

THE KINEMATIC DESIGN OF EPICYCLIC GEAR TRAINS

1. NOTATION AND UNITS

In this Notation, units are given for both SI and a familiar British system.

The sign convention adopted by this Item is that a negative sign implies rotation in the opposite sense to the input rotation.

1.1 Principal Symbols

		<i>SI</i>	<i>British</i>
d	diameter	m	ft
E	epicyclic gear train efficiency		
F	number of degrees of freedom		
Q	number of co-planar planet gears in any one single carrier gear train		
T	torque	N m	lbf ft
t	number of 'gear meshings' between planet gears and other gears, ignoring those planets which are kinematically redundant (see Section 8)		
U_o	fixed-carrier ratio		
u	gear ratio		
V	linear velocity of planet centre relative to the axis of rotation of the planet carrier	m/s	ft/min
v	linear velocity (pitch line)	m/s	ft/min
Y	total number of moving members which can independently rotate about the central axis, together with the total number of planet gears, ignoring those which are kinematically redundant		
Z	number of teeth		
Δ	transmission loss due to tooth mesh (power loss across the mesh expressed as a fraction of the input power)		
η	meshing efficiency		
ω	angular velocity	rad/s	rad/s

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1.2 Suffixes and Identifying Symbols

A	annulus gear
b, d, e, f, g, h	general lettering
C	planet carrier
P	planet gear
S	sun gear
$1 \dots 9$	general numbering

1.3 Combined Symbols

d_P	reference diameter of planet gear	m	ft
d_S	reference diameter of sun gear	m	ft
m_n	normal module pitch	m	ft
Z_A	number of teeth on annulus gear		
Z_{A1}, Z_{A2}	number of teeth on annulus gears 1 and 2 respectively		
Z_P	number of teeth on sun gear		
Z_{P1}, Z_{P2}	number of teeth on planet gears 1 and 2 respectively		
Z_S	number of teeth on planet gear		
Z_{S1}, Z_{S2}	number of teeth on sun gears 1 and 2 respectively		
η_t	total efficiency of gear train when allowing only for losses due to meshing		
ω_A	angular velocity of annulus gear	rad/s	rad/s
ω_{A1}, ω_{A2}	angular velocity of annulus gears 1 and 2 respectively	rad/s	rad/s
ω_C	angular velocity of planet carrier	rad/s	rad/s
ω_{C1}, ω_{C2}	angular velocity of planet carriers 1 and 2 respectively	rad/s	rad/s
ω_S	angular velocity of sun gear	rad/s	rad/s
ω_{S1}, ω_{S2}	angular velocity of sun gears 1 and 2 respectively	rad/s	rad/s

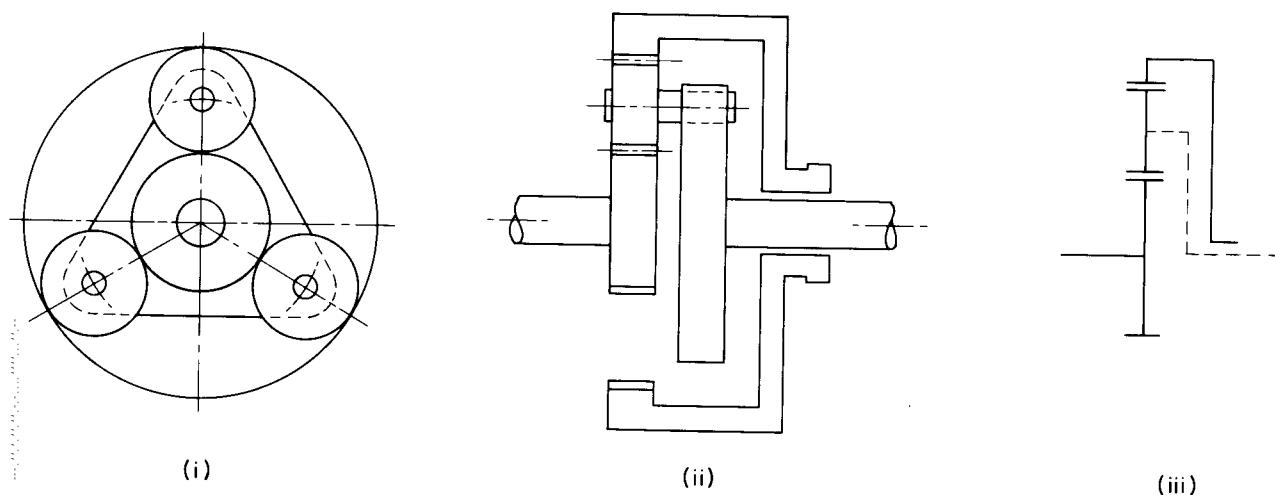
2. INTRODUCTION

This Item presents the user with two analytical methods for the kinematic design of epicyclic gear trains and is limited to a consideration of the kinematics and the efficiencies of the gear trains. However, Section 5 of this Item does give the formulae for determining the positions of multiple planets within an epicyclic gear train to ensure assemblability and Section 8 outlines a method for calculating the number of degrees of freedom in a gear train. Section 6 of this Item contains a tabular presentation of the efficiencies and achievable gear ratios of nine of the thirty four practical cases of a single-carrier gear which is here taken to mean a gear train containing two co-axial gears, one planet carrier and one or more planet gears. The remaining twenty five cases which are not in general use are presented in Appendix A. The use of bevel gears in epicyclic gearing is not covered by this Item and the data presented applies to full-depth involute tooth form having 20° pressure angle.

Epicyclic gear trains are used where space and/or weight saving is important. Power shifting is readily accommodated, and ratio changes are usually accomplished by the application and release of friction elements. The axial shifting of gears is not required thus avoiding gear clash and synchronizer problems. The use of multiple planets within an epicyclic gear train allows the use of a smaller driving gear and provided good load distribution is guaranteed this type of gear train is quieter than other types. The requirements for load distribution, load sharing and dynamic balancing in multiple planet trains, not covered by this Item, must be satisfied before completing a design.

In general, epicyclic gear trains have more than one planet gear meshed with the same sun gear and annulus gear. Sketch 2.1(i) shows diagrammatically a typical example where three coplanar planets are each meshed with the same sun and annulus gear. Sketch 2.1(ii) shows the side view of the same gear after the kinematically redundant gears have been removed and Sketch 2.1(iii) gives the schematic representation that is adopted throughout the rest of this Item. It should be noted that the kinematically redundant planets are provided for load carrying purposes and force and mass balancing.

