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Elastic local buckling stresses of thin-walled unstiffened circular cylinders under combined axial compression and internal pressure

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THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item, which supersedes ESDU Struct 04.01.01, was monitored and guided by the Structures Committee. This Committee first met in 1940 and now has the following membership:

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ELASTIC LOCAL BUCKLING STRESSES OF THIN-WALLED UNSTIFFENED CIRCULAR CYLINDERS UNDER COMBINED AXIAL COMPRESSION AND INTERNAL PRESSURE

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ELASTIC LOCAL BUCKLING STRESSES OF THIN-WALLED UNSTIFFENED CIRCULAR CYLINDERS UNDER COMBINED AXIAL COMPRESSION AND INTERNAL PRESSURE

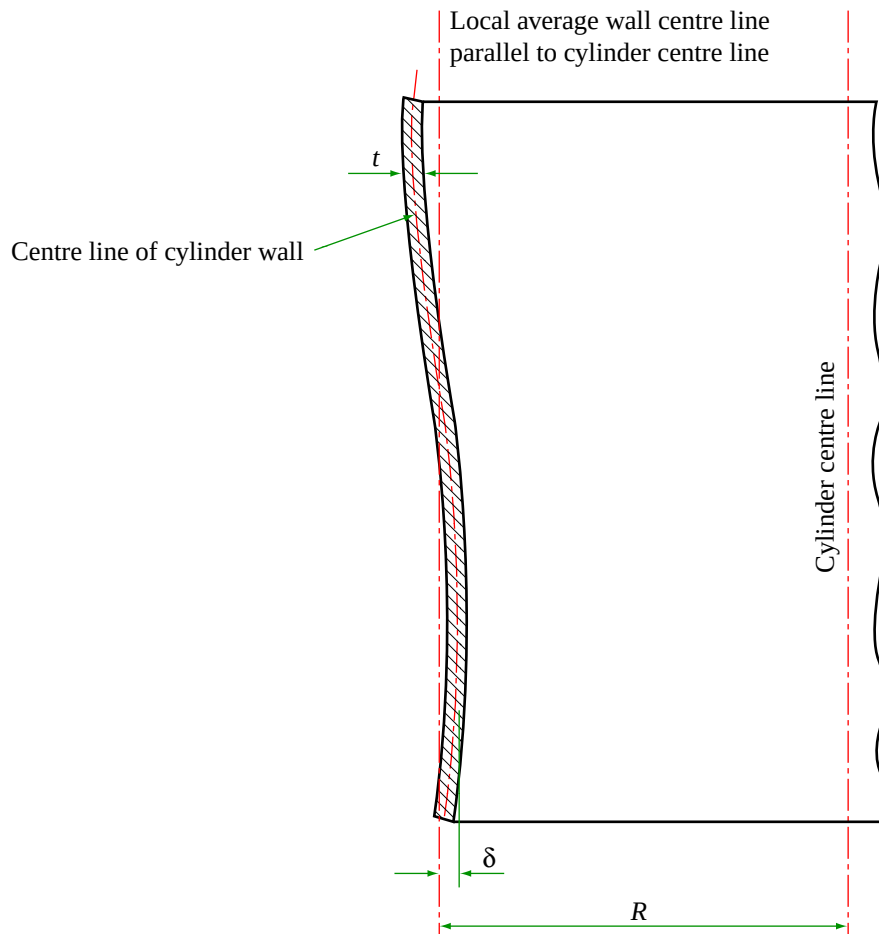
1. NOTATION

Both SI and British units are quoted, but any coherent system of units may be used.

c	confidence level, per cent		
E	modulus of elasticity of cylinder material	N/m ²	lbf/in ²
f_b	elastic buckling stress	N/m ²	lbf/in ²
f_{bt}	theoretical elastic buckling stress for geometrically perfect cylinder loaded in axial compression	N/m ²	lbf/in ²
	$f_{bt} = \frac{kE}{\left[3(1 - \nu^2)\right]^{1/2}} \left(\frac{t}{R}\right)$		
L	length of cylinder	m	in
k	coefficient in equation for f_{bt} correcting for end support conditions		
p	internal pressure	N/m ²	lbf/in ²
R	radius of cylinder (see Sketch 1.1)	m	in
S_{pc}	cumulative probability, per cent, that δ/t will not be exceeded by population estimated with confidence level c		
S_{qc}	cumulative probability, per cent, that δ/t will be exceeded by population estimated with confidence level c		
t	thickness of cylinder wall	m	in
δ	maximum inward initial deviation of mid-line of unloaded cylinder wall from straight axial mid-line generator of cylinder wall parallel to cylinder axis (see Sketch 1.1 and Section 3)	m	in
δ_e	effective value of δ when $p \neq 0$	m	in
ν	Poisson's ratio of cylinder material		
ρ_{Sc}	factor applicable to mean of sample to estimate δ/t for values of S_{pc} or S_{qc}		

Suffixes

It is convenient in calculation to replace the suffixes to ρ_{Sc} by the current values of S_{pc} or S_{qc} and c , for example $\rho_{99,95}$. This notation is also conveniently applied to other symbols as illustrated in Section 8.



