



Drag of circular cylinders normal to a flat plate with turbulent boundary layer for Mach numbers up to 3

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THE PREPARATION OF THIS DATA ITEM

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Mr H. Vogel	— British Aerospace, Weybridge-Bristol Division
Mr J. Weir	— Salford University.

* Corresponding Member

The construction and subsequent development of the Data Item was undertaken by

Mr P.D. Chappell	— Group Head.
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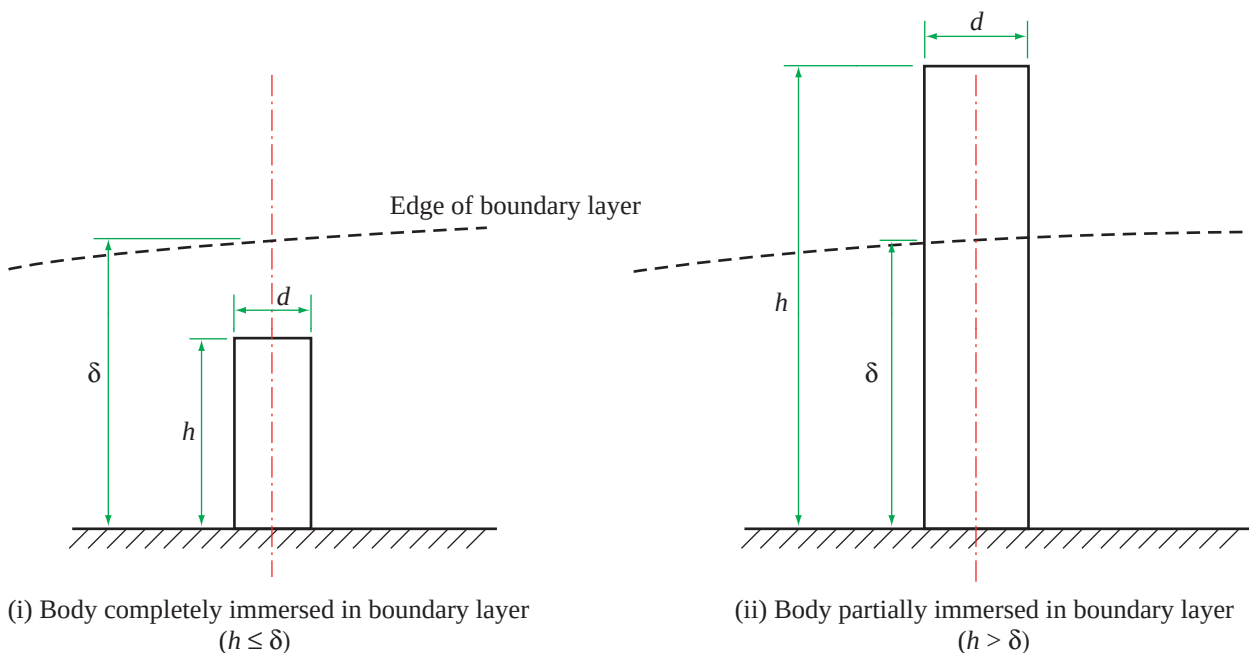
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1. NOTATION AND UNITS

		SI	British
a	coefficient in Equations (3.1) and (3.4)		
b	index in Equation (3.1)		
c	index in Equation (3.4)		
C_D	drag coefficient of cylinder in presence of boundary layer, based on kinetic pressure at edge of undisturbed boundary layer and frontal area, $D/(q_1 h d)$		
C_{Dref}	drag coefficient at $R_d = 2 \times 10^5$		
$C_{D\infty}$	drag coefficient of two-dimensional cylinder ($h/d \rightarrow \infty$) with longitudinal axis normal to uniform stream		
C_f	local skin friction coefficient in absence of cylinder, based on kinetic pressure at edge of boundary layer		
D	drag	N	lbf
d	cylinder diameter	m	ft
h	cylinder height	m	ft
M_1	Mach number at edge of undisturbed boundary layer or Mach number in free-stream		
q_1	kinetic pressure at edge of undisturbed boundary layer	N/m ²	lbf/ft ²
R_d	Reynolds number based on conditions at edge of undisturbed boundary layer (or in free-stream) and cylinder diameter		
R_h	Reynolds number based on conditions at edge of undisturbed boundary layer and cylinder height		
R_x	Reynolds number based on conditions at edge of undisturbed boundary layer and length, x , from virtual origin of turbulent boundary layer on flat plate		
u_τ	friction velocity $(\tau_w/\rho_w)^{1/2}$	m/s	ft/s
x	distance from virtual origin	m	ft

