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Pressure losses in curved ducts: single bends

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THE PREPARATION OF THIS DATA ITEM

The work on the first issue of this particular Item was in 1983 monitored and guided by the Internal Flow Panel, which had the following constitution:

Chairman	
Mr N.G. Worley	— Babcock Power Ltd
Members	
Mr J. Bacon	— Rolls-Royce Ltd
Dr T.W. Broyd	— Atkins Research and Development
Mr J. Campbell	— Ove Arp and Partners
Dr D.J. Cockrell	— Leicester University
Prof. D.J. Freeston*	— Auckland University, New Zealand
Dr G. Hobson	— GEC Turbine Generators Ltd, Rugby
Prof. J.L. Livesey	— University of Salford
Mr D.S. Miller	— British Hydromechanics Research Association
Dr D. Pollard	— GEC Power Engineering Ltd, Whetstone.

* Corresponding Member

The initial assessment of the available information and the subsequent development of the Item was undertaken by:

Mr R.A. Smith	— Independent
Mr B.C. Freeman	— Senior Engineer

As part of the continual process of review, the data and methods of this Item were in 1995 reassessed, extended and in part programmed by:

Mr K. Farnham	— Engineer.
---------------	-------------

The Internal Flow panel has the following membership:

Chairman	
Dr J.A. Eaton	— National University of Ireland, Galway
Vice-Chairman	
Mr D.A. Campbell	— Independent, UK
Members	
Prof P.R.N. Childs	— University of Sussex, UK
Mr S.C. Curzons	— Rolls-Royce plc, Derby, UK
Dr W.R. Geddes	— Nexia Solutions, UK
Dr S. Gilham	— Atkins, UK

(Continued on inside back cover)

PRESSURE LOSSES IN CURVED DUCTS: SINGLE BENDS

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PRESSURE LOSSES IN CURVED DUCTS: SINGLE BENDS

1. NOTATION AND UNITS

Two consistent sets of units are given in Section 1.1. Some conversion factors are given in Section 1.2.

1.1 Notation

		Units	
		SI	Imperial
A	internal cross-sectional area* of duct	m ²	ft ²
B	arc length of guide vane in mitre-bend cascade (see Figure 10a)	m	ft
b	internal dimension*, normal to plane of bend, of rectangular cross sectional (see Sketch 1.1)	m	ft
b	exponent used in Equation (11.11), defined in Equation (11.12)	—	—
C	chord of guide vane in mitre-bend cascade	m	ft
D	internal diameter* of duct of circular cross section	m	ft
D_E	equivalent diameter* of duct of non-circular cross section (also known as hydraulic diameter), $4 \times$ cross-sectional area/internal perimeter	m	ft
Dn	Dean number, $Re/\sqrt{2R/D}$	—	—
f	Fanning friction factor for fully-developed flow [†] in a straight pipe, $\Delta p_s/[4(\frac{1}{2}\rho V^2)(L/D_E)]$; alternative friction factor, λ , is given by $\lambda = 4f$	—	—
H	length of leading- and trailing-edge extensions of guide vanes in mitre-bend cascade (see Figure 10)	m	ft
h	internal dimension*, in plane of bend, of duct of rectangular cross section (see Sketch 1.1)	m	ft
K_b	component of gross static-pressure loss coefficient notionally attributed to friction in bend length (see Sections 4.1 and 5)	—	—
K_c	component of static-pressure loss coefficient notionally attributed to bend curvature (see Section 5)	—	—
K_d	component of static-pressure loss coefficient attributed to flow recovery in downstream tangent (see Section 5)	—	—
$K_{s, G}$	gross static-pressure loss coefficient (see Section 4.1)	—	—

For footnotes see end of Notation Section.

