

ELASTIC LOCAL BUCKLING STRESSES OF THIN-WALLED UNSTIFFENED CIRCULAR CYLINDERS UNDER COMBINED AXIAL COMPRESSION AND INTERNAL PRESSURE

1. NOTATION

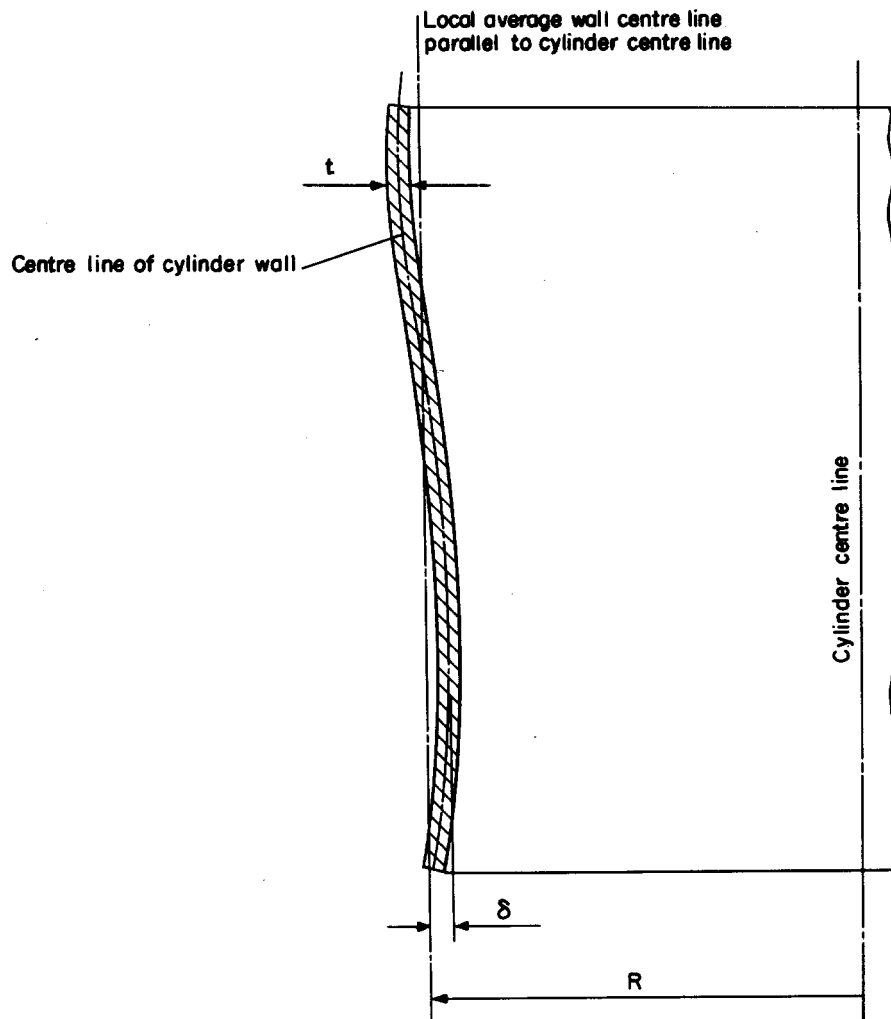
c	confidence level, per cent		
E	modulus of elasticity of cylinder material	N/m ²	lbf/in ²
f_b	elastic buckling stress	N/m ²	lbf/in ²
f_{bt}	theoretical elastic buckling stress for geometrically perfect cylinder loaded in axial compression	N/m ²	lbf/in ²
	$f_{bt} = \frac{kE}{[3(1 - \nu^2)]^{1/2}} \left(\frac{t}{R} \right)$		
L	length of cylinder	m	in
k	coefficient in equation for f_{bt} correcting for end support conditions		
p	internal pressure	N/m ²	lbf/in ²
R	radius of cylinder (see Sketch 1.1)	m	in
S_{pc}	cumulative probability, per cent, that δ/t will not be exceeded by population estimated with confidence level c		
S_{qc}	cumulative probability, per cent, that δ/t will be exceeded by population estimated with confidence level c		
t	thickness of cylinder wall	m	in
δ	maximum inward initial deviation of mid-line of unloaded cylinder wall from straight axial mid-line generator of cylinder wall parallel to cylinder axis (see Sketch 1.1 and Section 3)	m	in
δ_e	effective value of δ when $p \neq 0$	m	in
ν	Poisson's ratio of cylinder material		
ρ_{Sc}	factor applicable to mean of sample to estimate δ/t for values of S_{pc} or S_{qc}		

Suffixes

It is convenient in calculation to replace the suffixes to ρ_{Sc} by the current values of S_{pc} or S_{qc} and c , for example $\rho_{99,95}$. This notation is also conveniently applied to other symbols as illustrated in Section 8.

