

FAILURE CRITERIA FOR AN INDIVIDUAL LAYER OF A FIBRE REINFORCED COMPOSITE LAMINATE UNDER IN-PLANE LOADING

1. NOTATION

E_1, E_2	modulus of elasticity in 1- and 2-directions respectively (see Sketch 1.1), $E_1 = \frac{1}{S_{11}}$ and $E_2 = \frac{1}{S_{22}}$	N/m ²	lbf/in ²
G_{12}	shear modulus, $G_{12} = \frac{1}{S_{66}}$	N/m ²	lbf/in ²
Q	allowable shear stress of layer in 12-plane (see Sketch 1.1)	N/m ²	lbf/in ²
$\left. \begin{matrix} S_{11}, S_{22} \\ S_{12}, S_{21} \\ S_{66} \end{matrix} \right\}$	compliances of layer material (see Item No. 94003*) (note that $S_{12} = S_{21}$)	m ² /N	in ² /lbf
X	allowable direct stress in fibre direction	N/m ²	lbf/in ²
Y	allowable direct stress normal to fibre (in-plane)	N/m ²	lbf/in ²
ϵ	strain		
ϵ_Q	allowable shear strain in 12-plane (see Sketch 1.1)		
ϵ_X	allowable direct strain in fibre direction		
ϵ_Y	allowable direct strain normal to fibre (in-plane)		
θ	stress off-set angle measured from 1-axis	degree	degree
ν_1	longitudinal Poisson's ratio, ratio of transverse (in-plane) strain to longitudinal (fibre direction) strain due to an applied longitudinal direct stress		
	$\nu_1 = -\frac{S_{21}}{S_{11}}$		
ν_2	transverse Poisson's ratio, ratio of longitudinal strain to transverse strain due to an applied transverse direct stress		
	$\nu_2 = -\frac{S_{12}}{S_{22}}$		
σ	stress	N/m ²	lbf/in ²

For footnote see end of Notation

Issued June 1983
With Amendment A

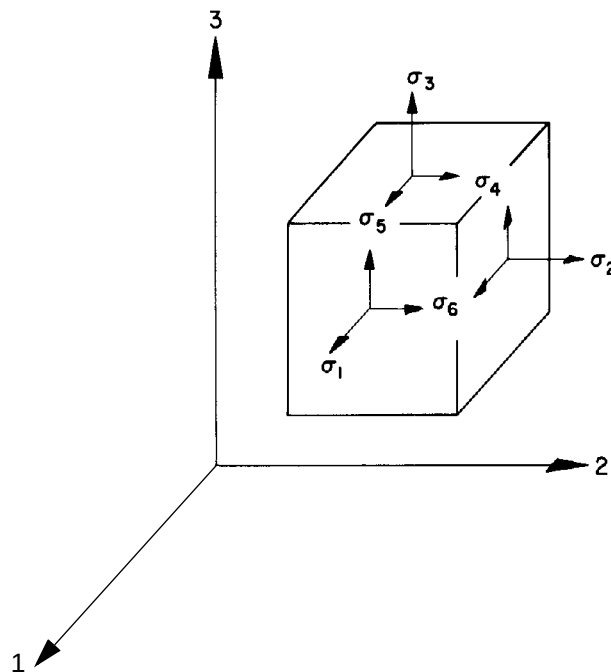
σ_1	direct stress in fibre direction	N/m^2	lbf/in^2
σ_2	direct stress normal to fibre (in-plane)	N/m^2	lbf/in^2
σ_6	shear stress in 12-plane	N/m^2	lbf/in^2
1, 2, 3	coordinate notation (see Sketch 1.1)		
<i>Suffixes</i>			
<i>c</i>	compression		
<i>t</i>	tension		
1, 2, 3	refer to coordinate system (see Sketch 1.1)		

* Data Item No. 94003 "Stiffnesses of laminated flat plates".

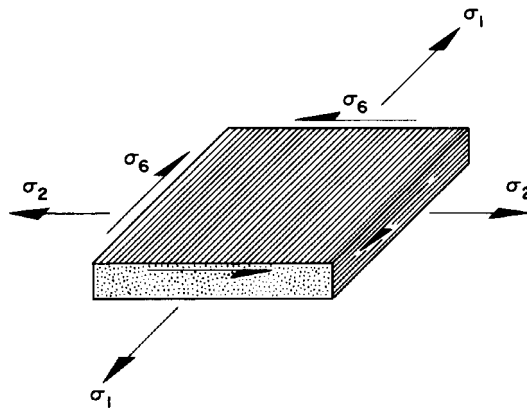
Other special notation is identified where used.