For the arrangements given in Tables 6.1 to 6.9, contained in Section 6, the range of ratios shown approximately correspond to those determined by arbitrarily choosing co-planar planet numbers in the range of 3 to 7.



Sketch 2.1

3. METHODS OF ANALYSIS

This section outlines two methods for the kinematic analysis of epicyclic gear trains and also defines the fixed-carrier ratio. The first method is tabular whereas the second is suitable for analysis by computer.

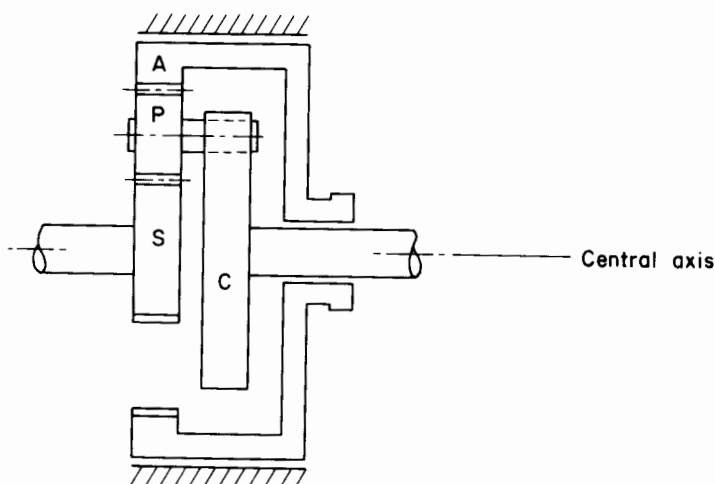
3.1 Fixed-carrier Ratio

The fixed-carrier ratio, U_o , is calculated by analysing the inversion of the mechanism which produces a simple gear train having a single degree of freedom, that is to say with the planet carrier assumed to be at rest and the two co-axial gears free to rotate. The fixed-carrier ratio is then equal to the ratio of the speeds of the two co-axial gears and can be expressed in terms of gear teeth numbers.

For example, consider the gear train shown in Sketch 3.1. If the planet carrier is considered to be fixed, the fixed carrier ratio is then equal to the ratio of the speeds of the two co-axial gears (labelled as S and A in Sketch 3.1) when they are both free to rotate. It should be noted that the speed of the co-axial gear which is fixed when the planet carrier is free to rotate always forms the denominator of the equation for the fixed-carrier-ratio. The fixed-carrier-ratio of the gear train shown in Sketch 3.1 is as follows.

$$|U_o| = \frac{\text{Speed of } S}{\text{Speed of } A} = \frac{\omega_S}{\omega_A} = \frac{Z_A}{Z_S}.$$

Note: The relative rotation of the gears must be strictly adhered to, and for the case outlined above the value of U_o would be negative. If an idler gear were to be placed between the sun and planet gear the resulting value of U_o would then be positive.



Sketch 3.1

3.2 Tabular Method

Sketch 3.1 shows a single carrier gear train with two degrees of freedom (see Section 8). The method adopted for the analysis of this particular gear train is a general one and may be used for the analysis of any two-degrees-of-freedom epicyclic gear train. The method is as follows.

- (i) Calculate the fixed-carrier-ratio, U_o , of the gear train as outlined in Section 3.1. The result of this is shown in Table 3.1.

TABLE 3.1

Action	Member			
	S	A	C	P
Input 1 to S with C fixed	1	$-\frac{Z_S}{Z_A}$	0	$-\frac{Z_S}{Z_P}$
Divide by $-\frac{Z_S}{Z_A}$	$-\frac{Z_A}{Z_S}$ (= U_o)	1	0	$\frac{Z_A}{Z_P}$

- (ii) Assign an input value equal to U_o to any member other than the fixed planet carrier, usually a sun gear (labelled S in Sketch 3.1). The resulting motions of the other gears are calculated and tabulated setting zeros for the non-rotating members and observing the algebraic signs for the direction of rotation of the rotating members. Table 3.2 shows the result of this analysis on the planetary gear train shown in Sketch 3.1 with the velocity of the planet gear given in terms of gear teeth numbers, Z.

TABLE 3.2

Action	Member			
	S	A	C	P
Input U_o to S	U_o	1	0	$-\frac{Z_S \cdot U_o}{Z_P}$

- (iii) The terms contained in Table 3.2 are manipulated to create the condition requiring analysis. The terms are manipulated by the addition or subtraction of any number or by the multiplication or division by any number. Each term must be manipulated in the same manner, i.e. if the value of 1 is added to the first term it must in turn be added to all of the other terms.

For example it may be required to find the ratio between the sun gear and the planet carrier of the gear train shown in Sketch 3.1 when the annulus gear is fixed. If A is to be fixed, the term for its rotational velocity in Table 3.2 must be manipulated to become zero. Hence, 1 is subtracted from all of the terms and the result of this action is shown in Table 3.3.