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$K_{s, N}$	net static-pressure loss coefficient (see Section 4.1)	—	—
k	index of isentropic expansion, equal to γ for ideal gas	—	—
L	length of duct or pipe measured along centre-line (see Sketches 1.1 to 1.3 and 4.1)	m	ft
$\dot{M}$	mass flow rate	kg/s	$\ddagger$ slug/s
Ma	mean Mach number, $V/\sqrt{kRT} = \dot{m}/\sqrt{k\rho p}$	—	—
$\dot{m}$	mass flux, $\dot{M}/A$	kg/s m ²	$\ddagger$ slug/s ft ²
n	guide vane location numbered from $n=0$ at inner corner for mitre-bend cascade (see Figure 10)	—	—
N	number of free-standing guide vanes in mitre-bend cascade (see Figure 10)	—	—
p	static pressure	Pa (N/m ²)	lbf/ft ²
Δp_s	static-pressure drop in the absence of terms due to gravity	Pa (N/m ²)	lbf/ft ²
p_t	total-pressure (mass-averaged)	Pa (N/m ²)	lbf/ft ²
Δp_t	total-pressure loss (mass-averaged)	Pa (N/m ²)	lbf/ft ²
R	radius of curvature ^{**} of bend centre-line (see Sketches 1.1 and 1.3)	m	ft
R_v	radius of curvature of guide vane in mitre-bend cascade (see Figure 10)	m	ft
Re	Reynolds number: $\rho VD/\mu$ or $\dot{m}D/\mu$ for circular pipes, $\rho VD_E/\mu$ or $\dot{m}(D_E/\mu)$ for non-circular ducts	—	—
S	pitch of guide vanes in mitre-bend cascade (see Figure 10)	m	ft
ΔS	increment of guide-vane pitch in mitre-bend cascade (see Figure 10)	m	ft
T	absolute static temperature	K	K
T_t	absolute total temperature	K	K
u	local absolute velocity	m/s	ft/s
V	mean flow velocity, $\dot{m}/\rho$	m/s	ft/s
Z	distance measured across corner diagonal in mitre-bend cascade (see Figure 10)	m	ft
γ	ratio of specific heat capacity at constant pressure to that at constant volume, for a gas	—	—
ε	effective roughness height of pipe surface (typical values are given in Table 12.1)	m	ft

For footnotes see end of Notation Section.

θ	overall turning angle of bend (see Sketches 1.1, 1.2 and 1.3)	degree	degree
$\Delta\theta$	turning angle of segments of composite mitre bend (see Sketch 1.3)	degree	degree
μ	dynamic viscosity of fluid	N s/m ²	lbf s/ft ²
ρ	density of fluid	kg/m ³	‡slug/ft ³
ϕ_1	Reynolds number correction factor for turbulent flow (see Equation (11.17) and Section 6)	—	—
ϕ_2	correction factor for short downstream tangents (see Section 5.2 and Equation (11.18))	—	—
ϕ_3	aspect ratio correction factor for a rectangular duct (see Section 5.3)	—	—
ϕ_4	surface roughness correction factor (see Sections 5 and 6)	—	—
ϕ_5	bend angle correction factor (see Equation (11.19))	—	—
ϕ_6	guide vane or splitter correction factor (Figure 8)	—	—

* The internal cross-sectional dimensions to be used in this Item must be actual values and not the nominal values quoted by manufacturers which may be considerably different.

† Fully-developed flow, for the purposes of this Item, is defined as flow having a constant pipe friction gradient.

‡ 1 slug = 32.174 lbfm.

** For composite mitre bends R is defined in Sketch 1.3.

Subscripts

a	refers to segment spacer length of composite mitre bend (see Sketch 1.3)
b	refers to developed length of a bend (see Sketch 4.1 and Equations (4.3) and (4.4))
d	refers to downstream tangent (see Sketches 1.1 to 1.3 and 4.1)
fd	refers to position at which fully-developed straight pipe flow exists (see Sketch 4.1)
min	refers to minimum value
n	refers to n th guide vane or splitter numbered from inside of bend ($n = 0$ on inside wall)
u	refers to upstream tangent (see Sketches 1.1 to 1.3 and 4.1)
1, 2	refer to stations upstream and downstream of bend or, in Section 7, to guide vane or splitters (see subscript n)

