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The synthesis of cam motion by blending segments

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The Mechanisms Committee, which first met in 1975, is continuing the work that was previously the responsibility of the former Gearing and Mechanisms Committee, which first met in November 1964.

THE SYNTHESIS OF CAM MOTION BY BLENDING SEGMENTS

CONTENTS

	Page
1. NOTATION AND UNITS	1
2. INTRODUCTORY NOTES	2
2.1 Reasons for Blending Cam Laws	3
2.1.1 Asymmetrical dwell-rise-dwell (DRD) motion	3
2.1.2 Motion with a period of constant velocity	3
2.1.3 Precision point	3
2.1.4 Dwell-rise-return-dwell (DRRD) motion	3
3. GENERAL DESCRIPTION OF BLENDING METHODS	4
3.1 The Common Tangent Intersect Point	4
3.2 The Blend Factor, j	5
3.3 Parameters used in the Blending Methods	7
4. ASYMMETRICAL DWELL-RISE-DWELL (DRD) MOTION	8
5. MOTION WITH A PERIOD OF CONSTANT VELOCITY	10
5.1 Asymmetrical DRD Motion with an Opening Ramp	10
5.2 A Symmetrical DRD Motion with an Opening Ramp and a Period of Constant Velocity	12
6. MOTION WITH A PRECISION POINT	15
6.1 Displacement Precision Point (Some Constant Velocity if Necessary)	15
6.2 Displacement Precision Point (no Constant Velocity)	17
7. DWELL-RISE-RETURN-DWELL (DRRD) MOTION	21
7.1 Symmetrical DRRD Motion	21
7.2 Asymmetrical DRRD Motion	23
7.3 Asymmetrical DRRD Motion with Optimised Accelerations	26
8. REFERENCES AND DERIVATION	30

9.	EXAMPLES	31
9.1	Example 1, Asymmetrical DRD Motion	31
9.2	Example 2, Asymmetrical DRD Motion with an Opening Ramp and a Period of Constant Velocity	33
9.3	Example 3, Asymmetrical DRD Motion with a Displacement Precision Point and no Constant Velocity	34
9.4	Example 4, Symmetrical DRRD Motion	37
9.5	Example 5, Asymmetrical DRRD Motion	40
9.6	Example 6, Asymmetrical DRRD Motion with Optimised Accelerations	42
APPENDIX A – DERIVATION OF THE TANGENT INTERSECT BLEND FACTOR, J		45
A.1	Derivation	45
APPENDIX B – FLOW CHART FOR DRRD MOTION WITH OPTIMISED ACCELERATIONS		48
APPENDIX C – THE GENERAL FORM OF THE SCCA (SINE-CONSTANT-COSINE ACCELERATION) CAM LAW		50
C.1	Description	50
APPENDIX D – TABLES OF PARAMETERS FOR EIGHT STANDARD DRD CAM LAWS		53
D.1	Tables	53

THE SYNTHESIS OF CAM MOTION BY BLENDING SEGMENTS

1. NOTATION AND UNITS

Throughout this Item the follower is assumed to or radians.

		<i>SI</i>	<i>British</i>
a, b, c	coefficients of SCCA cam laws (Appendix C)	—	—
C_{vmax}	coefficient of maximum velocity of a cam law	—	—
H	total follower displacement associated with one motion segment which may consist of blended portions	m	in
$H_{0, 1, 2, 3, \dots}$	total follower displacement corresponding to the complete cam law, a portion of which forms part (0,1,2,3,..) of the motion segment (see Sketch 4.2)	m	in
$h_{0, 1, 2, 3, \dots}$	intermediate follower displacement corresponding to one part (0,1,2,3,..) of the motion segment	m	in
j_o	blend factor for a segment of motion which begins at a point corresponding to $u = 0.00$, $w = 0.00$ and $w' = 0.00$	—	—
j_i	blend factor for a segment of motion which ends at a point corresponding to $u = 1.00$, $w = 1.00$ and $w' = 0.00$	—	—
$S_{0, 1, 2, 3, \dots}$	slope of displacement curve at blend point 0, 1, 2, 3, ..	m/degree	in/degree
u	normalised input displacement (cam rotation) for a portion of the blended profile	—	—
w	non-dimensional output displacement	—	—
w'	non-dimensional output velocity	—	—
w''	non-dimensional output acceleration	—	—
$\beta_{0, 1, 2, 3, \dots}$	cam angle of rotation between the intersect point and the blend point in a portion of the motion	degree	degree
γ	cam angle of rotation between certain blend points in a DRRD motion (see Sketch 7.2)	degree	degree
δ	cam angle of rotation between certain intersect points in a DRRD motion (see Sketch 7.2)	degree	degree
$\theta, \lambda, \mu, \nu$	cam angle of rotation of the blend points in a DRRD motion (see Sketch 7.3)	degree	degree

$\Phi_{0, 1, 2, 3, \dots}$	total cam angle of rotation corresponding to the complete cam law a portion of which forms part (0, 1, 2, 3,..) of the motion segment	degree	degree
$\phi_{0, 1, 2, 3, \dots}$	intermediate range of cam rotation corresponding to one part (0, 1, 2, 3,..) of the motion segment	—	—

Subscripts

c	refers to co-ordinates of a precision point (see Section 6)
$()_{0, 1, 2, 3, \dots}$	the parameter in the brackets relates to the portion of the motion given by 0, 1, 2, 3,.. outside the brackets

2. INTRODUCTORY NOTES

This document is one of the comprehensive series of Engineering Sciences Data Items (see References 1 to 16) that deals with the design of cam and follower systems. ESDU 82006 (Reference 2) in this series deals with the selection of symmetrical cam laws for dwell-rise-dwell (DRD) applications which cover the majority of motion requirements.

On occasions the designer may wish to exercise greater control over the motion characteristics of the follower than is possible by simply using a combination of symmetrical DRD cam laws and dwell segments. In such cases where a special motion is required, the methods presented in this Item may provide a solution.

This Item uses portions of symmetrical DRD cam law motion segments such as those described in ESDU 82006 (Reference 2) in combination with and/or superimposed on periods of constant velocity to satisfy special motion requirements of the types discussed in Section 2.1. It is not sufficient simply to join together arbitrarily selected portions of cam laws; the portions used must be carefully selected so that displacement and velocity are continuous from one cam law segment to another, *i.e.* the cam laws must be blended.