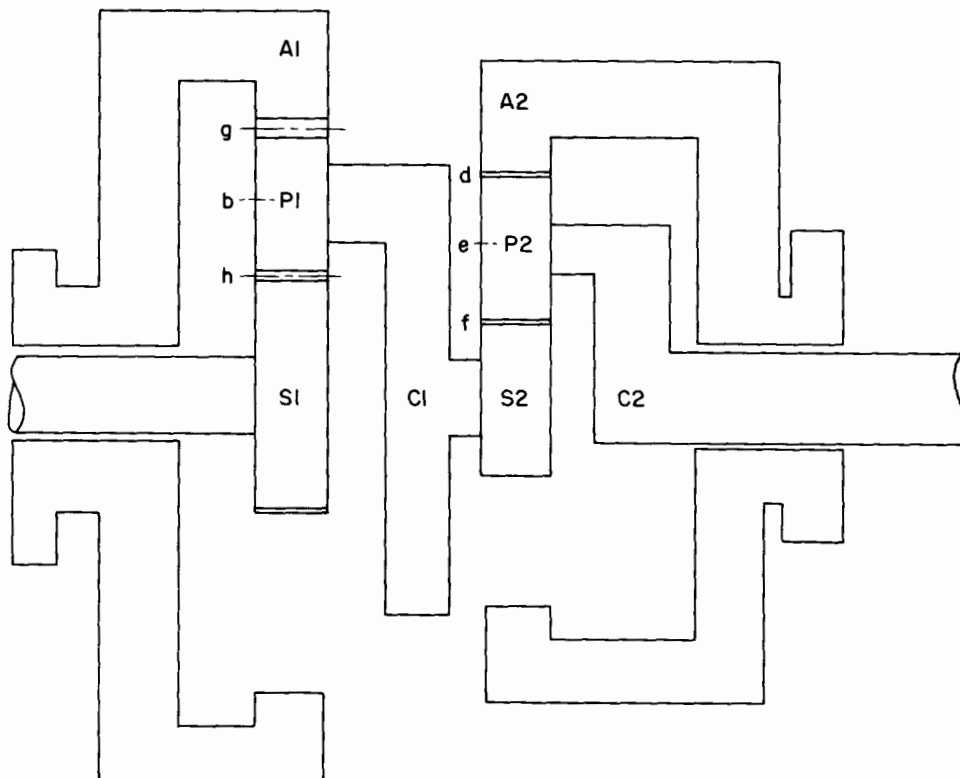
TABLE 3.3

Action	Member			
	S	A	C	P
Input U_o to S	U_o	1	0	$-\frac{Z_S \cdot U_o}{Z_P}$
- 1 (fixes A)	$U_o - 1$	0	- 1	$-\frac{Z_S \cdot U_o}{Z_P} - 1$

The ratio, u , between the sun gear and the planet carrier, for the case when the annulus is fixed, is equal to the ratio of their respective terms in Table 3.3.

$$u = \frac{U_o - 1}{-1} = 1 - U_o.$$

3.3 Pitch Line Velocity Method



Sketch 3.2

Sketch 3.2 shows a coupled epicyclic gear train (see Section 7) which has three degrees of freedom. As an alternative to the tabular method the procedure for the kinematic analysis of this gear train using the method of pitch line velocities is as follows (a flow chart of this process is contained in Appendix B).

- (i) Express the linear velocities of the planet centres, b and e , in terms of the unknown angular velocities, ω_{C1} , and ω_{C2} , of the planet carriers $C1$ and $C2$ as illustrated in Sketch 3.2. The velocity at b equals the angular velocity of the gear multiplied by the pitch-circle radius, where the pitch-circle radius = $Z \cdot m_n/2$ and m_n is the normal module pitch (mm).

It can be shown that the linear velocities of the planet centres are as follows,

$$\left. \begin{aligned} V_b &= (Z_{S1} + (Z_{A1} - Z_{S1})/2) \cdot \omega_{C1} \cdot m_{n1}/2 \\ V_e &= (Z_{S2} + (Z_{A2} - Z_{S2})/2) \cdot \omega_{C2} \cdot m_{n2}/2 \end{aligned} \right\} \quad (3.1)$$

- (ii) Express the linear velocities at the pitch points (labelled as g , h , d and f in Sketch 3.2) in terms of the unknown velocities of the corresponding sun gears (ω_{S1} and ω_{S2}), and the annulus gears (ω_{A1} and ω_{A2}).

$$\left. \begin{aligned} v_g &= Z_{A1} \cdot \omega_{A1} \cdot m_{n1}/2 \\ v_h &= Z_{S1} \cdot \omega_{S1} \cdot m_{n1}/2 \\ v_d &= Z_{A2} \cdot \omega_{A2} \cdot m_{n2}/2 \\ v_f &= Z_{S2} \cdot \omega_{S2} \cdot m_{n2}/2 \end{aligned} \right\} \quad (3.2)$$

- (iii) Relate the linear velocities of the planet centres to those of the pitch points.

$$\left. \begin{aligned} v_g - 2 \cdot V_b + v_h &= 0 \\ v_d - 2 \cdot V_e + v_f &= 0 \end{aligned} \right\} \quad (3.3)$$

- (iv) Substitute the equations obtained from procedures (i) and (ii) into the equations from procedure (iii) to obtain equations relating the angular velocities of the sun gears, the annulus gears and the planet carriers.

$$\left. \begin{aligned} Z_{A1} \cdot \omega_{A1} - 2 \cdot (Z_{S1} + (Z_{A1} - Z_{S1})/2) \cdot \omega_{C1} + Z_{S1} \cdot \omega_{S1} &= 0 \\ Z_{A2} \cdot \omega_{A2} - 2 \cdot (Z_{S2} + (Z_{A2} - Z_{S2})/2) \cdot \omega_{C2} + Z_{S2} \cdot \omega_{S2} &= 0 \end{aligned} \right\} \quad (3.4)$$

where $\omega_{S2} = \omega_{C1}$.

Equations (3.4) are valid for all ratios of the planetary gear train shown in Sketch 3.2. The gear ratios, u , can be found by setting the angular velocities of the fixed members to zero.

4. EFFICIENCY OF GEAR TRAINS

This section presents a method for calculating the efficiency of single carrier gear trains (see Section 6). It must be noted that the method adopted gives an approximate answer only, but is useful for giving comparative efficiencies for initial selection or when more pertinent data are not available. The efficiency of an epicyclic train can vary greatly according to the configuration and the members used for input and output.

The principle losses in the operation of a gear train are those due to sliding friction between the mating tooth surfaces, to oil churning within the train and to friction in the shaft support bearings. When considering the efficiencies given by this Item, the following assumptions have been made:

- (i) Losses due to gear meshes between external/internal gears are in general less than those between external/external gears,
- (ii) The pitch-line velocity of a gear has no influence on the overall efficiency of a gear train.
- (iii) The efficiency loss due to meshing is independent of the gear tooth form.
- (iv) Oil churning and bearing losses are not included.

4.1 Direction of Power Flow

To calculate the efficiency of an epicyclic gear train, the direction in which the power flows through the planet system must be determined. This is obtained by calculating the fixed-carrier ratio, U_o , of the gear train being considered (see Section 3.1). The direction of the power flow is then determined from Table 4.1.

