0003	-0.0003	0.0006	0.0082	-0.0099	-0.0015	0.0052	0.0009

TABLE 10.2 Coefficients F_{ij} , G_{ij} , H_{ij} : Tangent Ogive Forebody with Cylindrical Afterbody

$$F_{11} = F_{21} = F_{31} = 0$$

f	f_f	F_{12} $-H_{11}$ $-H_{33}$	F_{13}	F_{22} $-H_{21}$	F_{23}	F_{32} $-H_{31}$	F_{33}	G_{11}	G_{12} G_{21}	G_{13} G_{31}	G_{22}	G_{23} G_{32}	G_{33}	H_{12}	H_{13}	H_{22}	H_{23}	H_{32}
1	0.5	1.5708	-7.0686	0.5236	2.6507	0.6872	0.3534	1.3090	-0.0982	0.2487	0.0916	-0.0614	0.1730	2.0944	-2.4544	-0.3927	1.3614	0.2094
2	0.5	0.3927	-1.7671	0.1636	0.7547	0.2638	0.3700	0.3600	-0.0143	0.0737	0.0264	-0.0110	0.0494	0.5563	-0.5216	-0.0982	0.3601	0.0326
2	1.0	0.3927	-1.7671	0.1119	0.6075	0.1166	-0.0867	0.3083	-0.0307	0.0548	0.0207	-0.0182	0.0391	0.5046	-0.6688	-0.0982	0.3282	0.0645
3	0.5	0.1745	-0.7854	0.0776	0.3527	0.1345	0.2269	0.1648	-0.0044	0.0356	0.0125	-0.0037	0.0238	0.2521	-0.2145	-0.0436	0.1641	0.0104
3	1.0	0.1745	-0.7854	0.0622	0.3006	0.0825	0.0450	0.1495	-0.0102	0.0274	0.0103	-0.0070	0.0187	0.2368	-0.2666	-0.0436	0.1521	0.0224
3	1.5	0.1745	-0.7854	0.0480	0.2654	0.0473	-0.0516	0.1353	-0.0141	0.0239	0.0091	-0.0083	0.0171	0.2225	-0.3018	-0.0436	0.1449	0.0296
4	0.5	0.0982	-0.4418	0.0450	0.2036	0.0809	0.1479	0.0941	-0.0019	0.0210	0.0073	-0.0017	0.0141	0.1432	-0.1154	-0.0245	0.0936	0.0046
4	1.0	0.0982	-0.4418	0.0385	0.1799	0.0572	0.0595	0.0876	-0.0046	0.0169	0.0062	-0.0034	0.0113	0.1367	-0.1392	-0.0245	0.0879	0.0103
4	1.5	0.0982	-0.4418	0.0325	0.1620	0.0393	0.0039	0.0816	-0.0065	0.0146	0.0055	-0.0043	0.0101	0.1307	-0.1571	-0.0245	0.0840	0.0141
4	2.0	0.0982	-0.4418	0.0266	0.1484	0.0257	-0.0315	0.0757	-0.0081	0.0133	0.0051	-0.0047	0.0096	0.1248	-0.1707	-0.0245	0.0813	0.0168
6	0.5	0.0436	-0.1963	0.0206	0.0929	0.0384	0.0756	0.0424	-0.0006	0.0098	0.0034	-0.0005	0.0066	0.0642	-0.0489	-0.0109	0.0422	0.0014