In general, a cam law is a mathematical function defining the motion of the follower, and is of the form $w = f(u)$. In this Item, when a cam law is referred to by its generic name, *e.g.* Cycloidal or Modified Trapezoidal, the cam law formulae are those given in ESDU 82006 (Reference 2) under the relevant cam law heading. Appendix C to this Item presents the general formulae for the Sine-Constant-Cosine Acceleration (SCCA) family of cam laws. Cam laws of this family are defined by assigning values to the three parameters a , b , and c which relate to the duration of the sine, constant and cosine parts of the cam law acceleration characteristic respectively. Many of the well known “standard” cam laws are members of the SCCA family, and these are also shown in Appendix C along with the relevant values of the parameters, a , b , and c .

It is important to note that for a given cam law, $w = f(u)$, the actual profile depends upon the values assigned to the displacement and rotation parameters, H and Φ respectively. It is therefore possible to obtain different output motions with the same function $w = f(u)$ simply by altering the values of H and Φ .

The calculation procedures for pressure angle, radius curvature etc., that are presented in other Items of the cam series must still be applied to the special motions developed using this Item because each cam law equation covering any particular segment of the motion is retained in its standard form. However, care must be taken when determining the reference circle radius used in these calculations, and in general each blended portion will be associated with a different reference circle radius.

2.1 Reasons for Blending Cam Laws

The blending together of portions of standard cam laws can be used to satisfy requirements for special motions of the types discussed in the following paragraphs.

2.1.1 Asymmetrical dwell-rise-dwell (DRD) motion

This is the simplest case of cam law blending and is concerned with applications requiring a rapid acceleration and slow deceleration (or the converse) of the follower. In applications where a spring closure is employed, the spring force must be sufficient to maintain contact between the cam and the follower during the deceleration period where inertia forces are acting to separate them. If the deceleration period is extended, the maximum deceleration will be reduced, thus resulting in a smaller spring force being required to maintain contact.

2.1.2 Motion with a period of constant velocity

The period of constant velocity may occur at the beginning or end of the segment (opening or closing ramp) or at some other position within the rise (or return). The incorporation of a period of constant velocity is particularly useful in automatic assembly machines where the motion of the follower must be matched to that of another machine element. In such cases, the total lift in any segment may be a “flexible” parameter, being determined once the requirement(s) for constant velocity have been satisfied.

The introduction of a constant velocity period will decrease the maximum velocity in a motion segment, and hence decrease the slope of the cam profile making possible a smaller cam for the same total lift. A constant velocity period will however increase the peak acceleration resulting in increased dynamic forces.

2.1.3 Precision point

The most common requirement for a precision point is the case where the follower must pass through a point at a particular cam angle that does not lie on the displacement curve of one of the standard cam laws i.e. the portions of different motion segments must be blended to meet the condition corresponding to,

$$w = w_c \quad \text{at} \quad u = u_c.$$

Similar precision point requirements may be,

$$w' = w'_c \quad \text{at} \quad u = u_c,$$

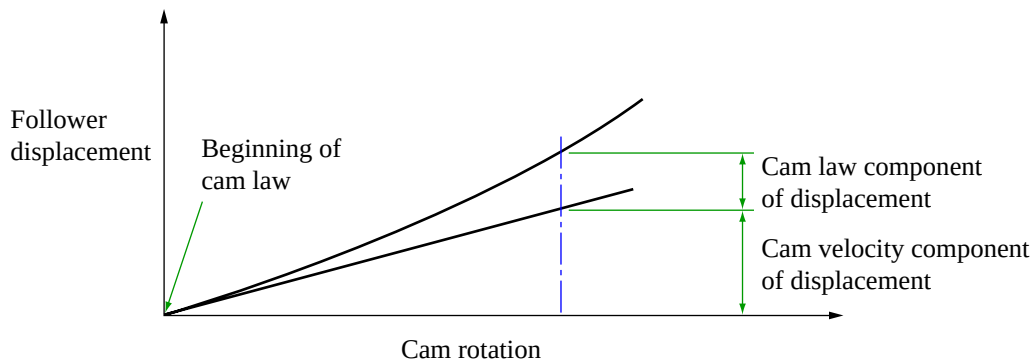
or
$$w'' = w''_c \quad \text{at} \quad u = u_c.$$

2.1.4 Dwell-rise-return-dwell (DRRD) motion

In this case there is no dwell between the rise and return segments and even for unfavourable circumstances the acceleration need not return to zero at the reversal point. This type of motion can be symmetrical or asymmetrical and may also have to satisfy one or more of the requirements described in Sections 2.1.1, 2.1.2 and 2.1.3.

3. GENERAL DESCRIPTION OF BLENDING METHODS

The special motion requirements described in Section 2.1 can be met by blending together portions of segments of standard DRD cam laws and, where necessary, segments of constant velocity. Generally, the follower displacement will be the sum of one or more cam law components and a constant velocity component (see Sketch 3.1). The constant velocity component will have a negligible effect upon the dynamic response of the system when in combination with a cam law component because it will not affect the acceleration.



Sketch 3.1 Combination of constant velocity ramp and cam law components of displacement

The various segments of cam law and constant velocity are blended together at blend points (BP). The main requirements at a blend point are that the follower displacement and velocity must be continuous[†]. In most cases, a further requirement is that the acceleration must be continuous, *i.e.* there must not be a step change in acceleration. All of the methods described in this Item apply even if there is a step change in acceleration.

If continuous acceleration is a requirement at the blend point, the best way of ensuring that this condition is met is to use a complete acceleration or deceleration loop of each of the cam laws employed.