Stress and strain convention: tensile stresses and strains are positive, compressive stresses and strains are negative, allowable stresses X , Y and Q (tensile and compressive) are positive.

Both British and SI units are quoted but any coherent system of units may be used.



Sketch 1.1 Coordinate system



Sketch 1.2 Stress system aligned with principal (fibre) axis of orthotropy for individual layer in plane stress

2. INTRODUCTION

2.1 General

Failure criteria for an individual unidirectional fibre reinforced layer of a composite laminate under in-plane loading are presented in tabular form. Notes on the application of the criteria, their limitations and the tests required for use of the criteria are also given. The strength criteria considered here are macromechanical values based on the average stresses or strains in an individual layer. These criteria may be used to assess the strength of a layer subjected to combined in-plane loading from standardised strength data obtained from tests appropriate to the particular materials and criterion used.

2.2 Application to Multilayer Laminates

The single layer criteria considered here may be incorporated in step-by-step analyses of the whole laminate and therefore, once the requisite strength data have been obtained, offer flexibility in lay-up design. Other criteria that consider the laminate as a whole usually require data for each lay-up configuration as well as for the layer material.

3. CHOICE OF CRITERIA

There are many different approaches to the strength assessment of an anisotropic laminate under combined loading and therefore many different failure criteria have been developed. Each criterion has been supported by experiment to some extent but none may be applied universally with complete confidence. This is due to the complex stress interactions within the wide range of fibre and matrix material combinations and laminate lay-up configurations available. The choice of criterion will usually be based on what strength data are available or can be readily obtained. This Item will assist the user to choose that which most suits his requirements.

In Section 4 a number of criteria are listed for the strength assessment of an individual fibre reinforced composite layer under in-plane loading, for which a condition of plane stress is assumed to prevail as is shown in Sketch 1.2. The criteria presented have been selected on the following bases.

- (1) representation of each of the types of criterion that are available,
 - (2) frequency of use (that is popularity)
- and (3) apparent accuracy.

4. TYPES OF FAILURE CRITERIA

Eight failure criteria for single fibre reinforced composite layers under in-plane loading have been selected on the above bases. These are listed in Table 4.1 and are described individually in Sections 5 to 11. Further examples of the criteria that are available are given in Appendix A.

The criteria given in Table 4.1 are grouped into three main types:

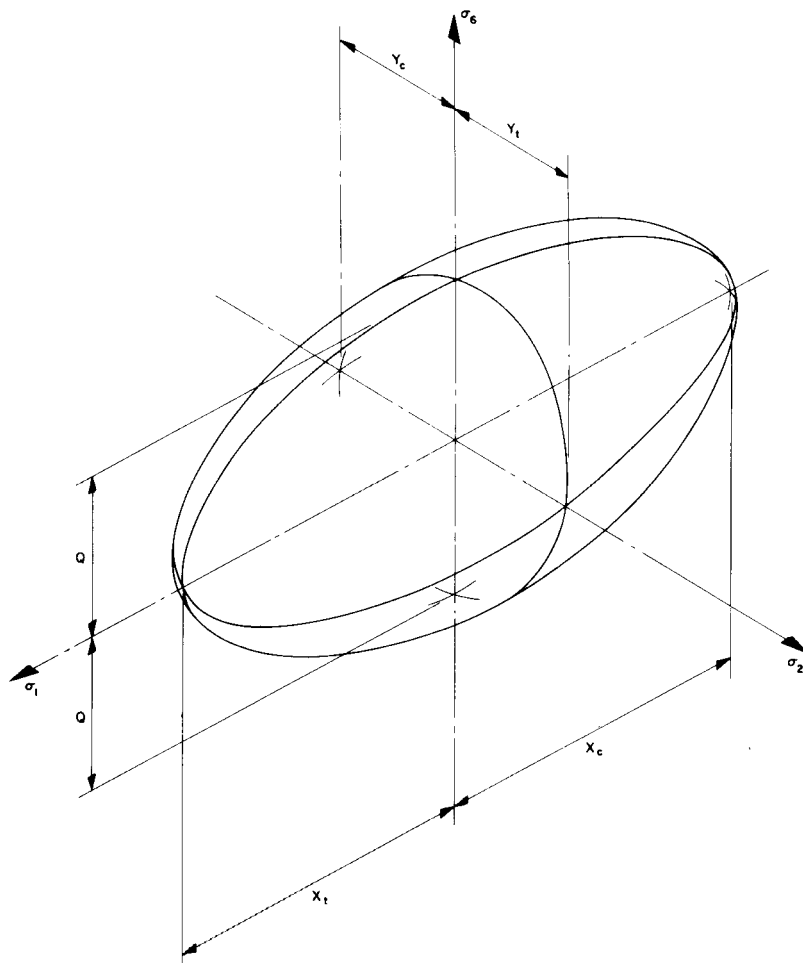
- 1 criteria each comprising a number of independent conditions,
 - 2 criteria each comprising a number of independent and interactive conditions
- and 3 criteria each comprising a number of fully interactive conditions.