Both SI and British units are quoted but any coherent system of units may be employed.

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Sketch 1.1 Section of cylinder wall showing definition of δ

2. GENERAL NOTES

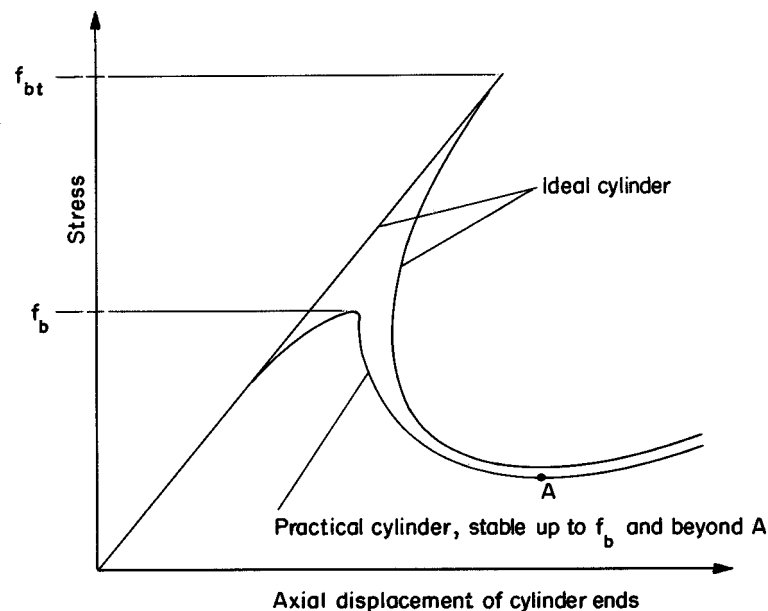
This Item provides data for estimation of the elastic local buckling stress of thin-walled unstiffened circular cylinders subject to axial load or to combined axial load and internal pressure; overall buckling is not included. The data relate to cylinders of uniform wall thickness manufactured from isotropic materials. Local buckling occurs as a series of buckle waves around and along the cylinder. Notes on overall buckling, which normally only occurs in long cylinders acting as struts, are provided in Data Item No. 78021*.

The axial load carrying capacity of unpressurised cylinders is strongly influenced by inward initial irregularities, that is small inward deviations from the ideal “theoretical” cylinder prior to loading. Sketch 2.1 shows the local buckling behaviour of ideal and practical cylinders. The buckling stress for ideal cylinders, f_{bt} , is given by

$$f_{bt} = \frac{kE}{[3(1-\nu^2)]^{1/2}} \left(\frac{t}{R} \right). \quad (2.1)$$

* Data Item No. 78021 “Guide to Items on the strength and stability of struts”.

In practical cylinders increasing load results in a slow growth of initial irregularities. As the load is increased the initial irregularities grow more rapidly and buckling occurs, in that mode which requires the least energy of deformation, at an initial buckling stress f_b . Thereafter the mode of deformation usually develops into a characteristic diamond pattern that can reach a stable condition at a much increased axial displacement of the cylinder ends, see point A on Sketch 2.1. This is rarely of structural significance since local yielding usually occurs at the boundaries between diamonds in which case the cylinder retains much of the post buckling deformation on removal of the load.