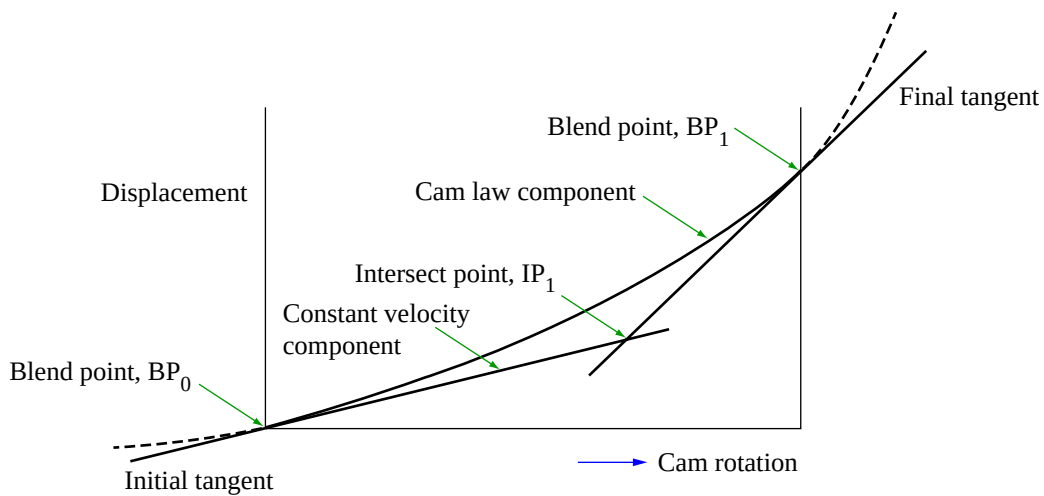
When a complete acceleration or deceleration loop is used, the acceleration will be zero at each end of the portion of the segment, and one end of the portion will have a zero velocity cam law component. If portions other than complete loops are used, the displacement and velocity will always be continuous by the methods presented in this Item, but continuity of acceleration will depend on the portions of cam law segments that are used.

3.1 The Common Tangent Intersect Point

At each blend point, a straight line can be drawn which touches the displacement curve and is tangential to both portions of the curve meeting at the point.

There will be one such common tangent at each end of each blended portion. Each common tangent intersects the adjacent common tangent at the intersect point, as shown in Sketch 3.2. The timing (or angular) co-ordinate of the intersect point can be found easily by a simple calculation from the normalised cam law formulae. This calculation makes use of the blend factor, j (see Section 3.2 and Appendix A). Thus, from known end conditions for the special motion required, the co-ordinates of the intersect point are found, from which a simple construction of straight lines will yield the displacement co-ordinate of the blend point. The exact values of the blend point co-ordinates are calculated from the straight line geometry of the resulting diagram.

[†] Continuous is here taken to mean that there is no step change in the function at the blend point but does not necessarily exclude the possibility of a step change in the higher order derivatives of the function.

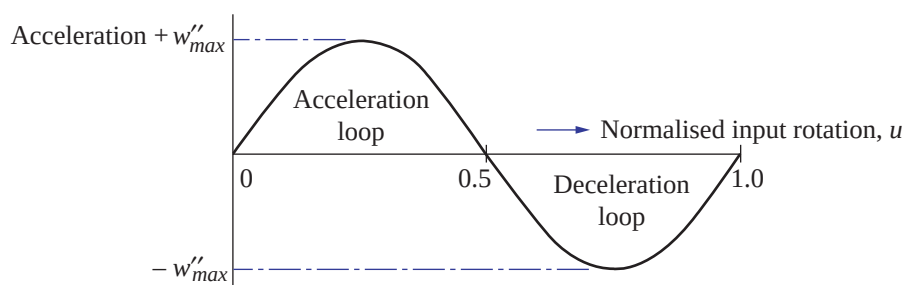


Sketch 3.2 Cam law component showing blend points and intersect point

3.2 The Blend Factor, j

All of the methods described in this Item make use of a “blend factor”, j , which is derived from the normalised displacement and velocity factors of the cam law at the end of the segment. (The full derivation of the blend factor is presented in Appendix A). In cases where a complete acceleration or deceleration loop is used, the blend factor, j , is equal to the maximum velocity coefficient, C_{vmax} .

Two blend factors, j_0 and j_1 , are used in this Item, but before these are defined it is necessary to understand how the portions of cam laws to be blended are specified. Sketch 3.3 shows how, for a symmetrical DRD cam law, the acceleration is zero at $u = 0.50$, and the magnitude of the maximum acceleration is equal to that of the deceleration.



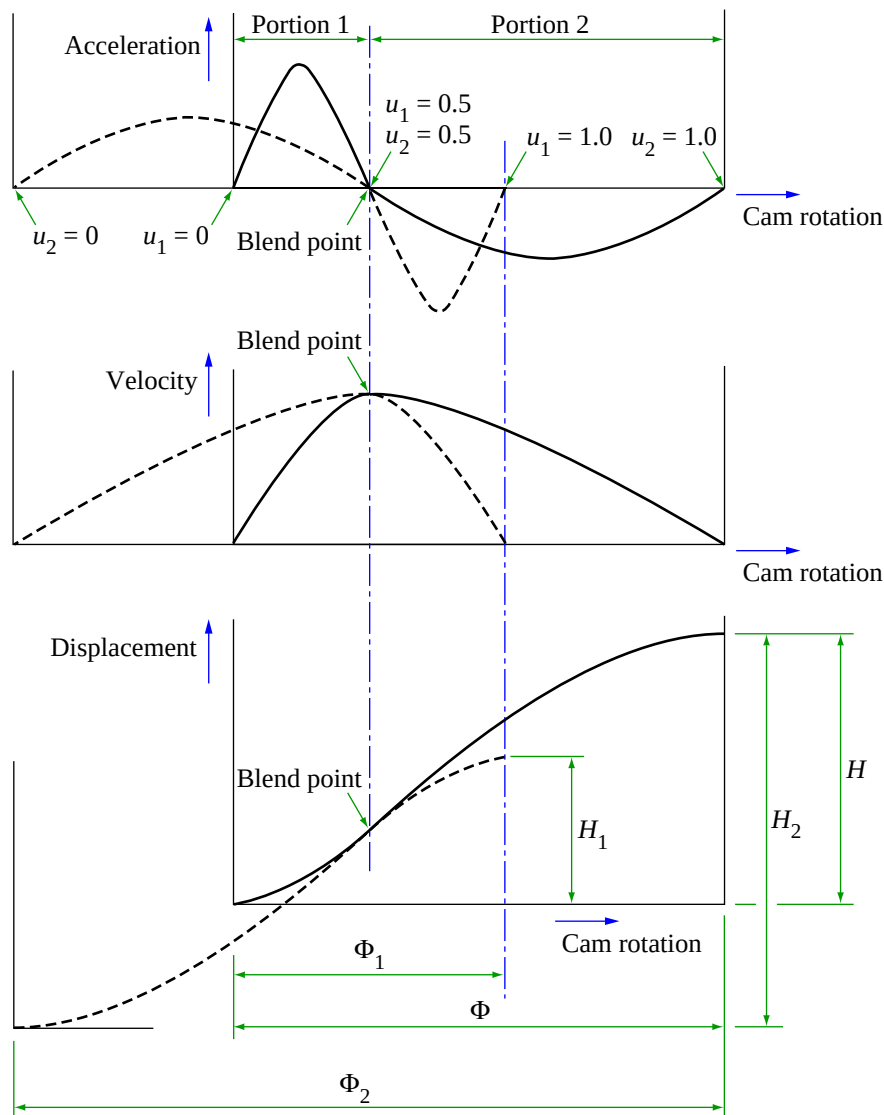
Sketch 3.3 Acceleration characteristic for a standard DRD cam law segment

Sketch 3.4 shows the acceleration curve for an asymmetrical DRD motion which is the simplest case of cam law blending. The acceleration characteristic is a combination of the acceleration loop of one cam law and the deceleration loop of another. At the blend point, the value of u for the first cam law is $u_1 = 0.50$ and for the second, $u_2 = 0.50$. The segment finishes at the end of the deceleration loop of the second cam law, at a value of $u_2 = 1.00$.