Superscript

'	a prime denotes a basic or reference value to which corrections are to be applied.
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1.2 Conversion Factors

Mass flow rate:

$$27.78 \times 10^{-3} \text{ lbm/s} = 1 \text{ lbm/h} = 8.634 \times 10^{-6} \text{ slug/s} = 12.60 \times 10^{-3} \text{ kg/s},$$

$$1 \text{ kg/s} = 2.2046 \text{ lbm/s}.$$

Pressure:

$$1 \text{ lbf/ft}^2 = 0.00694 \text{ lbf/in}^2 \text{ (psia)} = 32.174 \text{ pdl/ft}^2 = 16.02 \times 10^{-3} \text{ ft of water} = 47.880 \text{ Pa (N/m}^2\text{)}.$$

$$10.197 \times 10^{-6} \text{ kgf/(cm)}^2 = 1 \text{ Pa (N/m}^2\text{)} = 20.885 \times 10^{-3} \text{ lbf/ft}^2 = 1.01325 \times 10^{-5} \text{ atm}.$$

Dynamic viscosity:

$$10^3 \text{ cP} = 1 \text{ N s/m}^2 \text{ (or kg/m s)} = 20.885 \times 10^{-3} \text{ lbf s/ft}^2 \text{ (or slug/ft s)}$$

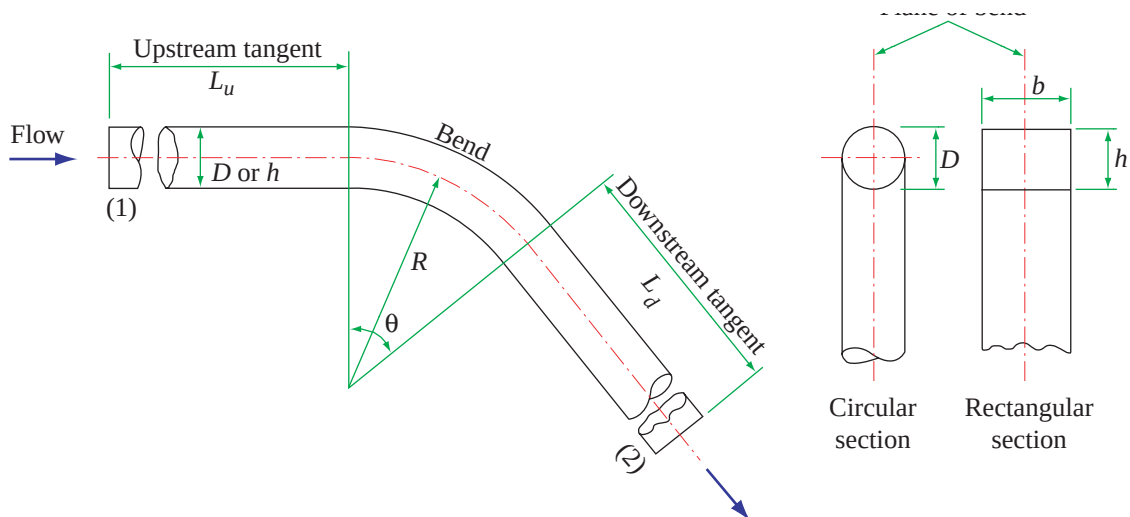
$$\text{Dynamic viscosity} = \text{kinematic viscosity} \times \text{density}$$

Density:

$$1 \text{ lbm/ft}^3 = 1728 \text{ lbm/in}^3 = 32.081 \times 10^{-3} \text{ slug/ft}^3 = 16.018 \text{ kg/m}^3,$$

