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Natural frequencies of rectangular, specially orthotropic laminated plates

Associated software: ESDUpac A8336

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THE PREPARATION OF THIS DATA ITEM

The work on this particular Item was monitored and guided by the Acoustic Fatigue Panel which had the following membership:

Chairman		
Mr J.A. Hay	—	British Aerospace plc, Aircraft Group, Weybridge – Bristol Division
Members		
Dr R.D. Adams	—	University of Bristol
Mr B.P. Broadley	—	Rolls-Royce Ltd, Derby
Mr R.J. Cummins	—	British Aerospace plc, Aircraft Group, Weybridge – Bristol Division
Mr D.C.G. Eaton*	—	The European Space Research and Technology Centre, Domeinweg, The Netherlands
Mr J.E. Marshall	—	Short Brothers Ltd, Belfast
Mr W.B. Roberts	—	British Aerospace plc, Dynamics Group, Hatfield Division
Prof. R.G. White	—	Institute of Sound and Vibration Research, University of Southampton.

* Corresponding Member

The Item was accepted for inclusion in the Acoustic Fatigue series by the Vibration and Structural Dynamics Steering Group which first met in July 1976 and now has the following membership:

Chairman		
Mr D.C.G. Eaton	—	The European Space Research and Technology Centre, Domeinweg, The Netherlands
Members		
Mr T.V. Lawson	—	University of Bristol
Dr M.R. Smith	—	University of Newcastle-upon-Tyne
Dr H.H.B.M. Thomas	—	Independent.

The work on this Item was carried out in the Aircraft Noise and Structural Dynamics Group of ESDU. The member of staff who undertook the technical work involved in the initial assessment of the available information and the construction and subsequent development of the Item was

Mr R.F. Lambert	—	Group Head.

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NATURAL FREQUENCIES OF RECTANGULAR, SPECIALLY ORTHOTROPIC LAMINATED PLATES

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NATURAL FREQUENCIES OF RECTANGULAR, SPECIALLY ORTHOTROPIC LAMINATED PLATES

1. NOTATION AND UNITS

a	length of plate in x-direction (see Sketch 1.1)	m	in
b	length of plate in y-direction, or arc length of curved plate (see Sketch 1.1)	m	in
D	flexural stiffness sub-matrix for plate [see Equation (2.2)]	N m	lbf in
E_α, E_β	lamina modulus of elasticity in α-, β-directions, respectively	N/m^2	lbf/in^2
f	natural frequency	Hz	Hz
$G_{\alpha\beta}$	shear stiffness of lamina in (α, β) direction	N/m^2	lbf/in^2
K	$\begin{bmatrix} \partial^2 w/\partial x^2 \\ \partial^2 w/\partial y^2 \\ 2\partial^2 w/\partial x \partial y \end{bmatrix}$ column sub-matrix of curvature components of plate [see Equation (2.2)]	m^{-1}	in^{-1}
M	$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix}$ column sub-matrix of bending moment intensities acting in xz- and yz-planes and twisting moment intensities acting about x- and y-axes, respectively	N m/m	lbf in/in
m	mode number in x-direction		
N	number of layers in plate		
n	mode number in y-direction		
R	plate mean radius	m	in
t	laminated plate thickness	m	in
w	deflection of plate normal to xy-plane	m	in
x, y	plate coordinate system (see Sketch 1.1)		
α, β	lamina coordinates aligned with principal axes of orthotropy of lamina (see Sketch 1.2)		
$\nu_{\alpha\beta}$	Poisson's ratio for lamina resulting from load applied in α-direction		

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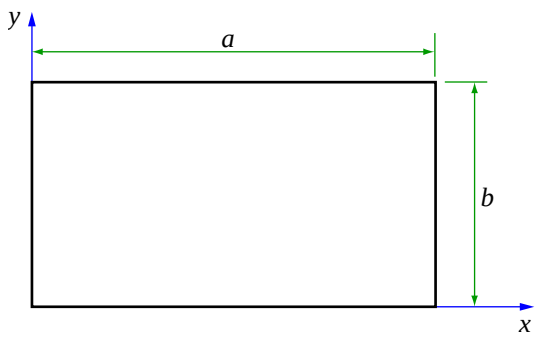
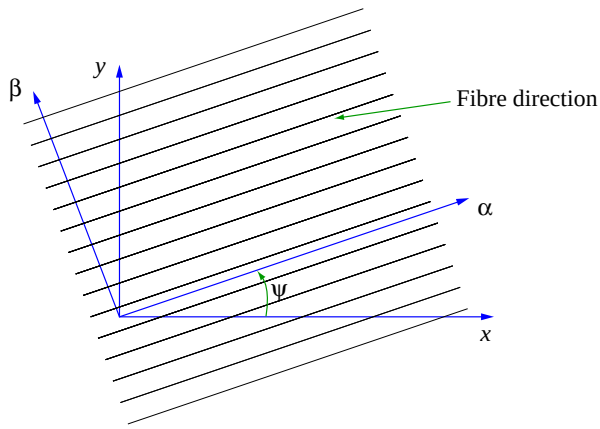
$\nu_{\beta\alpha}$	Poisson's ratio for lamina resulting from load applied in β-direction		
ρ	mean density of plate	kg/m^3	*
ψ	angle of principal axis of orthotropy, α, of lamina measured from plate principal axis, x (see Sketch 1.2)	degree	degree