The acceleration loop (portion 1) in Sketch 3.4 begins at $u = 0.00$, $w = 0.00$, $w' = 0.00$, and for this type of segment the blend factor, j_0 , is given by the expression[†],

$$(j_0)_1 = \frac{(u_1)_1 (w'_1)_1}{(w_1)_1}, \quad (3.1)$$

where $(u_1)_1$, $(w_1)_1$ and $(w'_1)_1$ are the normalised input rotation, and non-dimensional output displacement and velocity respectively at the blend point ($u_1 = 0.50$ in the case of Sketch 3.4) for the first cam law.



Sketch 3.4 Displacement, velocity and acceleration curves for an asymmetric motion

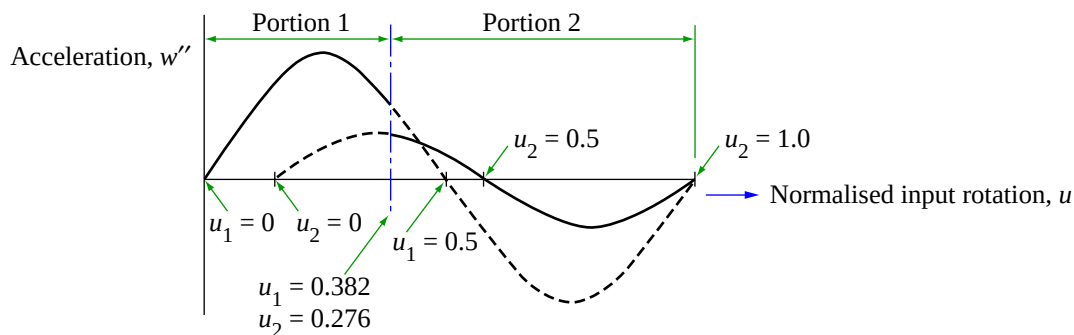
[†] where, for example, u_1 is the value of u corresponding to the end of any portion, and $(u_1)_1$ is the value of u corresponding to the end of portion 1.

The deceleration loop (portion 2) in Sketch 3.4 ends at $u = 1.00$, $w = 1.00$, $w' = 0.00$, and for this type of segment the blend factor, j_1 , is given by the expression[†],

$$(j_1)_2 = \frac{(1 - (u_0)_2)(w'_0)_2}{(1 - (w_0)_2)}, \quad (3.2)$$

where u_0 , w_0 and w'_0 are the normalised input rotation, non-dimensional output displacement and output velocity respectively at the blend point ($u_0 = 0.50$ in the case of Sketch 3.4) for the second cam law.

Sketch 3.4 illustrated the special case where complete acceleration and deceleration loops were used. Sketch 3.5 shows one of the infinity of other possible solutions where the value of u_1 and u_0 at the blend point are not both equal to 0.50. Although this sketch shows a discontinuity in the acceleration characteristic, this method ensures the vital continuity of the displacement and velocity characteristics. If there is such a step change in acceleration then, in the case of roller followers, slip is inevitable.



Sketch 3.5 Discontinuous acceleration characteristic

For Sketch 3.5, the relevant blend factors are,

$$\begin{aligned} \text{portion 1,} \quad (j_0)_1 &= \frac{(u_1)_1(w'_1)_1}{(w_1)_1}, & \text{where } (u_1)_1 &= 0.382 \\ \text{portion 2,} \quad (j_1)_2 &= \frac{(1 - (u_0)_2)(w'_0)_2}{(1 - (w_0)_2)}, & \text{where } (u_0)_2 &= 0.276. \end{aligned}$$

3.3 Parameters used in the Blending Methods

Appendix D of this Item contains tables of tabulated values for all of the various parameters required for the cam law blending methods. Tables are presented for each of the eight symmetrical DRD cam laws discussed in ESDU 82006 (Reference 2).

In all of the methods except the one for asymmetrical DRRD motion with optimised accelerations (Section 7.3) the only parameters required are the non-dimensional displacement and velocity, and the blend factor. The non-dimensional displacement and velocity are calculated using the cam law expressions given in Item 82006. The blend factor, j_0 , (Equation (3.1)) is used if the cam law portion begins at the position corresponding to $u = 0$, $w = 0$, $w' = 0$, and the blend factor, j_1 , (Equation (3.2)) is used if the cam law portion ends at the position corresponding to $u = 1$, $w = 1$, $w' = 0$.

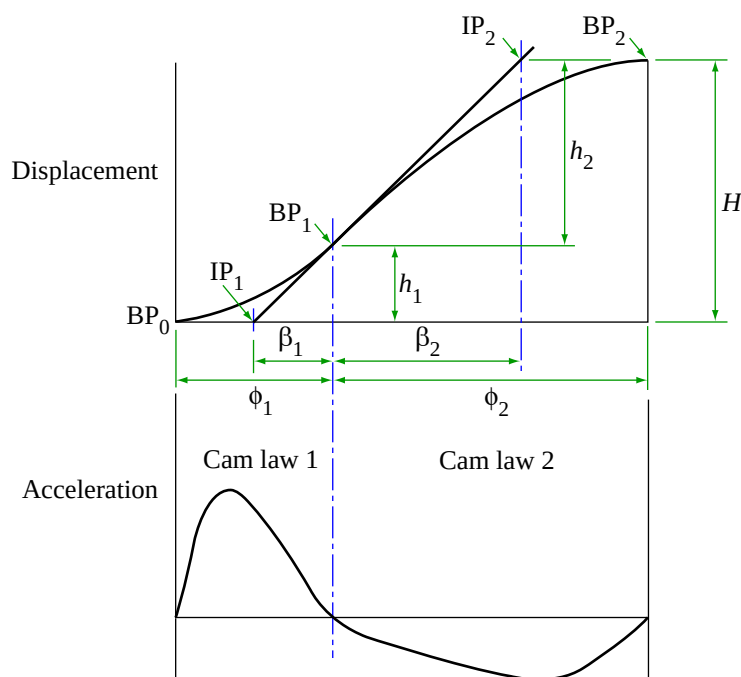
For the optimised acceleration case of asymmetrical DRRD motion, a flow chart is presented in Appendix B which can be used to generate all additional parameters required.

