

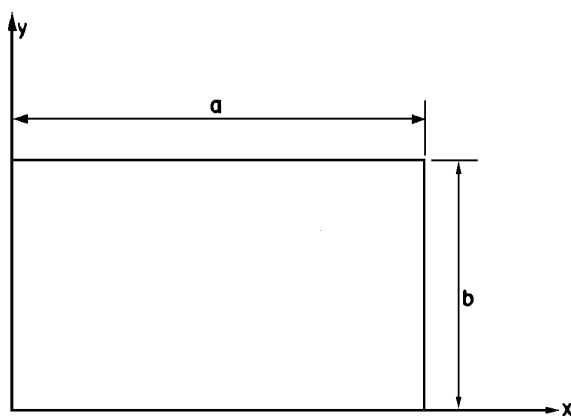
NATURAL FREQUENCIES OF RECTANGULAR, SPECIALLY ORTHOTROPIC LAMINATED PLATES

1. NOTATION AND UNITS

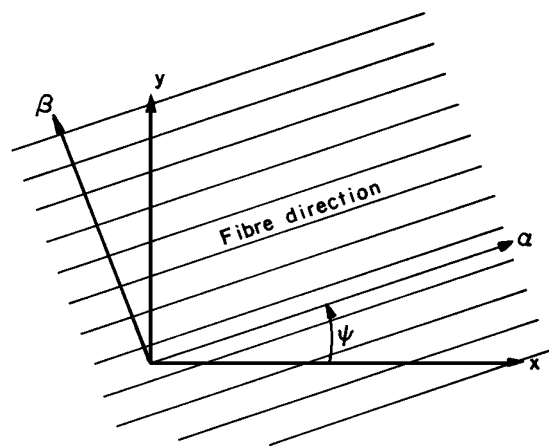
a	length of plate in x -direction (see Sketch 1.1)	m	in
b	length of plate in y -direction, or arc length of curved plate (see Sketch 1.1)	m	in
$\mathbf{D}$	flexural stiffness sub-matrix for plate [see Equation (2.2)]	N m	lbf in
E_α, E_β	lamina modulus of elasticity in α -, β -directions, respectively	N/m ²	lbf/in ²
f	natural frequency	Hz	Hz
$G_{\alpha\beta}$	shear stiffness of lamina in (α, β) direction	N/m ²	lbf/in ²
$\mathbf{K}$	$\begin{bmatrix} \partial^2 w / \partial x^2 \\ \partial^2 w / \partial y^2 \\ 2 \partial^2 w / \partial x \partial y \end{bmatrix}$ column sub-matrix of curvature components of plate [see Equation (2.2)]	m ⁻¹	in ⁻¹
$\mathbf{M}$	$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix}$ column sub-matrix of bending moment intensities acting in xz - and yz -planes and twisting moment intensities acting about x - and y -axes, respectively	N m/m	lbf in/in
m	mode number in x -direction		
N	number of layers in plate		
n	mode number in y -direction		
R	plate mean radius	m	in
t	laminated plate thickness	m	in
w	deflection of plate normal to xy -plane	m	in
x, y	plate coordinate system (see Sketch 1.1)		
α, β	lamina coordinates aligned with principal axes of orthotropy of lamina (see Sketch 1.2)		
$\nu_{\alpha\beta}$	Poisson's ratio for lamina resulting from load applied in α -direction		

$\nu_{\beta\alpha}$	Poisson's ratio for lamina resulting from load applied in β -direction		
ρ	mean density of plate	kg/m ³	*
ψ	angle of principal axis of orthotropy, α , of lamina measured from plate principal axis, x (see Sketch 1.2)	degree	degree

* A density value expressed in British units as lb/in³ has to be divided by 386.4 before being input into the computer program. (A force of 1 lbf acting on a mass of 1 lb produces an acceleration of 386.4 in/s².) For example, if the material density is 0.097 lb/in³, the value input into the computer program should be 2.51×10^{-4} lb s²/in⁴.