Each criterion relates to a single orthotropic layer in plane stress ($\sigma_3 = \sigma_4 = \sigma_5 = 0$). The simpler criteria given in Table 4.1, types 1a, 1b, 2b, 2c and 3a, have been derived on this assumption. Types 2a, 3b and 3c are simplified versions of criteria which have been formulated with a full (three-dimensional) laminate in mind. Only type 2a is based on an assessment of the detailed failure mechanisms of the layer, although criteria 2b, 2c and 3c do consider the fibre-matrix structure of the layer. Notes on the tests used to establish the allowable stresses required for use of the criteria are given in Section 12.

The coordinate system that is used for the criteria is presented in Sketches 1.1 and 1.2. Sketch 4.1 shows a three-dimensional surface representing a failure criterion that is a function of σ_1 , σ_2 and σ_6 . The criteria are given in a form that assumes an applied stress system aligned with the principal axes of orthotropy as shown in Sketch 1.2. Stresses applied at an angle θ to the principal axes of orthotropy should be transformed as illustrated in Data Item No. 94003.

TABLE 4.1 FAILURE CRITERIA

Type	Criterion	Critical conditions	See Notes, Section No.	Derivation
1. Independent conditions	(a) Maximum stress	$\sigma_1 = X_t \text{ or } -X_c$ or $\sigma_2 = Y_t \text{ or } -Y_c$ or $ \sigma_6 = Q$	5	13, 20, 23, 26
	(b) Maximum strain	$\epsilon_1 = \epsilon_{Xt} \text{ or } -\epsilon_{Xc}$ or $\epsilon_2 = \epsilon_{Yt} \text{ or } -\epsilon_{Yc}$ or $ \epsilon_6 = \epsilon_Q$	6	13, 15, 20, 22, 23, 26
2. Independent – partly interactive	(a) Grant-Sanders	$\sigma_1 = X_t \text{ or } -X_c$ plus tension/shear and compression/shear interaction formulae or $\epsilon_{m1} \text{ or } \epsilon_{m2} = \epsilon_{mc}$ or $ \sigma_6 = Q_f \text{ or } Q_m$	7	30, 31, 32
	(b) Puck	$\sigma_1 = X_t \text{ or } -X_c$ or $\left(\frac{\sigma_2}{Y}\right)^2 + \left(\frac{\sigma_6}{Q}\right)^2 = 1$	8	7, 16, 22
	(c) Puck modified	$\sigma_1 = X_t \text{ or } -X_c$ or $\frac{\sigma_2^2}{Y_t Y_c} + \sigma_2 \left(\frac{1}{Y_t} - \frac{1}{Y_c}\right) + \left(\frac{\sigma_6}{Q}\right)^2 = 1$	8	21, 22
3. Fully interactive	(a) Tsai-Hill	$\left(\frac{\sigma_1}{X}\right)^2 - \frac{\sigma_1 \sigma_2}{X^2} + \left(\frac{\sigma_2}{Y}\right)^2 + \left(\frac{\sigma_6}{Q}\right)^2 = 1$	9	1, 5, 13, 15, 16, 17, 20, 22
	(b) Tsai-Wu	$A_{11}\sigma_1^2 + 2A_{12}\sigma_1\sigma_2 + A_{22}\sigma_2^2 + A_{66}\sigma_6^2 + B_1\sigma_1 + B_2\sigma_2 = 1$	10	9, 13, 14, 15, 20, 22, 23, 26, 27
	(c) Puppo-Evensen	$\left(\frac{\sigma_1}{X}\right)^2 - \phi \frac{X\sigma_1\sigma_2}{Y} + \phi \left(\frac{\sigma_2}{Y}\right)^2 + \left(\frac{\sigma_6}{Q}\right)^2 = 1$ or $\phi \left(\frac{\sigma_1}{X}\right)^2 - \phi \frac{Y\sigma_1\sigma_2}{X} + \left(\frac{\sigma_2}{Y}\right)^2 + \left(\frac{\sigma_6}{Q}\right)^2 = 1$	11	11, 13, 16