[†] where, for example, u_0 is the value of u corresponding to the beginning of any portion,
and $(u_0)_2$ is the value of u corresponding to the beginning of portion 2.

4. ASYMMETRICAL DWELL-RISE-DWELL (DRD) MOTION (see Example 9.1)

Probably the most commonly encountered special motion requirement that can be satisfied by blending together portions of standard cam law segments is that of asymmetrical DRD motion where the acceleration and deceleration periods are of different duration. Given the total lift requirement and the cam angle of rotation for the acceleration and deceleration periods, any combination of symmetrical DRD cam laws can be blended to maintain continuous displacement and velocity characteristics.

Sketch 4.1 illustrates the notation and geometric construction. It should be noted that although this Sketch has been prepared using complete acceleration and deceleration loops, this method is equally applicable to any other possible combination of fractions, or portions, of cam laws (see Section 3.2).



Sketch 4.1 Asymmetrical DRD motion

The specified input requirements are given as follows.

- The cam angle of rotation, ϕ_1 , for the acceleration period.
- The cam angle of rotation, ϕ_2 , for the deceleration period.
- The total follower displacement, H , for the motion segment (acceleration + deceleration).
- The cam laws and proportions of them, $(u_1)_1$, and $(u_0)_2$, that are to be used for each period of the segment[†].

[†] If complete acceleration and deceleration loops are used then both $(u_1)_1$ and $(u_0)_2$ will be equal to 0.50 for symmetrical cam laws. Also, $(w_1)_1$ and $(w_0)_2 = 0.50$, and hence, $(j_0)_1$ and $(j_1)_2 = C_{vmax}$.

The stages of the blending method are given as follows.

- (i) Determine the values of $(w_1)_1$, $(w'_1)_1$ and subsequently, $(j_0)_1$, corresponding to the value of $(u_1)_1$ that has been specified for the end of the acceleration segment[†].
- (ii) Determine the values of $(w_0)_2$, $(w'_0)_2$ and subsequently, $(j_1)_2$ corresponding to the value of $(u_0)_2$ that has been specified for the beginning of the deceleration segment[†].
- (iii) Locate the positions of the intersect points IP_1 , and IP_2 (Sketch 4.1) by calculating,

$$\beta_1 = \frac{\phi_1}{(j_0)_1} \quad (4.1)$$

$$\text{and} \quad \beta_2 = \frac{\phi_2}{(j_1)_2} . \quad (4.2)$$

A straight line drawn between IP_1 and IP_2 will fix the position of the blend point, BP_1 , and will be tangential to the displacement curve at that point.

- (iv) The displacements h_1 and h_2 are determined by similar triangles,

$$h_1 = H\beta_1/(\beta_1 + \beta_2) \quad (4.3)$$

$$\text{and} \quad h_2 = H - h_1 . \quad (4.4)$$

- (v) Now calculate the total displacements and cam angles for the cam laws, segments of which are being blended to form the asymmetrical DRD motion.

$$H_1 = \frac{h_1}{(w_1)_1} , \quad (4.5)$$

$$\Phi_1 = \frac{\phi_1}{(u_1)_1} , \quad (4.6)$$

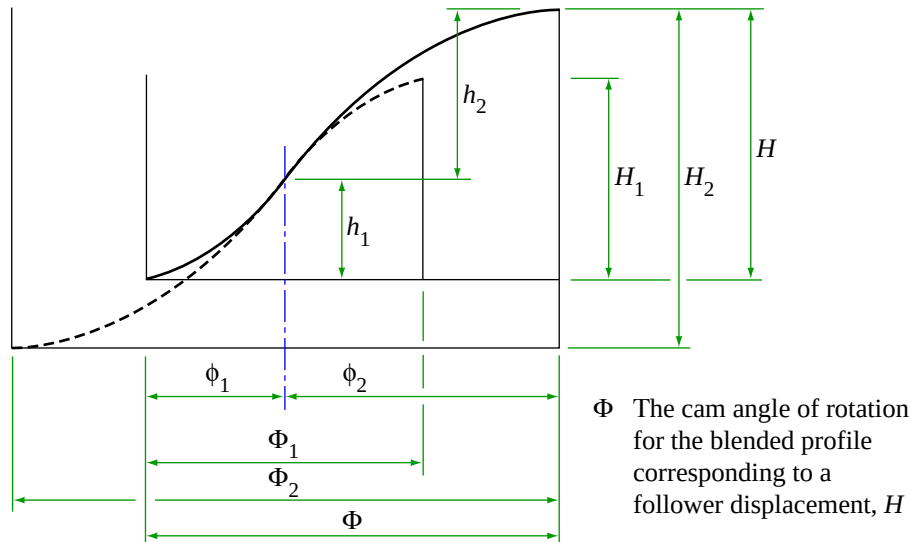
$$H_2 = \frac{h_2}{(w_0)_2} \quad (4.7)$$

$$\text{and} \quad \Phi_2 = \frac{\phi_2}{(u_0)_2} . \quad (4.8)$$

If complete acceleration and deceleration loops have been blended, the acceleration segment is half of a DRD motion of cam law 1, having period Φ_1 and displacement H_1 . The deceleration segment is half of a DRD motion of cam law 2, having period Φ_2 and displacement H_2 .

[†] where, for example, u_1 is the value of u corresponding to the end of any portion,
and $(u_1)_1$ is the value of u corresponding to the end of portion 1.

The implications of the summary paragraph above are illustrated in Sketch 4.2 which shows the two component DRD motions superimposed upon the resulting asymmetrical motion.



Sketch 4.2 Two component DRD motions superimposed upon a resulting asymmetrical motion

Note: With any asymmetric DRD motion in which both portions have the same blend factor at the blend point, the blend point will lie on the diagonal of the displacement diagram. This is always the case when both portions of the asymmetrical motion are complete loops of the same cam law (see also Section 6.2).