6	1.0	0.0436	-0.1963	0.0187	0.0854	0.0308	0.0456	0.0405	-0.0014	0.0084	0.0030	-0.0012	0.0055	0.0623	-0.0564	-0.0109	0.0403	0.0033
6	2.0	0.0436	-0.1963	0.0152	0.0739	0.0193	0.0070	0.0370	-0.0027	0.0067	0.0025	-0.0018	0.0046	0.0588	-0.0679	-0.0109	0.0377	0.0059
6	3.0	0.0436	-0.1963	0.0117	0.0657	0.0111	-0.0148	0.0335	-0.0036	0.0059	0.0022	-0.0021	0.0042	0.0554	-0.0761	-0.0109	0.0361	0.0075
8	1.0	0.0245	-0.1104	0.0110	0.0497	0.0190	0.0320	0.0232	-0.0006	0.0050	0.0018	-0.0005	0.0033	0.0355	-0.0301	-0.0061	0.0231	0.0015
8	2.0	0.0245	-0.1104	0.0095	0.0444	0.0137	0.0127	0.0217	-0.0012	0.0041	0.0015	-0.0009	0.0028	0.0340	-0.0354	-0.0061	0.0218	0.0027
8	3.0	0.0245	-0.1104	0.0080	0.0402	0.0095	-0.0001	0.0203	-0.0017	0.0036	0.0014	-0.0011	0.0025	0.0326	-0.0396	-0.0061	0.0209	0.0036
8	4.0	0.0245	-0.1104	0.0066	0.0369	0.0062	-0.0085	0.0188	-0.0020	0.0033	0.0013	-0.0012	0.0024	0.0311	-0.0429	-0.0061	0.0203	0.0043
12	2.0	0.0109	-0.0491	0.0046	0.0212	0.0075	0.0107	0.0101	-0.0004	0.0021	0.0007	-0.0003	0.0014	0.0155	-0.0143	-0.0027	0.0100	0.0009
12	3.0	0.0109	-0.0491	0.0042	0.0197	0.0060	0.0055	0.0096	-0.0005	0.0018	0.0007	-0.0004	0.0012	0.0151	-0.0158	-0.0027	0.0097	0.0012
12	4.0	0.0109	-0.0491	0.0038	0.0184	0.0048	0.0015	0.0092	-0.0007	0.0017	0.0006	-0.0005	0.0011	0.0147	-0.0171	-0.0027	0.0094	0.0015
16	2.0	0.0061	-0.0276	0.0027	0.0123	0.0047	0.0077	0.0058	-0.0002	0.0012	0.0004	-0.0001	0.0008	0.0089	-0.0076	-0.0015	0.0058	0.0004
16	3.0	0.0061	-0.0276	0.0025	0.0117	0.0040	0.0051	0.0056	-0.0002	0.0011	0.0004	-0.0002	0.0007	0.0087	-0.0083	-0.0015	0.0056	0.0005
16	4.0	0.0061	-0.0276	0.0024	0.0111	0.0034	0.0031	0.0054	-0.0003	0.0010	0.0004	-0.0002	0.0007	0.0085	-0.0089	-0.0015	0.0054	0.0007