Sketch 1.1 Plate dimensions



Sketch 1.2 Relationship between plate and layer axes

2. NOTES

2.1 General

This Data Item provides a means of estimating the lower natural frequencies of initially unstressed, specially orthotropic (see Section 2.2), rectangular plates. The data are particularly applicable to fibre reinforced composite laminated plates. Because of the large number of lay-up configurations that may be used in laminated plates it is not practical to provide natural frequency data in graphical form. Data are provided in the form of a computer program, known as ESDUpac A8336, written in Fortran 77. The Fortran source code with example input and output files can be found on the Compact Disc and on disc in the Software Volume.

The programs in this Item allow the estimation of the natural frequencies of plates having any combination of simply-supported and clamped edge conditions. These boundary conditions are illustrated in Figure 1.

An appropriate procedure is provided (Section 2.4) for the estimation of the fundamental natural frequency of slightly curved plates having all edges clamped. These singly-curved plate natural frequencies are obtained by factoring the computed flat plate fundamental natural frequencies.

Every reasonable effort has been made to ensure that the program performs the intended calculations satisfactorily. However, if you do have any difficulty in using it please notify ESDU International and we will do all we can to assist you to overcome the problem. In common with other software providers, ESDU

makes no representation as to the suitability or fitness of the program for any particular purposes and no liability for any loss occasioned by any persons as a direct or indirect result of use of the program whether arising from negligence or otherwise can be accepted. In no event shall ESDU or any individuals associated with the development of the program be liable for any damage, including loss of profit or consequential loss, arising out of or in connection with the program.

2.2 Specially Orthotropic Laminated Plates

Specially orthotropic plates are balanced laminates having layers of equal thickness, elastic properties and orientation that are symmetrically arranged about the plate central plane. The conditions for special orthotropy are detailed in Item No. 83035. In specially orthotropic plates, direct and shear stiffnesses are uncoupled; further, bending and twisting stiffnesses are uncoupled so that plane sections normal to the reference plane remain plane and normal to the reference plane during small amplitude deflections. Also, the in-plane and flexural stiffnesses are uncoupled so that for specially orthotropic laminates the flexural load deformation relationship (used in determining the plate natural frequencies) is

$$[\mathbf{M}] = [\mathbf{D}][\mathbf{K}] , \quad (2.1)$$

which, in its expanded form, becomes

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & 0 \\ 0 & 0 & D_{33} \end{bmatrix} \begin{bmatrix} \partial^2 w / \partial x^2 \\ \partial^2 w / \partial y^2 \\ 2 \partial^2 w / \partial x \partial y \end{bmatrix} . \quad (2.2)$$

The $\mathbf{D}$ -matrix in Equation (2.2) is symmetric so that $D_{12} = D_{21}$.

In this Item the principal axes of orthotropy of the specially orthotropic plates are assumed to be parallel to the xy -axes. In Item No. 82013 (Reference 5) the stacking sequence of layer angles to achieve specially orthotropic laminates is given for laminates with even or odd number of layers for both symmetric and anti-symmetric ply orientations about the central plane. In practice it is difficult to manufacture laminated plates that exactly conform to the conditions of special orthotropy, but many multilayered composite laminates approximate to those conditions which allows simple analysis procedures to be used to determine natural frequencies.

2.3 Mode Numbers

For any mode of vibration, the mode numbers m and n may be related to the nodal lines provided that these lines are parallel, or approximately parallel, to the plate sides. For these cases m and n are one plus the number of nodal lines crossing any line parallel to the x -axis and y -axis, respectively. In this definition plate edges, clamped or simply-supported, are not counted as nodal lines.

2.4 Effect of Plate Curvature

An approximate means of estimating the fundamental natural frequency ($m = n = 1$ mode) of slightly curved, rectangular laminated plates with all edges clamped is presented in Figure 2. On that Figure, the ratio of curved plate frequency to flat plate frequency is plotted against the curvature parameters b^2/Rt , where b is the arc length of the plate curved side. The curve is an empirical relationship but is based on the theoretical isotropic plate frequency ratios. It is applicable to all panel a/b ratios ($a/b > 1.0$) over the plotted range of b^2/Rt .

2.5 Accuracy and Limitations on Data

In this Item a means of estimating the approximate natural frequencies for initially unstressed, rectangular, specially orthotropic plates is provided.

The method of analysis is based on the Rayleigh-Ritz method in which the plate displacements are assumed to be represented by a finite series of terms involving functions that satisfy the approximate boundary conditions at the plate edges. In this Item the plate deflections are assumed to be represented by a single beam characteristic shape function in each of the x, y orthogonal directions. The estimated values of natural frequencies are upper bounds but, for the lower natural frequencies, are considered to be sufficiently accurate for practical engineering purposes.