The notation adopted within Table 4.1 is as follows:

I = the moving co-axial gear

J = the fixed co-axial gear

K = the moving planet carrier

TABLE 4.1

<i>Member</i>		<i>Direction of Power Flow</i>	
<i>Input</i>	<i>Output</i>	$I \rightarrow J$	$J \rightarrow I$
I	K	$U_o < 0$ $U_o > 1$	$0 < U_o < 1$
K	I	$0 < U_o < 1$	$U_o < 0$ $U_o > 1$

Note: $U_o = 1$ represents a case where there is no drive.

4.2 Efficiency Loss Due to Tooth Mesh

Sketches 4.1a and 4.1b give the values of the transmission losses, Δ , for external gears paired together and for internal gears paired with external gears.

The transmission losses are presented as a function of the speed ratio between the two gears being considered. The abscissae represent the ratios between the numbers of teeth on pairs of gears where the numerator of the ratio is always the number of teeth on the smaller gear.

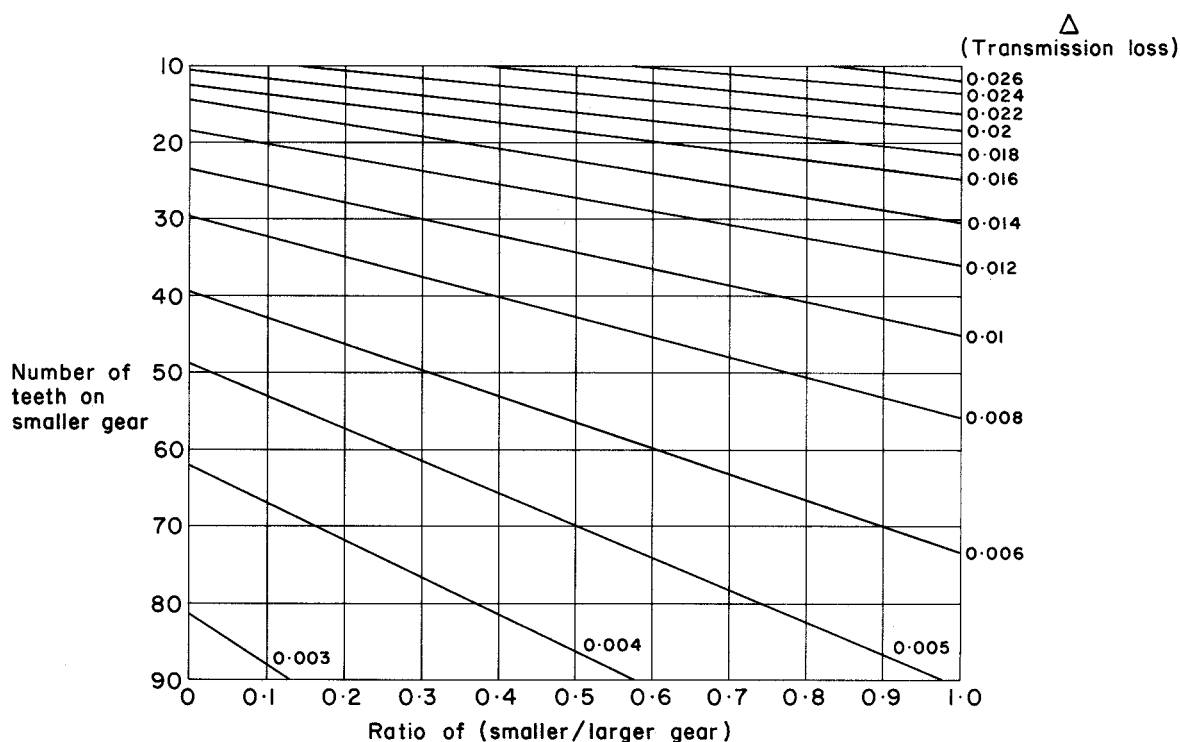
To calculate the transmission loss due to tooth mesh in a gear train, the efficiency losses at each of the individual meshing points within the gear train must be determined from Sketches 4.1a and 4.1b. Having obtained these values, the meshing efficiency, η , for each mesh can be determined from Equation (4.1).

$$\left. \begin{aligned} \eta_1 &= (1 - \Delta_1) \\ \eta_n &= (1 - \Delta_n) \end{aligned} \right\} \quad (4.1)$$

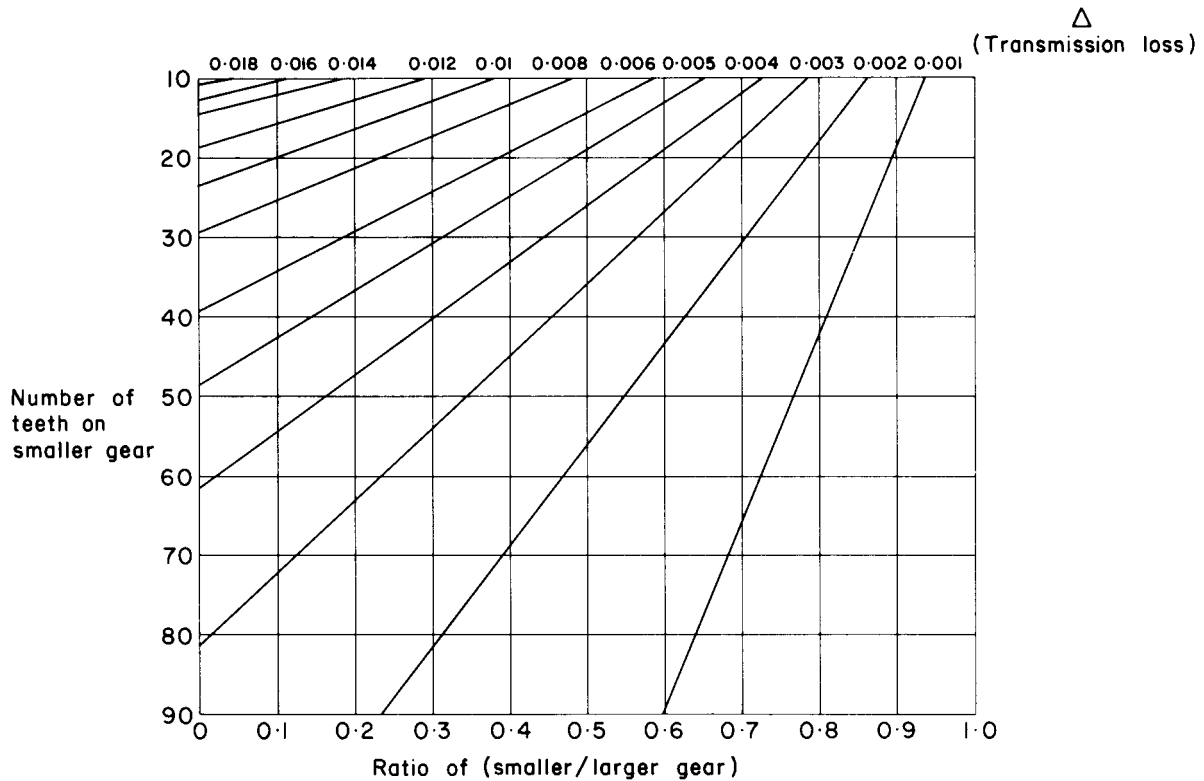
where 1 and n represent the first and n^{th} mesh respectively.