Sketch 4.1 Form of the failure surface for an orthotropic layer under in-plane stress

5. MAXIMUM STRESS CRITERION

In this criterion layer failure is judged to occur when any one of the three stresses σ_1 , σ_2 or σ_6 reaches the allowable value.

$$\left. \begin{aligned} \sigma_1 &= X_t \text{ or } -X_c \\ \text{or } \sigma_2 &= Y_t \text{ or } -Y_c \\ \text{or } |\sigma_6| &= Q \end{aligned} \right\} \quad (5.1)$$

Figure 1 illustrates the effect of an applied direct stress $\sigma_{\theta 1}$ at an angle θ to the 1-axis and presents test results for comparison. It can be seen that the critical stress is different for different angles of the applied stress. The discontinuities in the curves occur because interaction between the stresses is not considered.

6. MAXIMUM STRAIN CRITERION

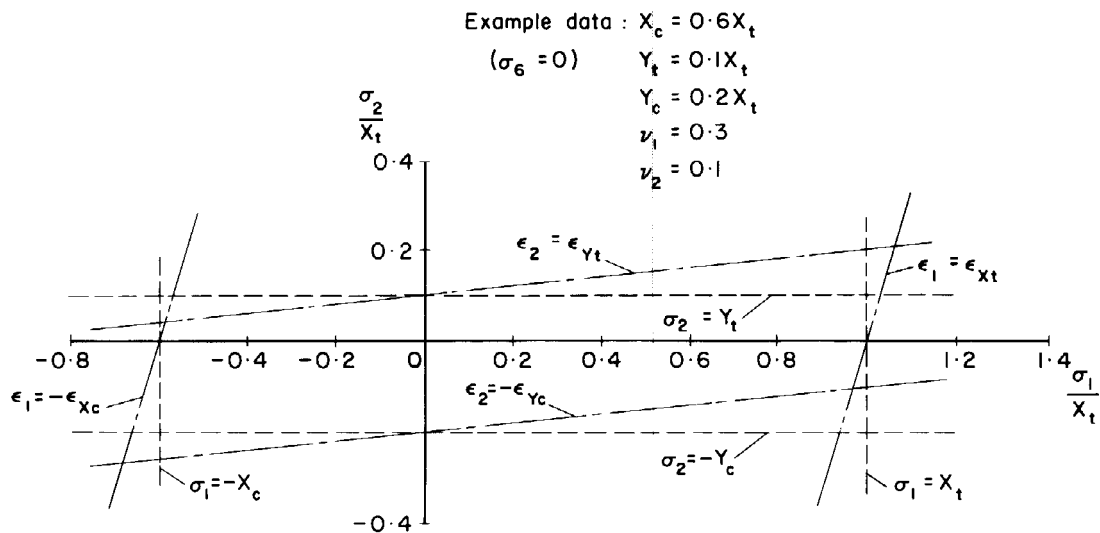
This criterion is applied in the same way as the maximum stress criterion, failure being assumed to occur when any one of the three strains ϵ_1 , ϵ_2 and ϵ_6 reaches the allowable value.

$$\left. \begin{array}{l} \epsilon_1 = \epsilon_{Xt} \text{ or } -\epsilon_{Xc} \\ \text{or} \quad \epsilon_2 = \epsilon_{Yt} \text{ or } -\epsilon_{Yc} \\ \text{or} \quad |\epsilon_6| = \epsilon_Q \end{array} \right\} \quad (6.1)$$

This may be given in terms of the stresses which, for a biaxial stress system, results in the following form.

$$\left. \begin{array}{l} \sigma_1 - \nu_1 \sigma_2 = X_t \text{ or } -X_c \\ \text{or} \quad \sigma_2 - \nu_2 \sigma_1 = Y_t \text{ or } -Y_c \\ \text{or} \quad |\sigma_6| = Q \end{array} \right\} \quad (6.2)$$

Figure 2 illustrates the effect of changing the angle of an applied stress $\sigma_{\theta 1}$ to the principal material axis and compares the criterion with test results. Sketch 6.1 compares the stress failure envelope forms for the maximum stress and strain criteria.