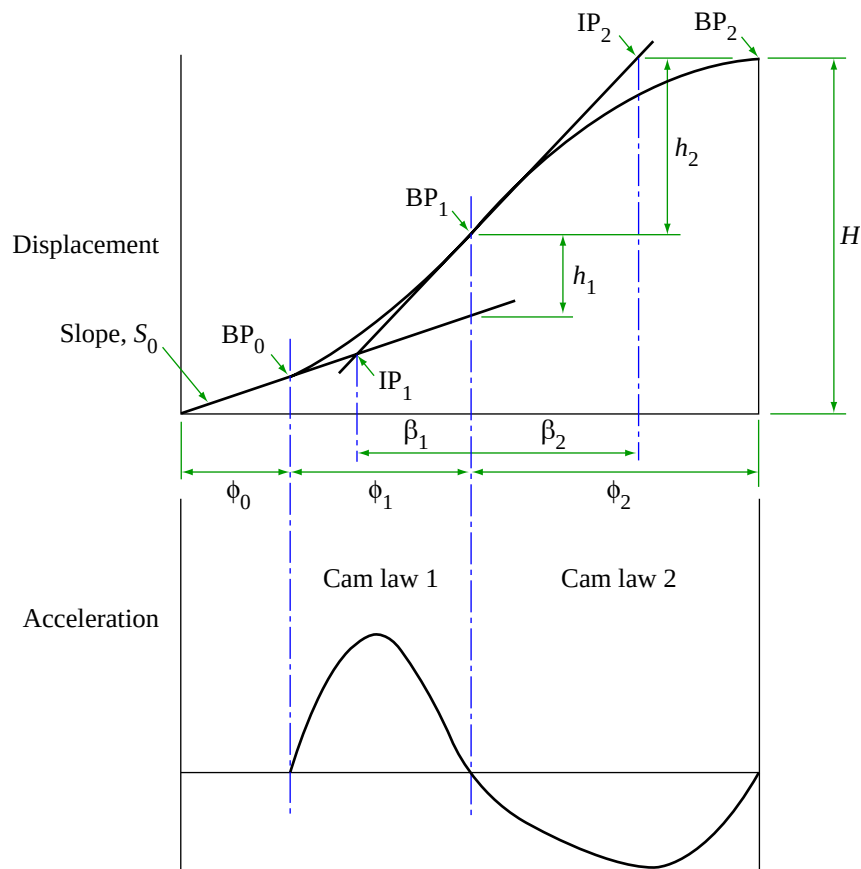
5. MOTION WITH A PERIOD OF CONSTANT VELOCITY

A period of constant velocity is sometimes required either at the beginning, the end or somewhere in the middle of a motion segment. If the constant velocity period is at the beginning or end of the segment it is termed an “opening ramp” or “closing ramp” respectively.

This Section of the Item contains two methods for blending cam laws with constant velocity, one with an opening ramp and the other with an opening ramp plus an additional period of constant velocity. Any other examples of this type of special motion, not specifically covered by these two methods can be solved using a combination of the data presented here and that presented in some of the other blending methods.

5.1 Asymmetrical DRD Motion with an Opening Ramp

Sketch 5.1 shows the geometric construction for this type of motion requirement. This Sketch has been prepared using complete acceleration and deceleration loops, although the method is equally applicable to any combination of proportions of cam laws (see Section 3.1).



Sketch 5.1 Asymmetrical DRD with an opening ramp

The specified input requirements for this method are given as follows.

- The slope of the ramp, S_0 , and the corresponding cam angle, ϕ_0 .
- The cam angle of rotation, ϕ_1 , for the acceleration period.
- The cam angle of rotation, ϕ_2 , for the deceleration period.
- The total follower displacement, H , for the motion segment (constant velocity + acceleration + deceleration).
- The cam laws and proportions of them, $(u_1)_1$, and $(u_0)_2$ that are to be used for the acceleration and deceleration periods.

The stages of the blending method are given as follows.

- Determine the values of $(w_1)_1$, $(w'_1)_1$, and $(j_0)_1$, corresponding to the selected value of $(u_1)_1$.
- Determine the values of $(w_0)_2$, $(w'_0)_2$ and $(j_1)_2$ corresponding to the selected value of $(u_0)_2$.
- Locate the positions of the intersect points IP_1 , and IP_2 by calculating,

$$\beta_1 = \frac{\phi_1}{(j_0)_1} \quad (5.1)$$

$$\text{and} \quad \beta_2 = \frac{\phi_2}{(j_1)_2} \quad (5.2)$$

A straight line drawn between IP_1 and IP_2 will pass through the blend point BP_1 and be tangential to the displacement curve at that point.

- (iv) The displacements, h_1 and h_2 are determined by similar triangles,

$$\frac{h_1 + S_0\beta_1}{\beta_1} = \frac{H - S_0(\phi_0 + \phi_1 - \beta_1)}{(\beta_1 + \beta_2)},$$

$$\text{which gives,} \quad h_1 = \frac{\beta_1}{(\beta_1 + \beta_2)} (H - S_0(\phi_0 + \phi_1 + \beta_2)) \quad (5.3)$$

$$\text{and} \quad h_2 = H - h_1 - S_0(\phi_0 + \phi_1). \quad (5.4)$$

- (v) Calculate the total cam angles and displacements for the cam law segments.