The total efficiency of the gear train, η_t , which is the efficiency of the gear train when allowing only for losses due to meshing, is determined from Equation (4.2).

$$\eta_t = \eta_1 \cdot \eta_2 \cdot \dots \cdot \eta_n \quad (4.2)$$



Sketch 4.1a EXTERNAL/EXTERNAL MESH



Sketch 4.1b EXTERNAL/INTERNAL MESH

4.3 Gear Train Efficiency

The epicyclic gear train efficiency is expressed by Equation (4.3).

For a given input torque,

$$E = \frac{\text{Output torque with efficiency losses}}{\text{Output torque without efficiency losses}} \quad (4.3)$$

The appropriate formulae for the output torques are given in Table 4.2. The formulae are presented according to which members are used to provide the input and the output motions, and the direction in which the power flows within the gear trains. The output torque is considered to be in the opposite sense to that of the input torque.

The notation adopted within Table 4.2 is as follows.

I = the moving co-axial gear

J = the fixed co-axial gear

K = the moving planet carrier,

TABLE 4.2

Member		Power-flow with Efficiency Losses		Power-flow without Efficiency Losses	
Input	Output	$I \rightarrow J$	$J \rightarrow I$	$I \rightarrow J$	$J \rightarrow I$
I	K	$T_K = T_I \cdot (U_o \cdot \eta_t - 1)$	$T_K = T_I \cdot \left(\frac{U_o}{\eta_t} - 1 \right)$	$T_K = T_I (U_o - 1)$	
K	I	$T_I = \frac{T_K}{U_o \cdot \eta_t - 1}$	$T_I = \frac{T_K \cdot \eta_t}{U_o - \eta_t}$	$T_I = \frac{T_K}{(U_o - 1)}$	

4.4 Efficiencies and Internally Transmitted Power

When calculating the efficiencies of certain epicyclic gears the user may obtain values which are substantially higher or lower than expected.

High Efficiencies

Analysis of a high reduction gear train may yield a high value for the total gear train efficiency, perhaps as much as 98 per cent. However, it is unlikely that this train will be practical or possible when the load carrying and the space requirements are considered. For improved load carrying capabilities an epicyclic gear will require multiple planets. The number of planet gears required will greatly affect the achievable ratio of an epicyclic gear train (see Section 5.1).

Low Efficiencies

In certain epicyclic gears the internally circulating power across the meshing gears can be considerably higher than the input power, perhaps as much as 50 times greater (see Reference 7) and in such cases most of the input power will be lost even if the meshing gears are 99 per cent efficient. Because of this the efficiencies of the meshing gears become critical. Low efficiencies imply heat generation and the possibility of the mechanism locking.

4.5 Reversibility

The reversibility of an epicyclic gear train is an important design consideration. If a gear train has a high efficiency when the output member becomes the input member, it may be considered to be reversible. A low efficiency in this direction would imply a tendency for the gear to be irreversible. In practice, whenever the input torque is removed from the input member, the output member then provides the input drive. If the gear train is very inefficient in its reverse direction, locking may occur. However, this condition may be required, as in the case of an indexing mechanism.

5. ASSEMBLABILITY OF MULTIPLE PLANETS

In general, when designing epicyclic gear trains the number of co-planar planets placed in each single planet carrier gear train may range from two up to as many as seven. The use of multiple planets in epicyclic gears reduces individual tooth contact forces and can balance out shaft bearing forces, but shows diminishing returns when using more than three co-planar planets. To design the planet carrier it is necessary to determine the number of planets and their spacing. If the correct spacing for the number of planets used is not observed the gear train will not assemble.

5.1 Number of Planets

In general, at least three co-planar planets are required for a practical epicyclic gear. The number of co-planar planets used in an epicyclic inversion places a restriction on the achievable gear ratio. Equations (5.1) give the maximum ratios between the number of teeth on the planet, sun and annulus gears, according to the number of co-planar planets, Q , required.

$$\left. \begin{aligned} \frac{Z_P}{Z_S} &= \frac{M}{1-M} - \frac{2}{Z_S} \\ \frac{Z_P}{Z_A} &= \frac{M}{1+M} - \frac{2}{Z_A} \end{aligned} \right\} \quad (5.1)$$

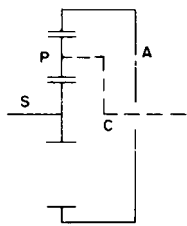
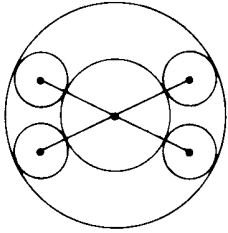
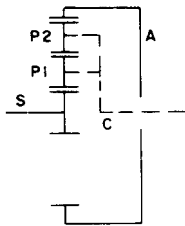
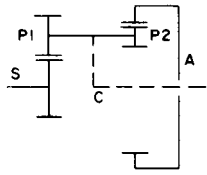
where $M = \sin \left\{ \frac{180}{Q} \right\}$ and $Q > 2$.

If the addendum modification is larger than normal, further account of interference is required.