Sketch 1.1 Section of cylinder wall showing definition of δ

2. GENERAL NOTES

This Item provides data for estimation of the elastic local buckling stress of thin-walled unstiffened circular cylinders subject to axial load or to combined axial load and internal pressure; overall buckling is not included. The data relate to cylinders of uniform wall thickness manufactured from isotropic materials. Local buckling occurs as a series of buckle waves around and along the cylinder. Notes on overall buckling, which normally only occurs in long cylinders acting as struts, are provided in ESDU 78021*.

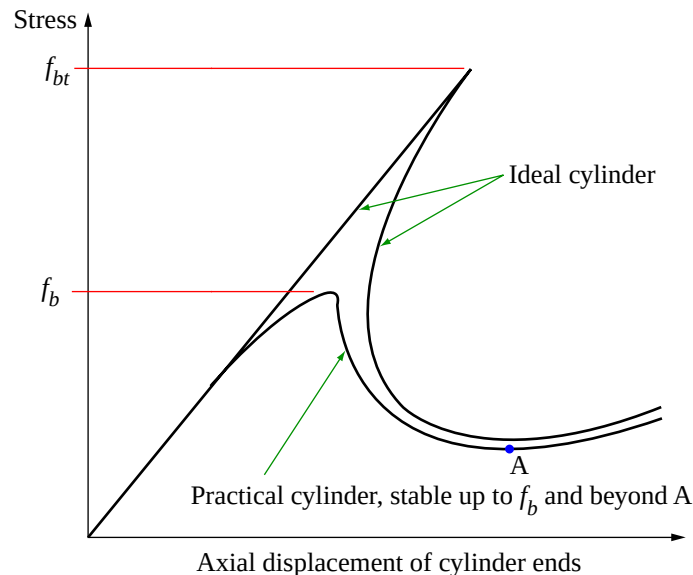
The axial load-carrying capacity of unpressurised cylinders is strongly influenced by inward initial irregularities, that is small inward deviations from the ideal “theoretical” cylinder prior to loading. Sketch 2.1 shows the local buckling behaviour of ideal and practical cylinders. The buckling stress for

* ESDU 78021 “Guide to Items on the strength and stability of struts”.

ideal cylinders, f_{bt} , is given by

$$f_{bt} = \frac{kE}{\left[3(1 - \nu^2)\right]^{1/2}} \left(\frac{t}{R}\right). \quad (2.1)$$

In practical cylinders increasing load results in a slow growth of initial irregularities. As the load is increased the initial irregularities grow more rapidly and buckling occurs, in that mode which requires the least energy of deformation, at an initial buckling stress f_b . Thereafter the mode of deformation usually develops into a characteristic diamond pattern that can reach a stable condition at a much increased axial displacement of the cylinder ends, see point A on Sketch 2.1. This is rarely of structural significance since local yielding usually occurs at the boundaries between diamonds in which case the cylinder retains much of the post-buckling deformation on removal of the load.



Sketch 2.1 Form of stress-displacement curves for ideal and practical cylinders

3. DEFINITION OF INITIAL IRREGULARITIES

For the purposes of buckling calculations it is convenient to divide the irregularities into two groups, axial and circumferential irregularities. Axial irregularities are the deviation of the wall centre line from a straight line, parallel to the axis of the cylinder, at the average radius of the wall centre line along the line. The maximum inward deviation anywhere along the cylinder for any axial line around the cylinder is termed δ . The influence of δ is such that $\delta = t/5$ is sufficient to reduce f_b to about half f_{bt} . Circumferential irregularities are deviations of the wall centre from the average circle. The effect on load-carrying capacity of circumferential irregularities that are maintained uniformly along the length of the cylinder is small, being approximately related to the reduced local curvature. In practical cylinders both axial and circumferential irregularities exist simultaneously, with the former dominating buckling.

Typical values of δ for specific constructions and size ranges may be obtained by measurement of previously manufactured cylinders. Estimates of δ may be obtained by placing a straight edge axially along the surface of the cylinder and measuring the maximum total inward irregularity of the surface with