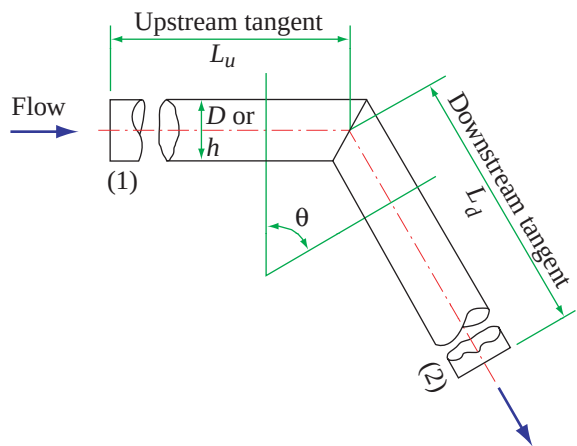
$$1 \text{ kg/m}^3 = 10^{-3} \text{ g/(cm)}^3 = 1.9403 \times 10^{-3} \text{ slug/ft}^3.$$

1.3 Bend Geometry

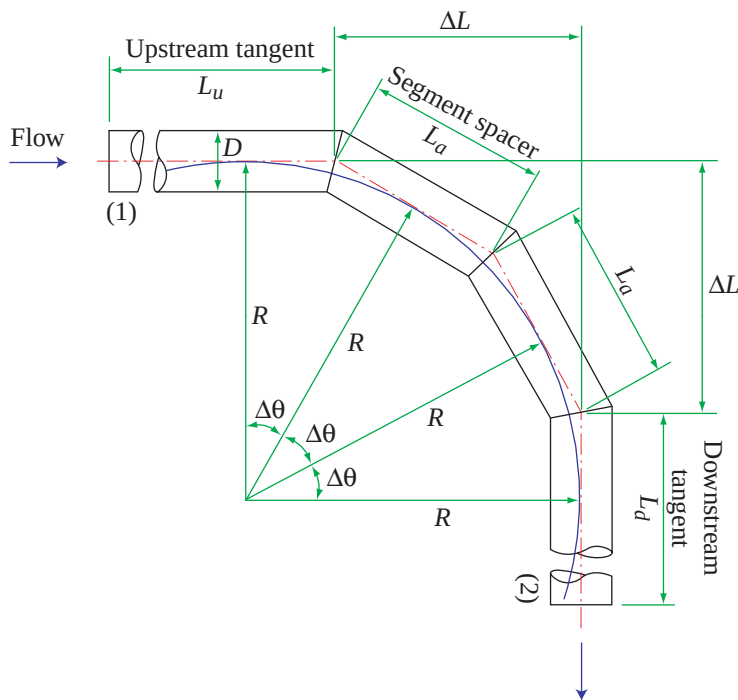


Sketch 1.1 Circular-arc bend

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Sketch 1.2 Single mitre bend



Bend type	2 × 45°	3 × 30°	4 × 22½°
layout			
R	1·207L _a	1·866L _a	2·514L _a
L _a	0·828R	0·536R	0·398R
ΔL	0·707L _a	1·366L _a	2·014L _a

$R = \frac{L_a}{2} \cot \frac{\Delta\theta}{2}, \Delta L = R - \frac{L_a}{2}$

Sketch 1.3 90° composite mitre bend. (A 3 × 30° bend is shown as an example)

2. INTRODUCTION

This Data Item presents pressure-loss data for flow through single* bends in pipe or duct systems. The types of bend considered are circular-arc and single and composite mitre bends. Only 90° composite mitre bends with equal segment angles and spacer lengths are covered. The geometries of the three types of bend are shown in Sketches 1.1 to 1.3 in Section 1.3.

The data are presented in terms of a non-dimensional static-pressure loss coefficient, K_s , which is used in the equations of Section 4 to determine the appropriate pressure loss.

The following parameters are considered:

- (i) Reynolds number, Re ,
- (ii) non-dimensional surface roughness of the bend and tangents, ε/D_E (assumed the same for both),
- (iii) cross-sectional shape (assumed constant),
- (iv) non-dimensional downstream tangent length, L_d/D_E ,
- (v) non-dimensional upstream tangent length, L_u/D_E ,

as well as the type of bend and the geometrical parameters characterising it.