5.2 Assemblability

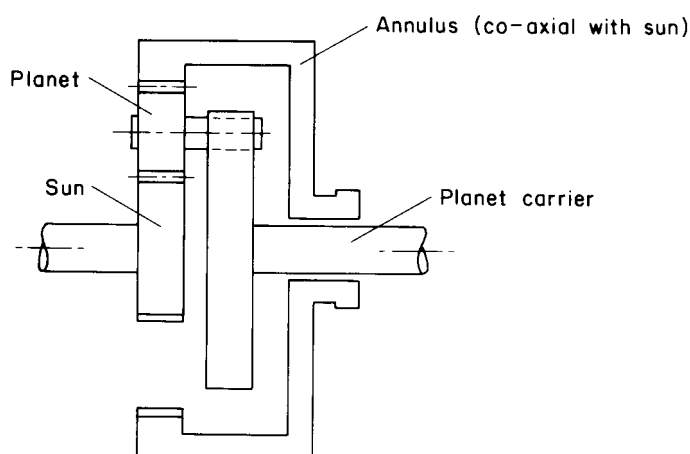
The required number of co-planar planets may not always assemble into a given gear train. The required formulae for assemblability are presented in Table 5.1.

TABLE 5.1

Type	Schematic	Equation of Assemblability $\begin{cases} Q = \text{No. of Planets} \\ Z = \text{No. of Teeth} \end{cases}$	Planet Spacing
Simple Planetary Gear		$\frac{Z_A + Z_S}{Q} = \text{An integer}$	Equal
Simple Planetary Gear	 Note:- With irregularly spaced planets, as shown in the schematic, force and mass balancing can be achieved if there is symmetry about two mutually perpendicular diameters.	Incremental angle $\beta = \frac{360}{Z_S + Z_A}$ Planets must be spaced at integer multiples of the incremental angle	Unequal
Simple Planetary Gear with Intermediate Planet Gear		$\frac{Z_A - Z_S}{Q} = \text{An integer}$	Equal
Compound Planets		$Z_S + \frac{Z_{P1} \cdot Z_A}{Z_{P2}} = \text{An integer}$	Equal

6. SINGLE-CARRIER GEAR TRAINS (TWO DEGREES OF FREEDOM)

A single-carrier gear train consists of two co-axial gears, one or more planet gears and one planet carrier, as illustrated in Sketch 6.1. This Section contains the nine configurations which are generally practical and allow the use of multiple co-planar planets (see Section 5).



Sketch 6.1

All elementary gear trains, such as the one shown in Sketch 6.1, have two degrees of freedom (see Section 8 for a method of calculating the number of degrees of freedom). To achieve a constant gear ratio the number of degrees of freedom of the train must be reduced to one by fixing the annulus gear, the planet carrier or the sun gear.

6.1 Information Given in the Tables

This section of the Item contains 9 tables, each one depicting a unique simple planetary gear train. The information contained in the tables is explained below and is illustrated with the aid of Sketch 6.2 overleaf.

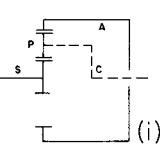
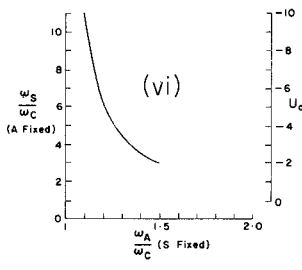
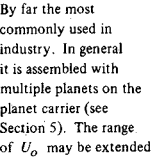
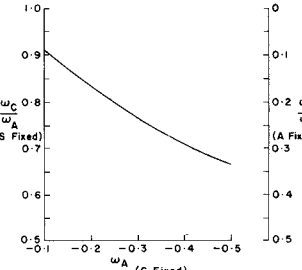
The areas of the table labelled in Sketch 6.2 are as follows.

- (i) Schematic representations of the simple planetary gear train being considered are given in this section of the table.
- (ii) This section of the table identifies which members are being used as the input, the output and the reaction (fixed).
- (iii) The gear ratio, u , of the case being considered (as defined in Section (ii) of the table) is given in this section of the table. A positive value indicates that the output gear rotates in the same direction as the input gear and a negative value indicates that it rotates in the opposite direction.
- (iv) This section of the table details the amount of rotation of the planet bearings, about their own axis, for one revolution of the output gear. A positive value indicates that the planet bearing rotates in the same direction as the output gear and a negative value indicates that it rotates in the opposite direction.
- (v) The sketches in this section of the table show the efficiencies (the graph ordinates) plotted as a

function of the gear ratios (the graph abscissae defined as the input member over the output member in Section (ii) of the table), for two differing values of the overall losses due to meshing, η_t . These two η_t values are the likely maxima and minima for the gear arrangement in question, and a linear interpolation can be used to find the efficiency for a given η_t value. The comparative efficiencies given do not include any losses due to the use of multiple planets or oil churning, hence, they are likely to be slightly higher than the practical values that can be achieved. However, these values do provide a good approximation for a given epicyclic gear train (see Examples).

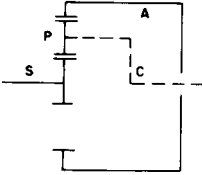
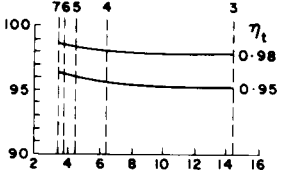
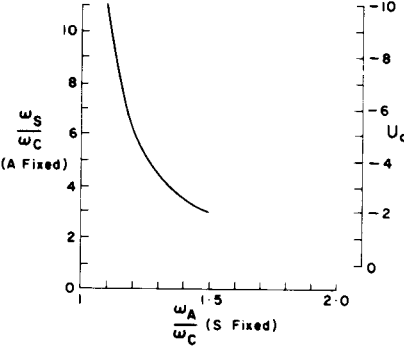
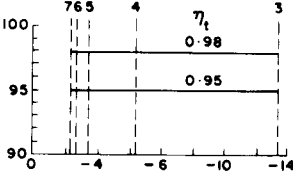
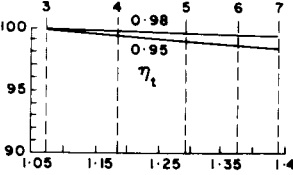
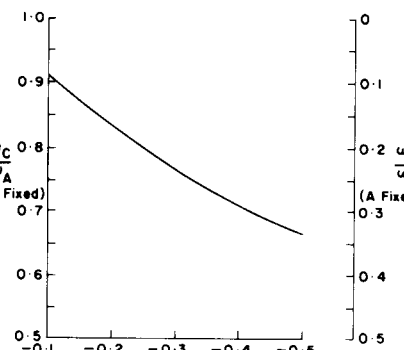
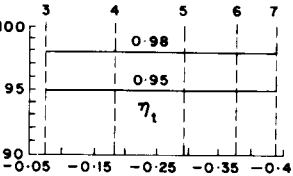
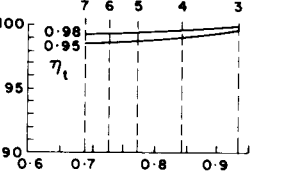
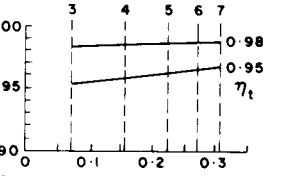
The number of co-planar planets used in an epicyclic inversion places a restriction on the achievable gear ratio. Where applicable, these restrictions have been shown as dotted lines on the efficiency graphs. These lines represent the maximum number of planets possible for the ratio specified, corresponding to the arrangement with the smallest overall diameter and based upon the requirement that each external gear has a minimum number of 10 teeth. It is possible however, for all of the tables contained in this section, other than Table 6.1, to assemble any number of planets for a given gear ratio by increasing the size of the overall arrangement.