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δ	local thickness of boundary layer on flat plate (thickness for local velocity in boundary layer equal to $0.99 \times$ free-stream velocity)	m	ft
ν_w	kinematic viscosity at wall	m^2/s	ft^2/s
ρ_w	density at wall	kg/m^3	slug/ft^3
τ_w	wall shear stress	N/m^2	lbf/ft^2



Sketch 1.1 Geometry

2. INTRODUCTION

This Item provides a means of estimating the drag due to a circular cylinder mounted with its longitudinal axis normal to a flat plate at zero incidence for Mach numbers up to 3. The cylinder can be either wholly or partially immersed in a turbulent boundary layer on the plate (see Sketch 1.1).

The complexity of the flow around a circular cylinder and its variation with cylinder geometry, Reynolds number and Mach number is such that no single drag prediction method could be expected to represent all possible geometries and flow conditions.

The availability of sufficient quantities of experimental data has enabled two ranges of cylinders to be catered for by separate empirically-based methods. Section 3 presents a method which is applicable to any cylinder whose height is not less than its diameter and which is not restricted with respect to the cylinder dimensions compared to the boundary layer thickness (but see Section 3.1). Section 4 presents a method applicable to low speeds for cylinders whose heights are generally small compared with their diameters and the boundary layer thickness.

The methods apply to air flow with zero heat transfer and zero pressure gradient on the plate which is assumed to be aerodynamically smooth. A general discussion of applicability and accuracy is given in Section 5 and worked examples are given in Section 7.

3. METHOD FOR $h/d \geq 1$

The method given in this Section applies broadly to any cylinder whose height is not less than its diameter. The method for estimating the drag of cylinders on a flat plate in the presence of a boundary layer is given in Section 3.1 for cylinders immersed in the boundary layer. Section 3.2 gives a method applicable to cylinders only partially immersed in the boundary layer.

3.1 Cylinders Immersed in the Boundary Layer, $h/\delta \leq 1$

Analysis of experimental data for the drag of circular cylinders mounted on a flat plate shows that $\log_{10}(C_D)$ is essentially linear with $\log_{10}(h/\delta)$ at a Reynolds number, R_d , of about 2×10^5 , which is adopted as a reference value in this Item, *i.e.*

$$C_{Dref} = a \left(\frac{h}{\delta} \right)^b. \quad (3.1)$$

For $M_1 \approx 0$ experimental data (Derivations 15 and 16) show that a and b are functions of d/δ only. The coefficient a is a weak function of d/δ and may be assumed constant over the range $0 \leq d/\delta \leq 1$. Thus, for $M_1 \approx 0$ and $d/\delta \leq 1$

$$a = 0.617 \quad (3.2)$$

and

$$b = 0.261 + 0.586 \left(\frac{d}{\delta} \right). \quad (3.3)$$

Data for $M_1 \geq 0.2$ (Derivations 10 and 11) are restricted to a small range of d/δ , centred around 0.1, and analysis shows that both a and b are dependent on M_1 . The coefficient a , presented in Figure 1, can be assumed independent of R_d and d/δ (at least for $d/\delta \leq 1$). The variation of the index b with M_1 for $R_d = 2 \times 10^5$ and $d/\delta = 0.1$ is shown in Figure 2. There is no substantial evidence as to the effect of d/δ on the index b for other than low speeds.