$$H_1 = \frac{h_1}{(w_1)_1}, \quad (5.5)$$

$$\Phi_1 = \frac{\phi_1}{(u_1)_1}, \quad (5.6)$$

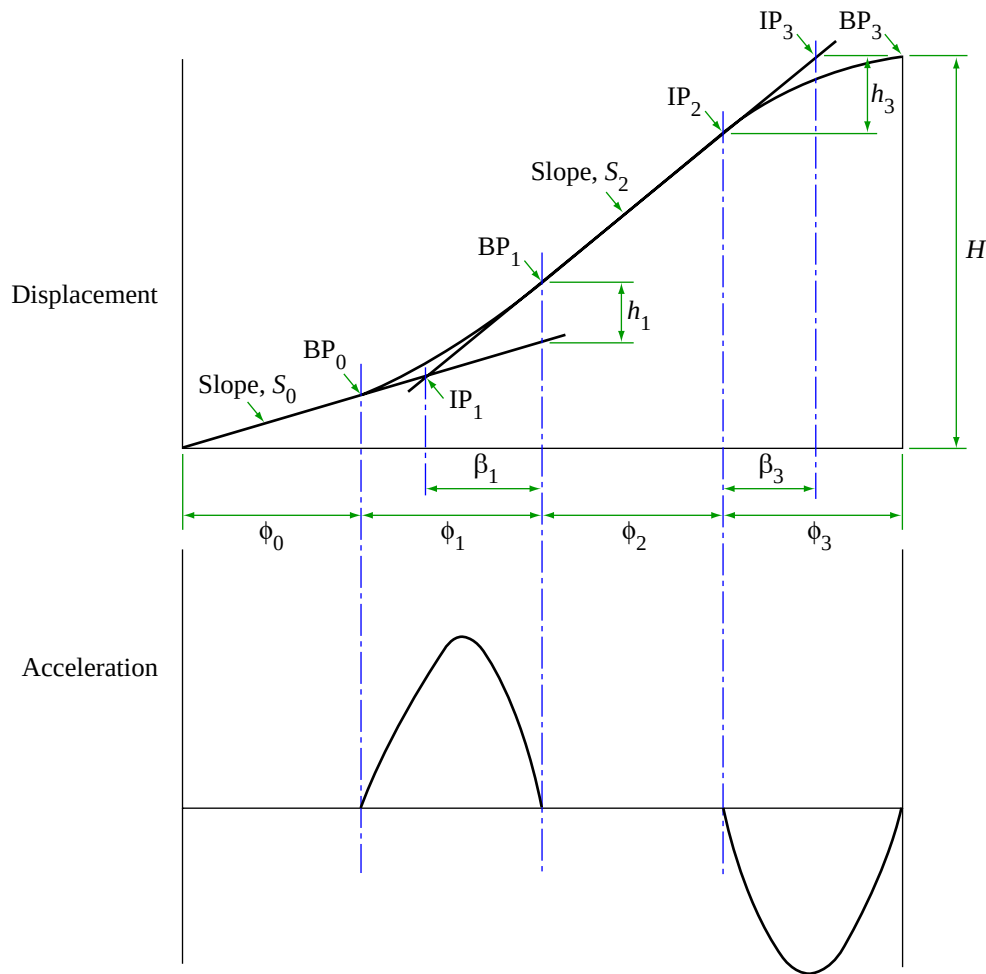
$$H_2 = \frac{h_2}{(w_0)_2} \quad (5.7)$$

$$\text{and} \quad \Phi_2 = \frac{\phi_2}{(u_0)_2}. \quad (5.8)$$

If complete acceleration and deceleration loops have been blended, the acceleration loop is half of a DRD motion of cam law 1, having period Φ_1 and displacement H_1 , added to a constant velocity of slope S_0 . The deceleration segment is half of a DRD motion of cam law 2, having period Φ_2 and displacement H_2 .

5.2 A Symmetrical DRD Motion with an Opening Ramp and a Period of Constant Velocity (see Example 9.2)

Sketch 5.2 shows the geometric construction and notation for this type of motion requirement. The method described below is not limited to blending complete acceleration and deceleration loops although for simplicity of presentation, this is the case illustrated.



Sketch 5.2 Asymmetrical DRD motion with an opening ramp and a segment of specified constant velocity

The specified input requirements for this method are given as follows.

- The slope of the ramp, S_0 , and the corresponding cam angle, ϕ_0 .
- The slope of the constant velocity period, S_2 , and the corresponding cam angle, ϕ_2 .
- The cam angle of rotation, ϕ_1 , for the acceleration period.
- The cam angle of rotation, ϕ_3 , for the deceleration period.
- The cam laws and proportions of them, $(u_1)_1$, and $(u_0)_3$ for the acceleration and deceleration periods.

The stages of the blending method are given as follows.

- Determine the values of $(w_1)_1$, $(w'_1)_1$ and $(j_0)_1$ corresponding to the selected value of $(u_1)_1$.
- Determine the values of $(w_0)_3$, $(w'_0)_3$ and $(j_1)_3$ corresponding to the selected value of $(u_0)_3$.

- (iii) Locate the positions of the intersect points IP_1 , and IP_3 by calculating,

$$\beta_1 = \frac{\phi_1}{(j_0)_1} \quad (5.9)$$

and
$$\beta_3 = \frac{\phi_3}{(j_1)_3} . \quad (5.10)$$

- (iv) Calculate h_1 from,

$$h_1 = \beta_1(S_2 - S_0) . \quad (5.11)$$

Knowing the slopes S_2 and S_0 , plus the displacement h_1 , the positions of the points IP_1 , BP_0 and BP_2 can be located. A straight line projected through IP_1 , and BP_1 will fix the positions of BP_2 , IP_3 and hence BP_3 , and will determine the follower displacement for the segment.

- (v) Calculate h_3 from,

$$h_3 = S_2\beta_3 . \quad (5.12)$$

- (vi) Calculate the total cam angles and displacements corresponding to the cam law segments used,

$$H_1 = \frac{h_1}{(w_1)_1} , \quad (5.13)$$

$$\Phi_1 = \frac{\phi_1}{(u_1)_1} , \quad (5.14)$$

$$H_3 = \frac{h_3}{(w_0)_3} \quad (5.15)$$

and
$$\Phi_3 = \frac{\phi_3}{(u_0)_3} . \quad (5.16)$$

- (vii) The overall follower displacement for the segment is given by,

$$H = S_0(\phi_0 + \phi_1 - \beta_1) + S_2(\beta_1 + \phi_2 + \beta_3) . \quad (5.17)$$

If complete acceleration and deceleration loops have been blended, the acceleration loop is half of a DRD motion of cam law 1, having period Φ_1 , and displacement H_1 , added to a constant velocity slope S_0 . The deceleration loop is half of a DRD motion of cam law 3, having period Φ_3 and displacement H_3 .

6. MOTION WITH A PRECISION POINT

The precision point is most likely to occur in the displacement characteristic of the motion, *i.e.* the follower must be in a particular position at a particular time or cam angle. The data presented in Section 5 show how cam laws can be blended to produce a given follower velocity at a particular cam angle (a velocity precision point).

Two methods that are presented in this section can be used to satisfy a displacement precision point requirement. The first method introduces a period of constant velocity and assumes that the precision point occurs at maximum velocity, *i.e.* at the end of the acceleration period. As explained in Section 2.1.2, the introduction of a period of constant velocity into the motion will increase the follower acceleration/deceleration and dynamic forces. In some circumstances, the second method can be used to provide the displacement precision point condition without the introduction of a period of constant velocity. This second method (see Section 6.2) will, if it can be applied, provide a dynamically more satisfactory solution than the first because it will decrease the maximum accelerations/decelerations.