Section 3 considers the limitations and uncertainty of the data in this Item and Section 4 outlines the definition of bend pressure losses and their calculation from the data presented.

The data for circular-arc bends are discussed in Section 5. These data are presented generally in equation form and the equations are summarised in Section 11. A graphical solution of these equations is provided and a computer program that uses the data to calculate the pressure loss, ESDUpac A8337, is introduced in Appendix A. Data for 90° smooth bends with long tangents are also presented graphically. The data for mitre bends are considered in Section 6; these data are presented in graphical form only. Section 7 presents information on guide vanes and splitters in bends.

Pipe systems are often made up from standard pipe fittings. Provided that the geometry conforms to that defined in Sketches 1.1 to 1.3 and the joints with the adjacent pipe are smooth, the appropriate data from Sections 5 or 6 may be applied. However, Section 8 provides approximate data for more general pipe bend fittings in terms of nominal size and typical descriptions used in the literature. The large uncertainty of these data may be acceptable in some circumstances especially if there is more than about 100 diameters of straight pipe associated with each bend.

A worked example is given in Section 9.

* Correction factors for two bends in close proximity (less than about 30 diameters apart) are given in Reference 12. Although the data presented in this Item can be applied for helical coils a more comprehensive treatment is given in Derivation 37.

3. APPLICABILITY AND UNCERTAINTY OF THE DATA

The data in this Item are applicable only to:

- (i) single-phase flow of Newtonian fluids,
- (ii) duct or pipe systems running full.

3.1 Uncertainties Arising in Application of the Data to Practical Systems

The data in this Item are derived from tests on systems constructed from purpose-made bends of constant cross-sectional area and shape with carefully made, internally flush tangent connections. While many practical systems approximate closely to these ideal systems others do not and departures from ideal generally increase the pressure loss.

The nature of the bend-tangent connection influences pressure losses considerably and special attention must be paid to this. Flanged or butt-welded couplings can give adequately flush internal surfaces provided that care is taken to align adjoining surfaces. When bend-tangent connections are not flush internally, the limited data³⁶ indicate that static-pressure loss coefficients are generally increased. For example, male threaded bends coupled via collars to the tangents and female threaded bends screwed directly to male threaded tangents have loss coefficients increased by as much as 50 per cent.

The cross sections of bends are often distorted in manufacture which usually results in an increase of pressure loss. If this is the case there will be additional uncertainties in pressure loss values as calculated from this Item.

The pipe diameters quoted by manufacturers are often nominal values which can be considerably different from the actual internal values on which this Item is based. If the actual internal dimensions are not known an additional uncertainty in pressure loss will result.

Uncertainty also arises from lack of precise knowledge of surface roughness; this will be minimised if measured friction factors are available but the effective roughness may vary with Reynolds number.

It is emphasised that the effects of upstream and downstream flow conditions on pressure losses may be considerable, especially when adjacent components are less than about $4D$ from the bend. Data for the pressure loss due to a bend followed closely by another bend are given in Reference 12 and for bend-diffuser interaction in Derivation 35 but generally data are not available to quantify the interference effects between a bend and another component. With an increasing spacer length between a bend and another component the interference effects decrease and are generally only of the order of ± 10 per cent of the pressure loss due to the two components in interference-free flow for spacer lengths greater than $30D$.

If the sum of the tangent lengths per bend in a system is greater than about 100 diameters, the pressure loss due to the straight parts of the pipe is the dominant loss and large uncertainties in bend losses can often be tolerated.