- (vi) The sketches in this section of the table show the relationships between the achievable gear ratios of the single carrier gear train being considered. The range of achievable gear ratios can be extended from those given in the tables; however, it must be noted that the efficiency of the gear train will differ according to which gear is used to provide the input. The efficiencies must be checked to ensure that the train is a practical proposition.

SCHEMATIC	MEMBER			RATIO u	PLANET BEARING SPEED				COMPARATIVE EFFICIENCIES	ACHIEVABLE RATIOS
	INPUT	FIXED	OUTPUT		P1	P2	P3	P4		
 $U_o = -\frac{Z_A}{Z_S}$ NOTES By far the most commonly used in industry. In general it is assembled with multiple planets on the planet carrier (see Section 5). The range of U_o may be extended with careful design.	S	A	C	$1-U_o$	$1-\frac{Z_A}{Z_S}$					
	C	A	S	U_o	$\frac{Z_A}{Z_S}$					
 $U_o = -\frac{Z_A}{Z_S}$ NOTES By far the most commonly used in industry. In general it is assembled with multiple planets on the planet carrier (see Section 5). The range of U_o may be extended with careful design.	A	S	C	$1-\frac{1}{U_o}$	$1-\frac{Z_S}{Z_A}$					
	C	S	A	$\frac{1}{U_o}$	$\frac{Z_S}{Z_A}$					
	S	A	C	$\frac{U_o}{U_o-1}$	$\frac{U_o-1}{U_o-(Z_S/Z_A)}$					
	A	S	C	$\frac{1}{1-U_o}$	$\frac{1}{(Z_S/Z_A)-1}$					

Sketch 6.2 Example of table (showing Table 6.1)

TABLE 6.1

	MEMBER			RATIO u	PLANET BEARING SPEED				COMPARATIVE EFFICIENCIES	ACHIEVABLE RATIOS
	INPUT	FIXED	OUTPUT		P1	P2	P3	P4		
SCHEMATIC				$1 - U_o$	$1 - \frac{Z_A}{Z_P}$					
$U_o = -\frac{Z_A}{Z_S}$										
NOTES	By far the most commonly used in industry. In general it is assembled with multiple planets on the planet carrier (see Section 5). The range of U_o may be extended with careful design.			U_o	$\frac{Z_A}{Z_P}$					
				$1 - \frac{1}{U_o}$	$1 + \frac{Z_S}{Z_P}$					
				$-\frac{1}{U_o}$	$-\frac{Z_S}{Z_P}$					
				$\frac{U_o}{1 - U_o}$	$\frac{(Z_A/Z_P) - U_o}{1 - U_o}$					
				$-\frac{1}{1 - U_o}$	$\frac{1 - (Z_A/Z_P)}{1 - U_o}$					