* A density value expressed in British units as lb/in^3 has to be divided by 386.4 before being input into the computer program. (A force of 1 lbf acting on a mass of 1 lb produces an acceleration of 386.4 in/s^2.) For example, if the material density is 0.097 lb/in^3, the value input into the computer program should be 2.51×10^{-4} lb s^2/in^4.



Sketch 1.1 Plate dimensions



Sketch 1.2 Relationship between plate and layer axes

2. NOTES

2.1 General

This Data Item provides a means of estimating the lower natural frequencies of initially unstressed, specially orthotropic (see Section 2.2), rectangular plates. The data are particularly applicable to fibre reinforced composite laminated plates. Because of the large number of lay-up configurations that may be used in laminated plates it is not practical to provide natural frequency data in graphical form. Data are provided in the form of a computer program, known as ESDUpac A8336.

The programs in this Item allow the estimation of the natural frequencies of plates having any combination of simply-supported and clamped edge conditions. These boundary conditions are illustrated in Figure 1.

An appropriate procedure is provided (Section 2.4) for the estimation of the fundamental natural frequency of slightly curved plates having all edges clamped. These singly-curved plate natural frequencies are obtained by factoring the computed flat plate fundamental natural frequencies.

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2.2 Specially Orthotropic Laminated Plates

Specially orthotropic plates are balanced laminates having layers of equal thickness, elastic properties and orientation that are symmetrically arranged about the plate central plane. The conditions for special orthotropy are detailed in ESDU 83035. In specially orthotropic plates, direct and shear stiffnesses are uncoupled; further, bending and twisting stiffnesses are uncoupled so that plane sections normal to the reference plane remain plane and normal to the reference plane during small amplitude deflections. Also, the in-plane and flexural stiffnesses are uncoupled so that for specially orthotropic laminates the flexural load deformation relationship (used in determining the plate natural frequencies) is

$$[\mathbf{M}] = [\mathbf{D}][\mathbf{K}] , \tag{2.1}$$

which, in its expanded form, becomes

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & 0 \\ 0 & 0 & D_{33} \end{bmatrix} \begin{bmatrix} \partial^2 w/\partial x^2 \\ \partial^2 w/\partial y^2 \\ 2\partial^2 w/\partial x \partial y \end{bmatrix} . \tag{2.2}$$

The **D**-matrix in Equation (2.2) is symmetric so that $D_{12} = D_{21}$.

In this Item the principal axes of orthotropy of the specially orthotropic plates are assumed to be parallel to the *xy*-axes. In ESDU 82013 (Reference 5) the stacking sequence of layer angles to achieve specially orthotropic laminates is given for laminates with even or odd number of layers for both symmetric and anti-symmetric ply orientations about the central plane. In practice it is difficult to manufacture laminated plates that exactly conform to the conditions of special orthotropy, but many multilayered composite laminates approximate to those conditions which allows simple analysis procedures to be used to determine natural frequencies.

2.3 Mode Numbers

For any mode of vibration, the mode numbers m and n may be related to the nodal lines provided that these lines are parallel, or approximately parallel, to the plate sides. For these cases m and n are one plus the number of nodal lines crossing any line parallel to the *x*-axis and *y*-axis, respectively. In this definition plate edges, clamped or simply-supported, are not counted as nodal lines.

2.4 Effect of Plate Curvature

An approximate means of estimating the fundamental natural frequency ($m = n = 1$ mode) of slightly curved, rectangular laminated plates with all edges clamped is presented in Figure 2. On that Figure, the ratio of curved plate frequency to flat plate frequency is plotted against the curvature parameters b^2/Rt, where b is the arc length of the plate curved side. The curve is an empirical relationship but is based on the theoretical isotropic plate frequency ratios. It is applicable to all panel a/b ratios ($a/b > 1.0$) over the plotted range of b^2/Rt.

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