The effect of Reynolds number on C_D is given as C_D/C_{Dref} in Figure 3 for $M_1 = 0.2, 0.8, 1.4, 2.2$ and 2.8 respectively. These Figures were obtained from analysis of the experimental data of Derivation 10. Values for intermediate Mach numbers may be obtained by cross-plotting the data from Figure 3. It can be assumed that for $M_1 \leq 0.1$ the effect of the variation in R_d is negligible over the range used in the Figures, at least for $h/\delta > 0.2$.

3.2 Cylinders Partially Immersed in the Boundary Layer, $h/\delta > 1$

Analysis of experimental data for cylinders protruding from the boundary layer (Derivations 10, 11 and 15) shows that $\log_{10}(C_D)$ is essentially linear with $\log_{10}(h/\delta)$, independent of R_d and d/δ (at least for $d/\delta \leq 1$), i.e.

$$C_D = a \left(\frac{h}{\delta} \right)^c. \quad (3.4)$$

Figure 1 provides data for the coefficient a while the index c is given in Figure 4 as a function of M_1 . Equation (3.4) may be applied provided that C_D is less than $C_{D\infty}$, the value appropriate to a two-dimensional cylinder in a uniform stream. If C_D evaluated using Equation (3.4) is greater than $C_{D\infty}$, then $C_{D\infty}$ must be used. The variation of $C_{D\infty}$ with M_1 , the Mach number in the free-stream, is presented in Figure 5. The curve was obtained from a correlation of experimental data (Derivations 1, 2, 3, 5 to 10, 11, 13 and 14) for two-dimensional cylinders in the main, although a few data were obtained by extrapolating drag data for finite length cylinders with large values of h/d . The peak and trough in the curve between $M_1 = 0.7$ and 0.8 is due to compressibility effects suppressing the instability of the wake which builds up for Mach numbers greater than the critical, $M_1 \approx 0.4$ (see Derivation 7). The correlation is largely independent of Reynolds number provided that M_1 is greater than about 0.4 . For M_1 less than about 0.4 the Reynolds number, R_d , based on free-stream conditions and cylinder diameter should be subcritical, i.e. $R_d \leq 2 \times 10^5$, approximately.

In order to use the method of Sections 3.1 and 3.2, data for the boundary layer thickness at the cylinder location will be required. Such data for flat plates are given in Figure 6, which was obtained from Item No. 68020 (Reference 17).

4. METHOD FOR $h/d \leq 0.5$, $h/\delta \leq 0.2$ AT LOW SPEEDS ($M_1 \leq 0.1$)

When a cylinder is small in height compared with its diameter ($h/d \leq 0.5$) and the boundary layer thickness ($h/\delta \leq 0.2$) the low speed data of Derivation 4 are shown in Reference 18 to be essentially correlated using the method adopted in Item No. 75031 (Derivation 12) for small scale two-dimensional forward-facing steps, i.e. C_D/C_f as a function of the roughness Reynolds number $u_\tau h/\nu_w$. The data are correlated by the expression

$$\frac{C_D}{C_f} = 60 \log_{10} \left(\frac{u_\tau h}{\nu_w} \right) - 80. \quad (4.1)$$

Appendix A of Item No. 75031 gives the relationship between $u_\tau h/\nu_w$ and the more practical parameters R_h , M_1 and C_f . For low-speed flows the relationship reduces to

$$\frac{u_\tau h}{\nu_w} = R_h \left(\frac{C_f}{2} \right)^{1/2} \quad (4.2)$$

which, when substituted into Equation (4.1), gives

$$\frac{C_D}{C_f} = 30 \log_{10} \left(\frac{C_f}{2} \right) + 60 \log_{10}(R_h) - 80. \quad (4.3)$$

If the C_D calculated using Equation (4.3) is negative, a value of $C_D = 0$ must be used.