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TABLE 6.2

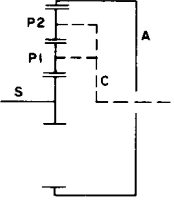
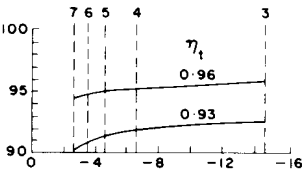
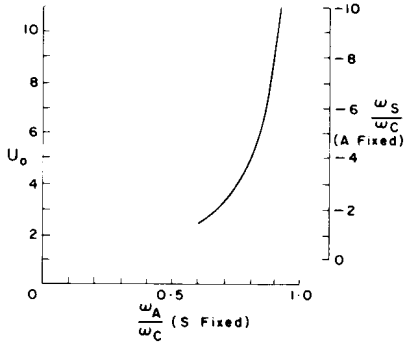
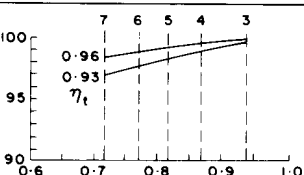
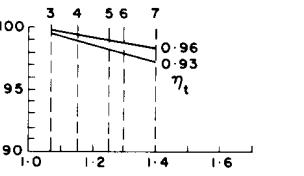
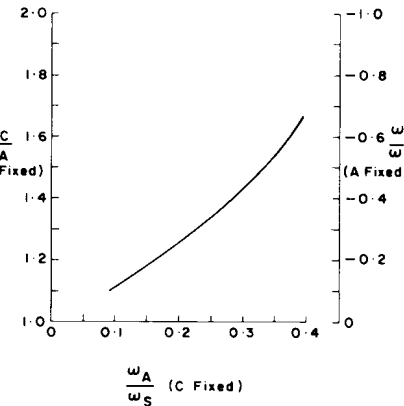
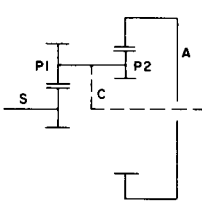
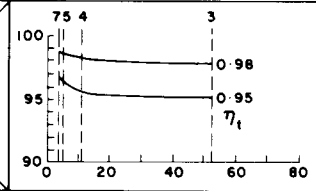
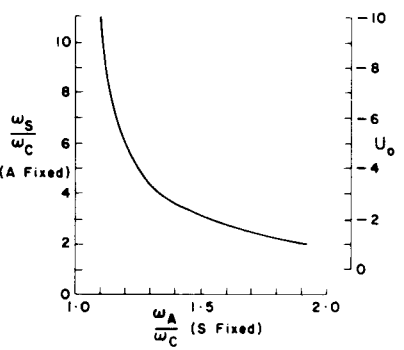
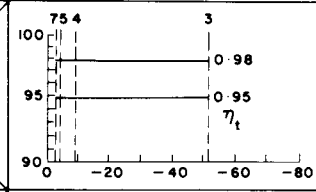
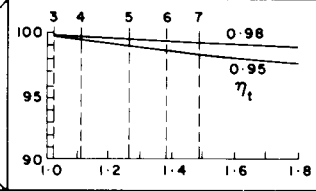
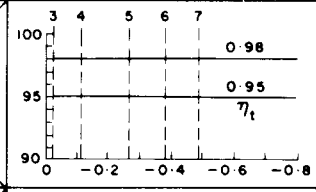
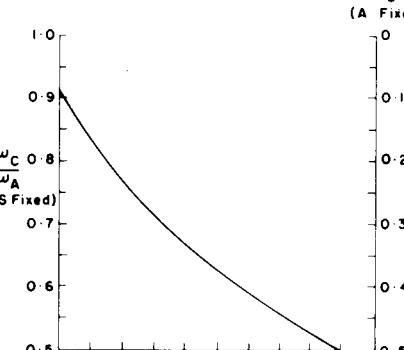
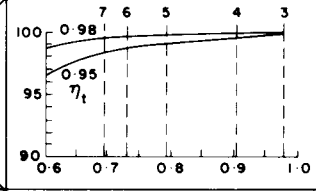
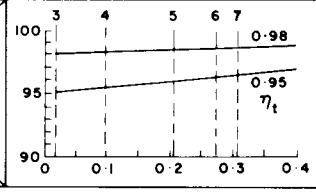
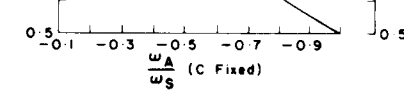
	MEMBER			RATIO u	PLANET BEARING SPEED				COMPARATIVE EFFICIENCIES	ACHIEVABLE RATIOS
	INPUT	FIXED	OUTPUT		P1	P2	P3	P4		
SCHEMATIC 	S	A	C	$1 - U_o$	$1 + \frac{Z_A}{Z_{P1}}$	$1 - \frac{Z_A}{Z_{P2}}$				
		C	A	U_o	$-\frac{Z_A}{Z_{P1}}$	$\frac{Z_A}{Z_{P2}}$				
NOTES The same gear as in Table 1, with the addition of a second idler gear (P2) to give the annulus (A) the same sense or rotation as the sun (S). Multiple planets can be assembled (see Section 5).	A	S	C	$-\frac{1}{U_o}$	$1 + \frac{Z_S}{Z_{P1}}$	$1 - \frac{Z_S}{Z_{P2}}$				
		C	S	$-\frac{1}{U_o}$	$-\frac{Z_S}{Z_{P1}}$	$\frac{Z_S}{Z_{P2}}$				
	C	S	A	$\frac{1}{1 - U_o}$	$-\frac{(Z_A/Z_{P1}) + U_o}{1 - U_o}$	$\frac{(Z_A/Z_{P2}) - U_o}{1 - U_o}$				
		A	S	$-\frac{1}{U_o - 1}$	$-\frac{(Z_A/Z_{P1}) + 1}{U_o - 1}$	$\frac{(Z_A/Z_{P2}) - 1}{U_o - 1}$				

TABLE 6.3

	MEMBER			RATIO u	PLANET BEARING SPEED				COMPARATIVE EFFICIENCIES	ACHIEVABLE RATIOS
	INPUT	FIXED	OUTPUT		P1	P2	P3	P4		
SCHEMATIC				$1 - U_o$	$1 - \frac{Z_A}{Z_{P2}}$					
$U_o = -\frac{Z_{P1} \cdot Z_A}{Z_S \cdot Z_{P2}}$										
NOTES	In general use within industry. The use of multiple planets on the carrier is possible (see Section 5). The value of the fixed carrier ratio (U_o) = -1 is almost achievable.			U_o	$\frac{Z_A}{Z_{P2}}$					
				$1 - \frac{1}{U_o}$	$1 + \frac{Z_S}{Z_{P1}}$					
				$\frac{1}{U_o}$	$-\frac{Z_S}{Z_{P1}}$					
				$\frac{U_o}{U_o - 1}$	$\frac{(Z_A/Z_{P2}) - U_o}{1 - U_o}$					
				$\frac{1}{1 - U_o}$	$\frac{1 - (Z_A/Z_{P2})}{1 - U_o}$					



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TABLE 6.4

	MEMBER			RATIO u	PLANET BEARING SPEED				COMPARATIVE EFFICIENCIES	ACHIEVABLE RATIOS
	INPUT	FIXED	OUTPUT		P1	P2	P3	P4		
SCHEMATIC										
$U_o = \frac{Z_A \cdot Z_{P2}}{Z_S \cdot Z_{P3}}$	S	A	C	$1 - U_o$	$\frac{Z_S \cdot U_o}{Z_{P1}} + 1$	$1 - \frac{Z_S \cdot U_o}{Z_{P2}}$				
NOTES	C	A		U_o	$1 - \frac{Z_S \cdot U_o}{Z_{P1}}$	$\frac{Z_S \cdot U_o}{Z_{P2}}$				
The same gear as in Table 3, with the addition of an idler gear (P1). Multiple planets can be assembled (see Section 5).	A	S	C	$1 - \frac{1}{U_o}$	$1 + \frac{Z}{Z_{P1}}$	$1 - \frac{Z}{Z_A}$				
	C	S		$\frac{1}{U_o}$	$1 - \frac{Z}{Z_{P1}}$	$\frac{Z}{Z_{P2}}$				
	S	A		$\frac{U_o}{U_o - 1}$	$1 - \frac{U_o \cdot (Z_S + Z_{P1})}{Z_{P1} \cdot (1 - U_o)}$	$\frac{U_o \cdot (Z_S - Z_{P2})}{Z_{P2} \cdot (1 - U_o)}$				
	C	A	S		$\frac{1}{1 - U_o}$	$1 - \frac{Z_S \cdot U_o + Z_{P1}}{Z_{P1} \cdot (U_o - 1)}$	$\frac{Z_S \cdot U_o - Z_{P2}}{Z_{P2} \cdot (U_o - 1)}$			