



ESDU 83035

Issued November 1983

With Amendment F

March 2012

Estimation of the stiffnesses and apparent elastic properties of laminated flat plates

Associated software: ESDUpac A8335



Endorsed by
The Royal Aeronautical Society

AVA Documents
Document Sharing Hub

ESDU DATA ITEMS

Data Items provide validated information in engineering design and analysis for use by, or under the supervision of, professionally qualified engineers. The data are founded on an evaluation of all the relevant information, both published and unpublished, and are invariably supported by original work of ESDU staff engineers or consultants. The whole process is subject to independent review for which crucial support is provided by industrial companies, government research laboratories, universities and others from around the world through the participation of some of their leading experts on ESDU Technical Committees. This process ensures that the results of much valuable work (theoretical, experimental and operational), which may not be widely available or in a readily usable form, can be communicated concisely and accurately to the engineering community.

We are constantly striving to develop new work and review data already issued. Any comments arising out of your use of our data, or any suggestions for new topics or information that might lead to improvements, will help us to provide a better service.

THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item was monitored and guided by the Acoustic Fatigue Panel on behalf of the Vibration and Structural Dynamic Steering Group.

Chairman

Mr J.A. Hay — British Aerospace plc, Aircraft Group, Weybridge

Members

Dr R.D. Adams — University of Bristol

Mr B.P. Broadley — Rolls-Royce Ltd, Derby

Mr R.J. Cummins — British Aerospace plc, Aircraft Group, Weybridge-Bristol Division

Mr D.C.G. Eaton* — The European Space Research and Technology Centre, Domeinweg, The Netherlands

Mr J.E. Marshall — Short Brothers Ltd, Belfast

Mr W.B. Roberts — British Aerospace plc, Dynamics Group, Hatfield Division

Prof. R.G. White — Institute of Sound and Vibration Research, University of Southampton

* Corresponding Member

The Vibration and Structural Dynamics Steering Group, which took over the work on acoustic fatigue previously monitored by the Fatigue Committee, first met in July 1976 and had the following membership:

Chairman

Dr D.C.G. Eaton — The European Space Research and Technology Centre, Domeinweg, The Netherlands

Members

Mr T.V. Lawson — University of Bristol

Dr M.R. Smith — University of Newcastle-upon-Tyne

Dr H.H.B.M. Thomas — Independent.

The technical work involved in the initial assessment of the available information and the construction and subsequent development of the Item was

Mr R.F. Lambert — Head of Noise and Structural Dynamics Group.

ESTIMATION OF THE STIFFNESSES AND APPARENT ELASTIC PROPERTIES OF LAMINATED FLAT PLATES

CONTENTS

	Page
1. NOTATION	1
2. INTRODUCTION	2
3. LAMINATED PLATE STIFFNESSES AND APPARENT ELASTIC PROPERTIES	3
4. SPECIALLY ORTHOTROPIC PLATES	4
5. ESTIMATION OF LAYER ELASTIC PROPERTIES	5
5.1 Layer Volume Fraction	6
5.2 Layer Modulus of Elasticity in α -direction	7
5.3 Layer Poisson's Ratios	7
5.4 Layer Modulus of Elasticity in β -direction	7
5.5 Layer Shear Modulus, $G_{\alpha\beta}$	8
6. PROGRAM TO ESTIMATE LAMINATED PLATE STIFFNESSES AND APPARENT ELASTIC PROPERTIES (THE CURRENT VERSION OF ESDUPAC A8335 IS 4.3)	8
6.1 Data Input	8
6.1.1 General	9
6.1.2 Estimation of lamina elastic properties	9
6.1.3 Lamina properties	10
6.2 Data Output	10
7. DERIVATION AND REFERENCE	11
7.1 Derivation	11
7.2 Reference	11
8. EXAMPLES	11
8.1 Estimation of Composite Plate Stiffnesses and Elastic Properties using the Computer Program Described in Section 6	11
8.2 Estimation of Elastic Properties of a Lamina at a Changed Fibre Volume Fraction	15
8.3 Sample Input Files	16
9. TABLE	19
FIGURES	20 to 22