3.2 Accuracy of the Data

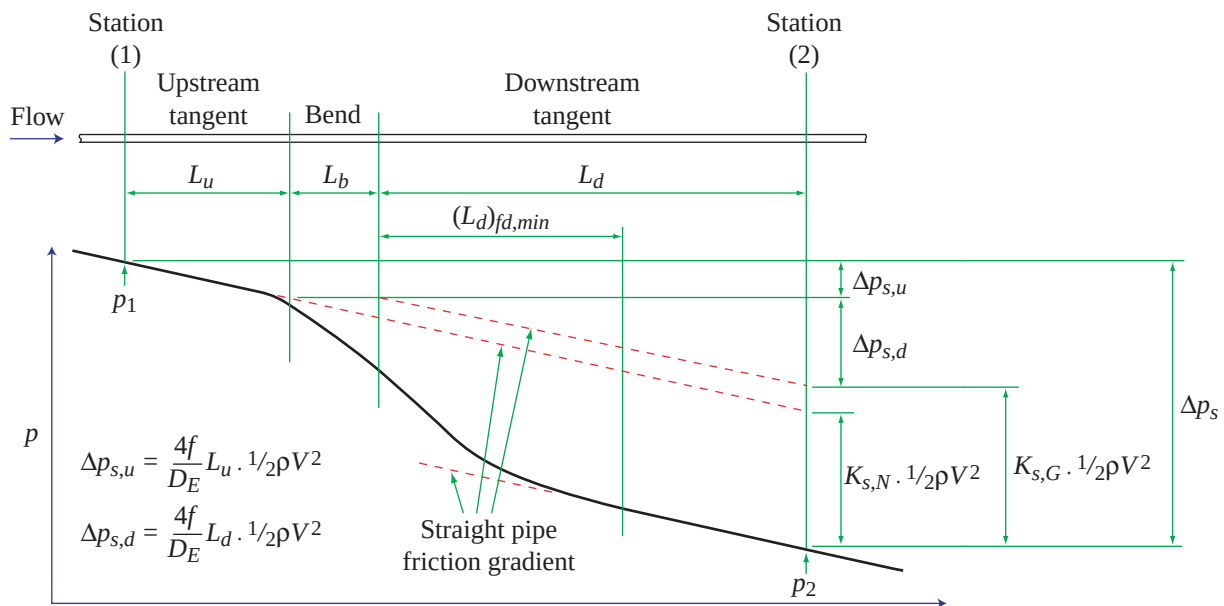
A large body of experimental data on pressure losses in bends exists but exhibits a large scatter because of uncertainty about factors such as upstream flow conditions, upstream and downstream measurement locations, surface roughness and accuracy of cross section.

The majority of the more reliable experimental data for the gross static-pressure loss coefficient, $K_{s,G}$, lie within about ± 15 per cent of the values predicted by this Item but errors as large as 30 per cent occur in some cases. For short circular-arc bends the uncertainty is largest for small R/D ($R/D < 1$), $\theta \neq 90^\circ$, high surface roughness and short upstream or downstream tangents. The data for laminar flow show increasing uncertainty as Re decreases. For mitre bends the uncertainty is large for $Re < 10^5$, especially for composite mitres, and increases with increasing spacer length in composite mitres. The uncertainty is also large for non-circular cross sections.

The data for standard pipe fittings are particularly uncertain because of differences in geometry, tangent connections and surface finish. Errors up to 100 per cent are possible.

4. CALCULATION OF PRESSURE LOSS

Sketch 4.1 shows a typical variation of the mean static pressure along a duct including a single circular-arc bend between lengths of straight pipe. The bend is here assumed to lie in a horizontal plane so that there are no pressure changes from gravitational effects. The influence of the bend extends a short distance (up to about $1D$) upstream of the bend entry and up to 50 or so diameters downstream* of the bend exit for circular-arc bends. In this downstream length the distorted flow recovers to a fully-developed state. Very close to the bend ($\frac{1}{2}$ diameter upstream and 2 or 3 downstream) and within the bend, the static pressure varies across the pipe, being greater on the outside of the bend, but elsewhere it is substantially constant across the section.