In calculating natural frequencies shear and rotary inertia effects are neglected. Also displacements are assumed to be small relative to the plate thickness, and the plate is assumed to be thin relative to the plate dimensions (*i.e.* the ratios a/t and b/t should be greater than about 20).

In theory, more accurate values of natural frequencies could be obtained by using more terms in the series representing the displacements in the Rayleigh-Ritz analysis. However, it should be realised that the conditions for special orthotropy are rarely exactly achieved in practice. Also because of the complexities involved in the manufacture of composite laminates it is likely that the tolerance on the elastic properties, used in estimating natural frequencies, would be greater than those for isotropic materials.

Figure 3 shows a comparison of the estimated and measured fundamental natural frequencies ($m = n = 1$ mode) of single plates within a stiffened panel array. In estimating the natural frequencies the plate dimensions were set equal to the distances between the attachments of the adjacent stiffening members, and a clamped edge condition was assumed on all plate edges (*i.e.* edge condition 1). Data are for carbon^{*}/epoxy laminated plates having more than four layers.

3. COMPUTATION OF NATURAL FREQUENCIES

The computer program on ESDUpac A8336 diskette provides a means of estimating the natural frequencies of flat, specially orthotropic plates having any combination of simply-supported and clamped edge conditions. These panel edge conditions are illustrated in Figure 1.

The program may be entered with known elements of the **D** stiffness matrix, or the laminate lay-up and individual lamina properties may be input from which the **D**-matrix is evaluated. When the **D**-matrix is evaluated, it is checked to see if it satisfies the conditions for special orthotropy[†]. If the elastic properties of a carbon-fibre lamina are not known, they may be estimated from the constituent fibre and matrix properties using Item No. 83035.

In using the program, care should be taken to ensure that the relationship between the plate x -, y -axes and the lamina α -, β -axes is as shown in Sketch 1.2. Also since the ratio a/b may be greater or less than 1, note that m and n are mode numbers in the x - and y -directions, respectively, and do not necessarily relate to the longer and shorter panel sides. Further it should be noted that D_{11} and D_{12} relate to bending in the xz -plane and the yz -plane, respectively.

* In reports published in the USA, carbon fibre is generally referred to as graphite fiber.

† It should be noted that the full requirements for special orthotropy cannot be assessed on the flexural stiffness matrix alone. See Item No. 83035 for an exact definition of special orthotropy.

3.1 Data Input

The input data may be entered either from the keyboard or from a file. If data are to be input from a file, the 'C' against one line in the program controlling routine, clearly identified within the program, must be deleted. For input from a file, the input file 'IN8336' must be independently created. Using a file input allows data correction, or case data changes, to be made using file editing facilities. Entering case data from the keyboard provides the opportunity for the program to give visual requests for each item of data as it is required.

The form of the input file IN8336 is illustrated by the input file listed for the example in Section 5.

Data are input in free format in the following order and any coherent system of units may be used.

3.1.1 General

Three lines of run title (75 characters allowed per line).

A blank line must be inserted for any of the three title lines not used*.

Filename for output (12 characters allowed).

3.1.2 Plate stiffness data

Are elements of the **D**-matrix to be calculated?

Answer 1 (YES) or 0 (NO).

If the **D**-matrix is to be calculated go on to Section 3.1.3.

Elements of the **D**-matrix.

$D_{11}, D_{12}, D_{22}, D_{33}$

Plate thickness, plate mean density[†].

t, ρ

Go to Section 3.1.4 for the next input.

3.1.3 Data for calculation of **D**-matrix

Number of layers in plate[‡].

N

Plate layers are automatically numbered from 1 to N , starting at an outer layer.

Is the laminate symmetric?

Answer 1 (YES) or 0 (NO)

For specially orthotropic plates considered in this Item, plates must be symmetric or anti-symmetric in their

* A blank line is created by pressing the enter key with the cursor at the left hand edge of the screen.

† See footnote at the end of the Notation section, Section 1.

‡ In the Fortran program the dimension statement for variable Q limits the maximum value of N . The maximum value of N is set to 50 in the program.