ESTIMATION OF THE STIFFNESSES AND APPARENT ELASTIC PROPERTIES OF LAMINATED FLAT PLATES

1. NOTATION

A	in-plane stiffness sub-matrix for plate [see Equation (3.1)]	N/m	lbf/in
B	coupled in-plane flexural stiffness sub-matrix for plate [see Equation (3.1)]	N	lbf
D	flexural stiffness sub-matrix for plate [see Equation (3.1)]	N m	lbf in
E	fibre or matrix modulus of elasticity	N/m ²	lbf/in ²
E_x, E_y	plate moduli of elasticity in x-, y-directions, respectively	N/m ²	lbf/in ²
E_α, E_β	lamina moduli of elasticity in α -, β -directions, respectively	N/m ²	lbf/in ²
G_{xy}	shear stiffness of plate in (x, y) direction	N/m ²	lbf/in ²
$G_{\alpha\beta}$	shear stiffness of lamina in (α , β) direction	N/m ²	lbf/in ²
K	$\begin{bmatrix} -\partial^2 w / \partial x^2 \\ -\partial^2 w / \partial y^2 \\ -2\partial^2 w / \partial x \partial y \end{bmatrix}$ column sub-matrix of curvature components of plate [see Equation (3.1)]	m ⁻¹	in ⁻¹
M	$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix}$ column sub-matrix of bending moment intensities acting in xz- and yz-planes and twisting moment intensities acting about x- and y-axes, respectively [see Equation (3.1)]	N m/m	lbf in/in
m	mass fraction		
N	$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix}$ column sub-matrix of direct load intensities and shear load intensities acting in neutral plane of plate, referred to x-, y-axes [see Equation (3.1)]	N/m	lbf/in
n	number of layers in plate		
V	volume fraction		
w	deflection of plate in z-direction	m	in

Issued November 1983
With Amendment F, March 2012 – 22 pages

α, β	lamina coordinate system aligned with principal axes of orthotropy of lamina (see Sketch 3.2)		
ε	$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{bmatrix}$	column matrix of mid-plane direct strains and shear strain acting in neutral plane of plate, referred to x-, y-axes	
η	non-dimensional parameter (see Figure 1)		
ν_{xy}	Poisson's ratio for plate resulting from load applied in x-direction		
ν_{yx}	Poisson's ratio for plate resulting from load applied in y-direction		
$\nu_{\alpha\beta}$	Poisson's ratio for lamina resulting from load applied in α -direction		
$\nu_{\beta\alpha}$	Poisson's ratio for lamina resulting from load applied in β -direction		
ρ	density	kg/m ³	*
ψ	angle of principal axis of orthotropy, α , for lamina, measured from plate principal x-axis (see Sketch 3.2)	degree	degree

Suffixes

f	fibre
k	known value
m	matrix

* Fibre and matrix densities are used in Equations (5.2) and (5.3). In both equations only density ratios are used so that density may be expressed in British units as lbf/in³ or lb s²/in⁴.

2. INTRODUCTION

Fibre reinforced composite materials are being increasingly utilised in the design of lightweight structures such as those used in aircraft. The particular advantages of such materials are their high strength-to-weight and stiffness-to-weight ratios. A further advantage of fibre reinforced composites is that the strength and stiffness properties may be tailored to meet requirements by the use of multilayered laminates with appropriate relative orientations of the principal fibre direction of the composite layers. This Item provides a computer program for the estimation of the stiffnesses and apparent elastic properties of composite material laminated plates.