Sketch 2.1 Form of stress-displacement curves for ideal and practical cylinders

3. DEFINITION OF INITIAL IRREGULARITIES

For the purposes of buckling calculations it is convenient to divide the irregularities into two groups, axial and circumferential irregularities. Axial irregularities are the deviation of the wall centre line from a straight line, parallel to the axis of the cylinder, at the average radius of the wall centre line along the line. The maximum inward deviation anywhere along the cylinder for any axial line around the cylinder is termed δ . The influence of δ is such that $\delta = t/5$ is sufficient to reduce f_b to about half f_{bt} . Circumferential irregularities are deviations of the wall centre from the average circle. The effect on load carrying capacity of circumferential irregularities that are maintained uniformly along the length of the cylinder is small, being approximately related to the reduced local curvature. In practical cylinders both axial and circumferential irregularities exist simultaneously with the former dominating buckling.

Typical values of δ for specific constructions and size ranges may be obtained by measurement of previously manufactured cylinders. Estimates of δ may be obtained by placing a straight edge axially along the surface of the cylinder and measuring the maximum total inward irregularity of the surface with a depth gauge. An initial estimate of δ is then half the measured depth. Further notes on δ and its application are provided in Sections 4 and 5.

4. EFFECT OF INITIAL AXIAL IRREGULARITIES, δ , ON BUCKLING STRESS

Figure 1 provides a curve of f_b/f_{bt} against δ/t or δ_e/t for cylinders subjected to axial load. This curve is derived from a theoretical analysis of the buckling of unpressurised cylinders containing initial irregularities (Derivation 8). It is based upon the assumption that the irregularity is axisymmetric in the form of axisymmetric buckling modes and that the resulting buckling mode is non-symmetric having both axial and circumferential buckling waves. Further the analysis only includes the first order effects of imperfections so that Figure 1 is strictly only valid when δ/t is less than about one. The curve has been extended to $\delta/t = 2$ and that part of the curve is shown dashed. This simple theoretical model is not capable of relating to the wide range of initial irregularities met in practical cylinders. Clearly the same value of δ may be associated with initial irregularities of different form and circumferential disposition so that appreciable variations in the buckling stress for any one value of δ/t are to be expected. Because of this, direct use of Figure 1 with measured values of δ should be treated with caution and is not recommended. Measured values of δ should be compared with the values implied by experiments, see Section 5.

For δ/t less than about 0.1 the curve of Figure 1 rises very steeply. This region is shown dashed on the figure because it does not apply to practical cylinders unless substantial internal pressure is present.

Local buckling of cylinders is not very sensitive to end conditions except in very short cylinders. Suitable values of k for most practical cylinders are 0.85 for simply-supported ends and 0.92 for clamped ends. Values of f_{bt} obtained using these values of k are applicable provided that $L^2/(Rt)$ is greater than about 50. For simply-supported end conditions limited analysis with $R/t = 100$ indicates that $k = 0.85$ is approximately correct if $L^2/(Rt)$ is greater than about 15. For very short cylinders, where $L^2/(Rt)$ is substantially below the values noted above, k increases with decreasing L . For extremely short cylinders, $L^2/(Rt)$ approaching unity, f_b tends to f_{bt} which approaches the buckling stress for a wide plate with free edges and the appropriate edge conditions on the loaded ends. Thus, for simply-supported ends,

$$f_b \rightarrow f_{bt} = \frac{\pi^2 E}{12(1 - \nu^2)} \left(\frac{t}{L} \right)^2 \quad (4.1)$$

and for clamped ends,

$$f_b \rightarrow f_{bt} = \frac{\pi^2 E}{3(1 - \nu^2)} \left(\frac{t}{L} \right)^2. \quad (4.2)$$

Figure 1 may be applied to ring stiffened cylinders where the rings provide the end conditions of at least simple-support. In such cases L may be taken to be the distance between rings.

5. INITIAL IRREGULARITIES DEDUCED FROM EXPERIMENTS

Figure 2 provides guidance on the value of δ/t inferred from tests on practical cylinders. The points on the figure have been derived from calculation of f_b/f_{bt} , using the results of experiments to determine f_b , and then obtaining the corresponding value of δ/t from Figure 1. These values of δ/t are then plotted against RL/t^2 to obtain Figure 2. Because the end conditions of the test cylinders were not always specified k has been assumed to be 0.9 throughout this analysis.

The data provided in Figure 2 are divided into two groups. The larger group relates to cylinders manufactured using normal engineering production methods. The smaller group, shown boxed on Figure 2, relates to specially manufactured laboratory specimen cylinders that are carefully loaded at their exact