6.1 Displacement Precision Point (Some Constant Velocity if Necessary)

The specified input requirements are given as follows.

- (a) The total lift, H , for the motion segment,
- (b) The total cam angle, Φ , for the motion segment,
- (c) The co-ordinates, (ϕ_1, h_1) of the precision point,
- (d) The cam laws and proportions of them, $(u_1)_1$ and $(u_0)_3$ that are to be used.

The stages of the blending method are given as follows.

- (i) Determine the values of $(w_1)_1$, $(w'_1)_1$, and $(j_0)_1$, corresponding to the selected value of $(u_1)_1$.
- (ii) Determine the values of $(w_0)_3$, $(w'_0)_3$ and $(j_1)_3$ corresponding to the selected value of $(u_0)_3$.
- (iii) Locate the intersect point, IP_1 , by calculating,

$$\beta_1 = \frac{\phi_1}{(j_0)_1} . \quad (6.1)$$

- (iv) Determine the slope of the constant velocity period,

$$S_1 = \frac{h_1}{\beta_1} . \quad (6.2)$$

- (v) Calculate ϕ_3 from,

$$\phi_3 = \left[\phi_4 - \frac{(H - h_1)}{S_1} \right] / \left[1 - \frac{1}{(j_1)_3} \right] \quad (6.3)$$

$$\text{and hence} \quad \beta_3 = \frac{\phi_3}{(j_1)_3}, \quad (6.4)$$

$$h_3 = S_1 \beta_3, \quad (6.5)$$

$$\phi_2 = \phi_4 - \phi_3 \quad (6.6)$$

$$\text{and} \quad h_2 = H - h_3 - h_1. \quad (6.7)$$

(vi) Determine the total cam angles and displacements corresponding to the cam law segments used,

$$H_1 = \frac{h_1}{(w_1)_1}, \quad (6.8)$$

$$\Phi_1 = \frac{\phi_1}{(u_1)_1}, \quad (6.9)$$

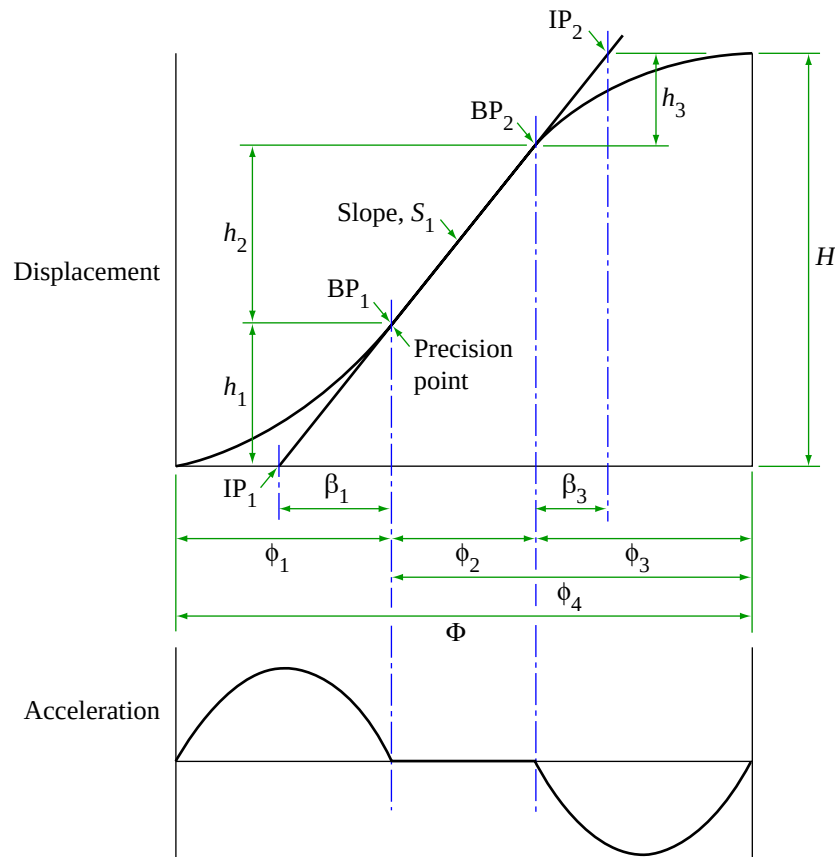
$$H_3 = \frac{h_2}{(w_0)_3} \quad (6.10)$$

$$\text{and} \quad \Phi_3 = \frac{\phi_3}{(u_0)_3}. \quad (6.11)$$

If complete acceleration and deceleration loops have been used, the acceleration loop is half of a DRD motion of cam law 1, having period Φ_1 and displacement H_1 . The deceleration loop is half of a DRD motion of cam law 2, having period Φ_3 and displacement H_3 . These two loops are connected by a period of constant velocity of duration ϕ_2 degrees and slope S_1 .

Sketch 6.1 shows one of many possible solutions to this type of problem, and in this case the precision point is located at the beginning of the constant velocity period. If the precision point lies nearer to the end than the beginning of the motion, then it should be matched at the end of the constant velocity period. There are of course many other possible solutions, in which the precision point is matched by the constant velocity period at a position other than at one of its ends.

One limitation of this method is that a simple three portion solution (as shown in Sketch 6.1) will not apply if the intersect point, IP_2 , is located outside the right hand boundary of the displacement diagram. In such cases it will be necessary to either add an extra acceleration portion or to modify the first portion of the motion so that the tangent is steeper (velocity is greater) at BP_1 or else BP_1 is relocated nearer to the start of the motion.



Sketch 6.1 Asymmetrical DRD motion with a displacement precision point and some constant velocity

6.2 Displacement Precision Point (no Constant Velocity), (see Example 9.3)

As stated in Section 4, the blend point in an asymmetrical DRD motion will lie on the diagonal of the displacement diagram as long as both the acceleration and deceleration loops have the same blend factors at that point. This will always be the case when both loops are half portions of the same cam law.

Sketch 6.2 shows a typical family of DRD motion displacement curves each with a different degree of asymmetry. In each case, the acceleration and deceleration portions are each half of the same cam law (Cycloidal Motion), and the blend point lies on the diagonal of the displacement diagram. Sketch 6.3 shows the extreme curves (acceleration loop only and deceleration loop only) for the eight standard cam laws in Item 82006.