A particular class of laminated plates, referred to as specially orthotropic laminates, is discussed in Section 4. These plates are balanced laminates, in which layers of equal thickness, elastic properties and orientation are symmetrically disposed about a central plane. However, not all laminated plates that exhibit

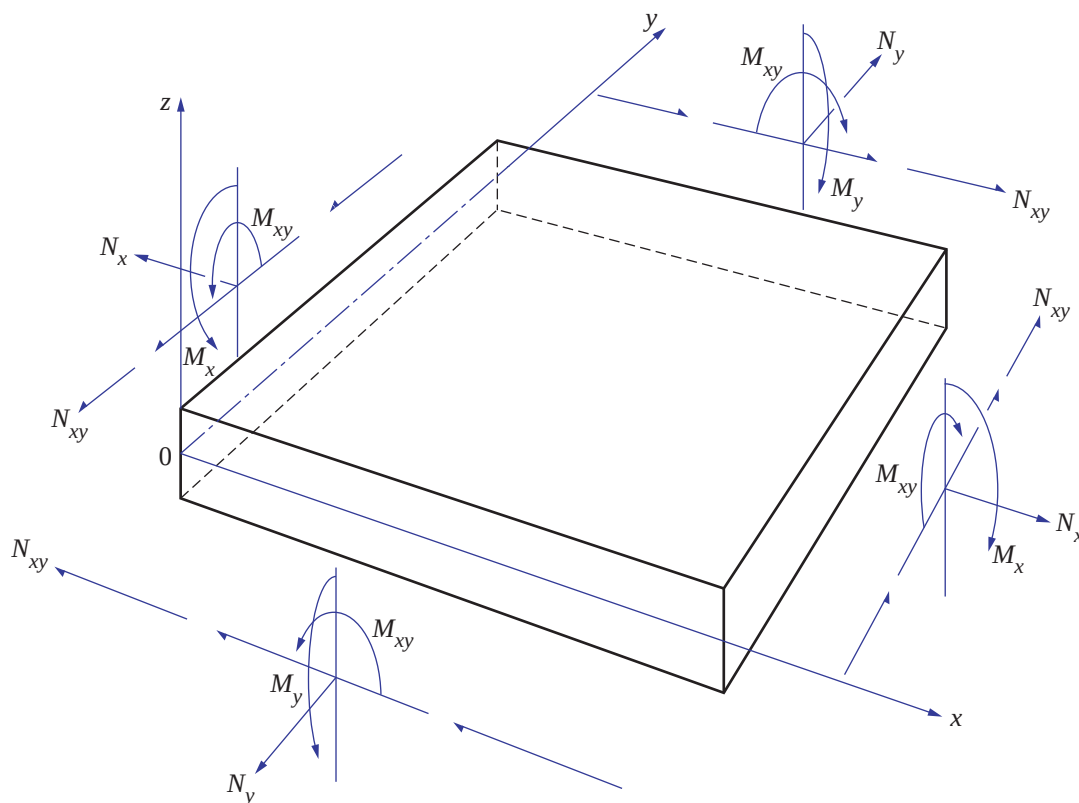
those characteristics are specially orthotropic (see Section 4 for a precise definition). The behaviour of multilayered specially orthotropic laminates under in-plane and flexural loads is closely approximated by that of an orthotropic plate under similar loading (*i.e.* plates bend and stretch without twisting).

In Section 3, the relationship between the in-plane and flexural load deformation is given which defines the laminated plate stiffness sub-matrices. That Section also gives examples of the types of laminate that can be analysed using this Item. The estimation of layer elastic properties is discussed in Section 5 and an approximate method is provided. The use of the program to estimate stiffnesses and elastic properties of laminated plates is described in Section 6, and input and output for the program are provided for a typical example in Section 8. A computer program, referred to as ESDUpac A8335*, accompanies this Item.

3. LAMINATED PLATE STIFFNESSES AND APPARENT ELASTIC PROPERTIES

The in-plane and flexural load deformation relationships for a plate consisting of an assembly of layers may be written, using the sign convention shown in Sketch 3.1, as follows.

$$\begin{bmatrix} \mathbf{N} \\ \mathbf{M} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon} \\ \boldsymbol{\kappa} \end{bmatrix} \quad (3.1)$$