Sketch 4.1 Typical mean static-pressure variation through a single circular-arc bend

In order to measure the most general value of bend pressure loss the pressure measurement stations ((1) and (2) in Sketch 4.1) must be in regions of fully-developed flow, *i.e.* at least $2D$ upstream and $50D$ downstream of the bend. An additional length of straight pipe upstream of (1) will also be required to ensure that the flow has recovered from any upstream disturbance. If these conditions are not met, the measured bend loss becomes a function of the specific conditions at Stations (1) and (2).

For the calculation of pressure loss, however, provided that adequate allowance can be made for the effects of short tangents, the location of Stations (1)[†] and (2) is unrestricted.

4.1 Static-pressure Drop, Incompressible Flow

In the absence of gravitational terms the static-pressure loss between Station (1) in fully-developed flow (as defined in Section 1.2) upstream* of a bend and a downstream Station (2) (Sketch 4.1) can be expressed as the sum of two terms:

$$p_1 - p_2 = \Delta p_s = \frac{1}{2} \rho V^2 K_{s,G} + \frac{1}{2} \rho V^2 \frac{4f}{D_E} (L_u + L_d). \quad (4.1)$$

* Table 12.2 gives some estimates of the downstream length required for recovery to fully-developed straight pipe flow.

† If Station (1) is within $2D$ of the bend, the pressure p_1 , is effectively that given by an extrapolation of the upstream pipe friction gradient.

The first term is the gross static-pressure loss due to the bend which may be obtained from the gross pressure loss coefficient, $K_{s,G}$, given in this Item as a function, principally, of bend type, θ , R/D_E , ε/D_E , L_d/D_E and Re . The second term is a pressure loss due to friction in the upstream and downstream tangents, calculated with the assumption of fully-developed flow throughout. The friction factor f may be obtained* as a function of Re and ε/D_E from the equations of Section 11.1 or from Figure 1.

An alternative formulation uses the net static-pressure loss coefficient, $K_{s,N}$, and gives

$$\Delta p_s = \frac{1}{2} \rho V^2 K_{s,N} + \frac{1}{2} \rho V^2 \frac{4f}{D_E} (L_u + L_b + L_d), \quad (4.2)$$

where the friction loss term now includes a notional friction loss in a straight pipe of length L_b equal to the bend centre-line length. For circular-arc bends,

$$L_b = \theta \pi R / 180 \quad (4.3)$$

with θ in degrees, for composite mitre bends,

$$L_b = \Sigma L_a, \quad (4.4)$$

and for single mitre bends, $L_b = 0$. The gross and net pressure loss coefficients are related by:

$$K_{s,G} = K_{s,N} + 4fL_b/D_E. \quad (4.5)$$

Pressure loss coefficients for circular-arc bends are discussed in Section 5, for mitre bends in Section 6, for bends with guide vanes in Section 7 and approximate data for “standard pipe fittings” in Section 8.

4.2 Static-pressure Drop, Compressible Flow

If the pressure drop of a gas is greater than 5 per cent of the absolute pressure, the assumption of incompressible flow may lead to significant error even for low Mach numbers ($Ma_1 < 0.2$, say) because of density changes through the system.

For flows in which $p_2 > 4\dot{m}^2/\rho_2$, ($Ma_2 < 0.45$ approximately), the assumption of isothermal flow ($\Delta T = 0$ or in practice $\Delta T < 0.1T$ approximately) can be made. The downstream pressure can then be calculated from Equation (4.6) if the upstream density and pressure are known or, if the downstream density and pressure are known, the upstream pressure can be calculated from Equation (4.7). Equations (4.6) and (4.7) are:

$$\frac{p_1^2 - p_2^2}{p_1} = \frac{\dot{m}^2}{\rho_1} \left[K_{s,G} + \frac{4f}{D_E} (L_u + L_d) + 2 \log_e \left(\frac{p_1}{p_2} \right) \right] \quad (4.6)$$

$$\text{and} \quad \frac{p_1^2 - p_2^2}{p_2} = \frac{\dot{m}^2}{\rho_2} \left[K_{s,G} + \frac{4f}{D_E} (L_u + L_d) + 2 \log_e \left(\frac{p_1}{p_2} \right) \right]. \quad (4.7)$$

Since the ratio p_1/p_2 is not known, an iterative solution is needed. As a first step the term $\log_e(p_1/p_2)$ may be neglected. Often this term is negligible in comparison with the remaining terms in the square bracket.