TABLE 10.3 Coefficients F_{ij} , G_{ij} , H_{ij} : Ellipsoidal Forebody with Cylindrical Afterbody

$$F_{11} = F_{21} = F_{31} = 0$$

f	f_f	F_{12} $-H_{11}$ $-H_{33}$	F_{13}	F_{22} $-H_{21}$	F_{23}	F_{32} $-H_{31}$	F_{33}	G_{11}	G_{12} G_{21}	G_{13} G_{31}	G_{22}	G_{23} G_{32}	G_{33}	H_{12}	H_{13}	H_{22}	H_{23}	H_{32}
1	0.5	1.5708	-7.0686	0.5236	2.6507	0.6872	0.3534	1.3090	-0.0982	0.2487	0.0916	-0.0614	0.1730	2.0944	-2.4544	-0.3927	1.3614	0.2094
2	0.5	0.3927	-1.7671	0.1636	0.7547	0.2638	0.3700	0.3600	-0.0143	0.0737	0.0264	-0.0110	0.0494	0.5563	-0.5216	-0.0982	0.3601	0.0326
2	1.0	0.3927	-1.7671	0.1309	0.6627	0.1718	0.0884	0.3272	-0.0245	0.0622	0.0229	-0.0153	0.0432	0.5236	-0.6136	-0.0982	0.3403	0.0524
3	0.5	0.1745	-0.7854	0.0776	0.3527	0.1345	0.2269	0.1648	-0.0044	0.0356	0.0125	-0.0037	0.0238	0.2521	-0.2145	-0.0436	0.1641	0.0104
3	1.0	0.1745	-0.7854	0.0679	0.3200	0.1018	0.1134	0.1551	-0.0081	0.0305	0.0111	-0.0057	0.0207	0.2424	-0.2473	-0.0436	0.1566	0.0179
3	1.5	0.1745	-0.7854	0.0582	0.2945	0.0764	0.0393	0.1454	-0.0109	0.0276	0.0102	-0.0068	0.0192	0.2327	-0.2727	-0.0436	0.1513	0.0233
4	0.5	0.0982	-0.4418	0.0450	0.2036	0.0809	0.1479	0.0941	-0.0019	0.0210	0.0073	-0.0017	0.0141	0.1432	-0.1154	-0.0245	0.0936	0.0046
4	1.0	0.0982	-0.4418	0.0409	0.1887	0.0660	0.0925	0.0900	-0.0036	0.0184	0.0066	-0.0027	0.0123	0.1391	-0.1304	-0.0245	0.0900	0.0082
4	1.5	0.0982	-0.4418	0.0368	0.1760	0.0533	0.0516	0.0859	-0.0050	0.0167	0.0061	-0.0034	0.0114	0.1350	-0.1430	-0.0245	0.0872	0.0109
4	2.0	0.0982	-0.4418	0.0327	0.1657	0.0430	0.0221	0.0818	-0.0061	0.0155	0.0057	-0.0038	0.0108	0.1309	-0.1534	-0.0245	0.0851	0.0131
6	0.5	0.0436	-0.1963	0.0206	0.0929	0.0384	0.0756	0.0424	-0.0006	0.0098	0.0034	-0.0005	0.0066	0.0642	-0.0489	-0.0109	0.0422	0.0014
6	1.0	0.0436	-0.1963	0.0194	0.0882	0.0336	0.0567	0.0412	-0.0011	0.0089	0.0031	-0.0009	0.0059	0.0630	-0.0536	-0.0109	0.0410	0.0026
6	2.0	0.0436	-0.1963	0.0170	0.0800	0.0255	0.0284	0.0388	-0.0020	0.0076	0.0028	-0.0014	0.0052	0.0606	-0.0618	-0.0109	0.0391	0.0045
6	3.0	0.0436	-0.1963	0.0145	0.0736	0.0191	0.0098	0.0364	-0.0027	0.0069	0.0025	-0.0017	0.0048	0.0582	-0.0682	-0.0109	0.0378	0.0058
8	1.0	0.0245	-0.1104	0.0112	0.0509	0.0202	0.0370	0.0235	-0.0005	0.0052	0.0018	-0.0004	0.0035	0.0358	-0.0289	-0.0061	0.0234	0.0011
8	2.0	0.0245	-0.1104	0.0102	0.0472	0.0165	0.0231	0.0225	-0.0009	0.0046	0.0016	-0.0007	0.0031	0.0348	-0.0326	-0.0061	0.0225	0.0020
8	3.0	0.0245	-0.1104	0.0092	0.0440	0.0133	0.0129	0.0215	-0.0012	0.0042	0.0015	-0.0009	0.0028	0.0337	-0.0358	-0.0061	0.0218	0.0027
8	4.0	0.0245	-0.1104	0.0082	0.0414	0.0107	0.0055	0.0205	-0.0015	0.0039	0.0014	-0.0010	0.0027	0.0327	-0.0383	-0.0061	0.0213	0.0033
12	2.0	0.0109	-0.0491	0.0048	0.0220	0.0084	0.0142	0.0103	-0.0003	0.0022	0.0008	-0.0002	0.0015	0.0158	-0.0134	-0.0027	0.0103	0.0007
12	3.0	0.0109	-0.0491	0.0045	0.0210	0.0073	0.0103	0.0100	-0.0004	0.0020	0.0007	-0.0003	0.0014	0.0155	-0.0145	-0.0027	0.0100	0.0009
12	4.0	0.0109	-0.0491	0.0042	0.0200	0.0064	0.0071	0.0097	-0.0005	0.0019	0.0007	-0.0004	0.0013	0.0152	-0.0155	-0.0027	0.0098	0.0011
16	2.0	0.0061	-0.0276	0.0028	0.0127	0.0051	0.0092	0.0059	-0.0001	0.0013	0.0005	-0.0001	0.0009	0.0089	-0.0072	-0.0015	0.0059	0.0003
16	3.0	0.0061	-0.0276	0.0027	0.0122	0.0046	0.0074	0.0058	-0.0002	0.0012	0.0004	-0.0001	0.0008	0.0088	-0.0077	-0.0015	0.0057	0.0004
16	4.0	0.0061	-0.0276	0.0026	0.0118	0.0041	0.0058	0.0056	-0.0002	0.0012	0.0004	-0.0002	0.0008	0.0087	-0.0081	-0.0015	0.0056	0.0005