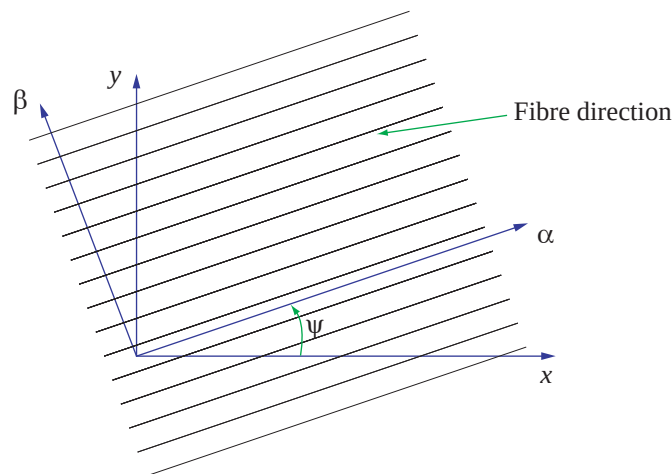
Sketch 3.1 Loading sign convention

* The full ESDUpac number has an appended V followed by two digits. The digits indicate the ESDUpac version number. For example, the full ESDUpac number for version 4.3 of ESDUpac A8335 is A8335V43. The version appendage will not be used when referring to the ESDUpac in the text for this Item.

It is essential that the sign convention used in the evaluation of the stiffness matrices is identical to that used in the theory for subsequent analyses in which the calculated stiffness matrices are used. For example, a reversal of the moment sign convention shown in Sketch 3.1 will leave the **A** and **D** matrices unchanged but will reverse the sign of the **B** matrix without changing the magnitude of the matrix elements.

The properties of the plate are referred to a set of reference axes (xyz). The $z = 0$ plane is chosen as the central plane as this minimises the coupling terms between in-plane loading and bending. The properties of an individual layer within the plate are referred to the principal axes of orthotropy of the layer (α, β). The relationship between the plate axes and layer axes is shown in Sketch 3.2. In plates that behave orthotropically the xy -axes are chosen as the principal axes of orthotropy.

The basic formulae and matrix calculation procedure used in the computer programs in this Item, for the plate stiffnesses, **A**, **B** and **D**, are given in ESDU 94003 (Derivation 5). The apparent elastic properties, derived from the elements of the **A** matrix, are the values of Young's moduli, shear stiffness and Poisson's ratios that may be used when the laminate is assumed to act as an orthotropic plate.



Sketch 3.2 Relationships between plate and layer axes

Examples of the types of laminated plates, for which the stiffnesses and elastic properties may be found using the program in Section 6, are listed.

- (a) Laminated fibre reinforced plates composed of layers having different properties, thicknesses and/or orientations of the principal axes of orthotropy.
- (b) Laminated plates composed of combinations of layers of fibre reinforced materials and isotropic materials.
- (c) Sandwich panels with isotropic cores and face plates. The face plates of these sandwich panels may also be of the types (a) or (b).

4. SPECIALLY ORTHOTROPIC PLATES

Specially orthotropic laminates have principal axes of orthotropy that are, by convention, chosen as the xy -axes. When stiffnesses are referred to these axes they satisfy the following three conditions.

- (1) The in-plane stiffness sub-matrix **A** has direct and shear stiffnesses uncoupled so that A_{13} and A_{23} are zero.

- (2) The flexural stiffness sub-matrix **D** has bending and twisting stiffnesses uncoupled so that D_{13} and D_{23} are zero
- (3) The in-plane and flexural stiffness sub-matrices are uncoupled, that is **B** = 0.

For a specially orthotropic laminate the in-plane and flexural load deformation relationships [see Equation (3.1)] are,

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 & 0 & 0 \\ A_{21} & A_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{33} & 0 & 0 & 0 \\ - & - & - & - & - & - \\ 0 & 0 & 0 & D_{11} & D_{12} & 0 \\ 0 & 0 & 0 & D_{21} & D_{22} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \\ -\partial^2 w / \partial x^2 \\ -\partial^2 w / \partial y^2 \\ -2\partial^2 w / \partial x \partial y \end{bmatrix} \quad (4.1)$$

In Equation (4.1) the **A** and **D** sub-matrices are symmetric so that $A_{12} = A_{21}$, $A_{13} = A_{31}$, $A_{23} = A_{32}$ and $D_{12} = D_{21}$, etc.