Sketch 6.1 Stress envelopes for maximum stress and strain criteria

7. GRANT-SANDERS CRITERION

This method uses a number of separate criteria, some simple and others involving interaction between stress and strain components, to assess the possibility of failure in each of the distinct modes identified by a study of individual layers in multilayer laminates.

For single layers, the simple criteria are:

$$\left. \begin{aligned} \sigma_1 &= X_t \text{ or } -X_c \\ \epsilon_{m1} \text{ or } \epsilon_{m2} &= \epsilon_m \\ |\sigma_6| &= Q_f \text{ or } Q_m \end{aligned} \right\} \quad (7.1)$$

where Q_f is shear strength across fibres,
 Q_m is matrix material shear strength parallel to fibres,
 ϵ_{m1} is matrix material direct strain in fibre direction,
 ϵ_{m2} is matrix material direct strain normal to fibres (in-plane)
 and ϵ_m is equivalent allowable matrix material direct strain given by the maximum matrix material tensile stress divided by the material modulus of elasticity.

For combined tension and shear:

$$\left(\frac{\epsilon_6}{\epsilon_{mQ}} \right)^2 + \left(\frac{\epsilon}{\epsilon_m} \right)^2 = 1, \quad (7.2)$$

where $\epsilon = \epsilon_1$ or ϵ_2 and ϵ_{mQ} is the matrix allowable shear strain (at stress Q_m).

For combined compression and shear (σ_1 negative):

$$\frac{|\sigma_6|}{KQ_m} - \frac{\sigma_1}{X_c} = 1, \quad (7.3)$$

where K is a factor determined by experiment (Derivation 30); usually $K \approx 1.5$.

This method requires more parameters to be determined by testing than the others presented in this Item. This extensive testing is only relevant and justifiable when the method is used, as it is intended, for assessing failure in multidirectional laminates. The method is included here to illustrate the relative complexity of failure criteria that are based on distinct observed failure modes.

8. PUCK CRITERION

8.1 Simple Criterion

This criterion is derived from the assumption that fibre (longitudinal) failures are determined by σ_1 and matrix failures are determined by σ_2 and σ_6 .

$$\text{or } \left. \begin{array}{l} \sigma_1 = X_t \text{ or } -X_c \\ \left(\frac{\sigma_2}{Y}\right)^2 + \left(\frac{\sigma_6}{Q}\right)^2 = 1 \end{array} \right\} \quad (8.1)$$

In the second expression Y should be replaced by Y_t or Y_c according to the sign of σ_2 .

8.2 Modified Criterion

When using the Puck criterion it has been found that where there are great differences between the tensile and compressive strengths of the material a more satisfactory fit to experimental data can be obtained by taking a single ellipse through the failure points of Y_t , Y_c and Q (Derivations 21 and 22).

Hence the modified criterion:

$$\text{or } \left. \begin{array}{l} \sigma_1 = X_t \text{ or } -X_c \\ \frac{\sigma_2^2}{Y_t Y_c} + \sigma_2 \left(\frac{1}{Y_t} - \frac{1}{Y_c} \right) + \left(\frac{\sigma_6}{Q} \right)^2 = 1 \end{array} \right\} \quad (8.2)$$

This criterion combines simple computation with simple testing while allowing interaction of stresses in both tension and compression, although it allows fewer interactions than the Tsai-Wu criterion and is less flexible than the Puppo-Evensen criterion. In Figures 6 and 7 the failure envelopes for these criteria when $\sigma_6 = 0$ and $\sigma_1 = 0$, respectively, are compared to those for the Tsai-Hill, Tsai-Wu and Puppo-Evensen criteria.

9. TSAI-HILL CRITERION

This criterion is developed from the more general Hill yield criterion for anisotropic materials (Derivation 1). For a single homogeneous orthotropic layer in plane stress the criterion becomes

$$\left(\frac{\sigma_1}{X}\right)^2 - \frac{\sigma_1 \sigma_2}{X^2} + \left(\frac{\sigma_2}{Y}\right)^2 + \left(\frac{\sigma_6}{Q}\right)^2 = 1, \quad (9.1)$$

where X should be replaced by X_t or X_c and Y should be replaced by Y_t or Y_c according to the signs of σ_1 and σ_2 respectively.

Unlike the previous criteria this is a single expression producing an ellipsoidal (three-dimensional) failure surface. In Figure 3 this criterion is shown as a failure envelope in the σ_1 , σ_2 plane. Curves are given for various values of the direct strength ratio (X/Y). The curves are obtained by rearranging Equation (9.1)