The analysis of laminated plates is simplified if the conditions for special orthotropy are satisfied. Also, because end load and shear, and bending and twisting moments, are all uncoupled, distortion during manufacture is minimised. In ESDU 82013 (Reference 6) the stacking sequence of layer angles to achieve specially orthotropic laminates is given for laminates with an even or odd number of layers for both symmetric and anti-symmetric arrangements about the central plane. In practice it is difficult to manufacture laminated plates that exactly conform to the conditions of special orthotropy, but many multilayered composite laminates approximate to these conditions.

5. ESTIMATION OF LAYER ELASTIC PROPERTIES

Essential inputs into the computer program for the estimation of laminated plate stiffnesses are the elastic constants of the individual layers. These properties may be found by direct measurement on a multilayered composite having all fibres aligned in one direction. In order to obtain reliable elastic constants this unidirectional-fibre laminate should be manufactured using the same procedures as those for normal production laminated plates. If measurement of the individual layer properties is not practicable, the properties may be estimated using approximate relationships (presented later in this Section) based on the individual fibre and matrix elastic properties and their volume fractions. These approximate relationships may also be used to estimate changes in the known elastic properties of an individual lamina at different fibre volume fractions. However, it should be noted that the approximate relationships for lamina elastic properties are not applicable to those in which the fibres are woven.

To estimate the elastic properties of a lamina the following basic assumptions must be made.

- (a) For the lamina.
The lamina is assumed to be macroscopically homogeneous, linearly elastic, generally orthotropic or transversely orthotropic and initially stress-free.

- (b) For the fibres.
The fibres are assumed to be linearly elastic, homogeneous, regularly spaced and perfectly aligned.
- (c) For the matrix.
The matrix is assumed to be isotropic, linearly elastic and homogeneous.

Further, the fibre and matrix are assumed to be free of voids and the bonding between the fibres and matrix is assumed to be perfect.

Consideration of the assumptions needed to estimate lamina elastic properties illustrates the number of factors that should be maintained constant in the manufacture of a lamina. It follows that there could be significant statistical variation of the composite lamina elastic properties compared with those for isotropic materials. It is therefore common practice to estimate composite lamina elastic properties using simple approximate relationships empirically derived from the more complicated micromechanics results. This approach, which is considered sufficiently accurate for most engineering applications when the fibre volume fraction does not approach unity, is described in the following sub-sections.

Elastic properties, other than the longitudinal elastic modulus, of individual fibres are not easily measured. Fibre elastic properties are often found by measuring the appropriate elastic property of a multilayered unidirectional-fibre laminate and of the matrix material, and then estimating the fibre property using the prediction equation. In particular, the procedure is generally used to obtain the fibre transverse modulus of elasticity. Fibre properties found in that way are appropriate for use in the prediction equations but may not be suitable for other purposes. It should be noted that the same prediction equations should be used both to obtain the fibre properties from measurements, and to predict the lamina elastic properties using those derived fibre properties. In Table 9.1, elastic properties of some commonly used fibre and matrix materials are listed. These properties may be used for the estimation of lamina properties using the procedures given in this Item.

5.1 Layer Volume Fraction

The fibre volume fraction, V_f , and the matrix volume fraction, V_m , in a lamina without voids, satisfy the relationship

$$V_f + V_m = 1. \quad (5.1)$$

The estimation procedures in this Item are given in terms of the volume fractions. If the composition of a lamina is specified in terms of the fibre and matrix mass fraction, m_f and m_m respectively, these are readily converted into volume fractions using the relationships

$$V_f = \frac{m_f}{m_f + m_m (\rho_f / \rho_m)} \quad (5.2)$$

and

$$V_m = \frac{m_m}{m_f (\rho_f / \rho_m) + m_m}, \quad (5.3)$$

where ρ_f and ρ_m are the fibre and matrix material densities respectively.