* Table 12.2 gives some estimates of the downstream length required for recovery to fully-developed straight pipe flow

Alternatively the graphical approach in ESDU 74029¹¹, which avoids iteration, may be adapted by including the bend pressure loss with the pipe friction in a similar manner to Equations (4.6) and (4.7).

For flows which cannot be considered isothermal and for higher Mach numbers upstream of the bend, the assumption of adiabatic flow ($\Delta T_t = 0$) is more realistic and the appropriate method of calculation in ESDU 74029 can be used, similarly adapted, provided that the Mach number is less than about 0.6 before the bend. It should be remembered that the flow will choke at the downstream end of the system and this may limit the maximum achievable Mach number upstream of the bend.

These calculation methods assume that the bend loss coefficient, $K_{s,G}$, is independent of Mach number. Whilst this assumption is valid^{22,41} for circular-arc bends with $R/D \geq 1$, it is not valid for bends with $R/D < 1$ in which significant flow separation occurs. It may be expected to be similarly invalid for mitre bends. Use of the method for bends with $R/D < 1$ and for mitre bends will increasingly underestimate the pressure loss as the Mach number increases.

4.3 Total-pressure Loss

For system loss* calculations the total-pressure loss for incompressible flow can be taken as equal to the static-pressure drop,

$$\Delta p_t = \Delta p_s. \quad (4.8)$$

For compressible flow the total pressure p_t can be related to the static pressure p by

$$\frac{p_t}{p} = \left(1 + \frac{\gamma-1}{2} Ma^2\right)^{\frac{\gamma}{\gamma-1}}. \quad (4.9)$$

From the known conditions at one station, the calculated Δp_s and the assumption of isothermal flow or adiabatic flow, the Mach numbers Ma_1 and Ma_2 can be calculated[†]. The total-pressure loss can then be calculated from the static pressure drop.

5. STATIC-PRESSURE LOSS COEFFICIENTS FOR CIRCULAR-ARC BENDS

Circular-arc bends may be divided into two groups: long bends, in which a large region of fully-developed curved flow exists and contributes the majority of the pressure loss, and short bends, in which little or no fully-developed curved flow exists and other contributions to pressure loss are important. The pressure loss caused by short bends is significantly affected by upstream and downstream conditions but that caused by long bends is not.

Figure 2 shows the boundary between long and short bends, based on data²⁶ for turbulent flow. For laminar flow, the flow development is slower than for turbulent flow and the boundary will be higher. The few data available for laminar flow are insufficient to define the boundary. Most practical bends in pipe or duct systems of up to 90° angle will lie in the short bend region but some 180° return bends may lie within the cross-hatched boundary for turbulent flow. For pipes with roughness ratios, ϵ/D , less than 10^{-4} , the calculation methods for long and short bends give similar results but for very rough surfaces appreciable differences can occur and the more conservative pressure loss estimate should be taken.

* If the downstream tangent discharges to atmosphere or into a large space, an additional pressure loss, equal to $\frac{1}{2}\rho V^2$, must be added to obtain the overall system loss.

† A number of useful compressible flow relationships are given in ESDU 74028¹⁰.

In this Data Item the gross pressure loss coefficient, $K_{s,G}$, for circular-arc bends has been correlated by expressing it as the sum of three contributions:

$$K_{s,G} = K_b + K_c + K_d, \quad (5.1)$$

where K_b represents a notional friction loss in the bend length calculated as if it was a length of straight pipe, K_c represents a pressure loss attributed primarily to the curvature of the bend and K_d represents a pressure loss attributed primarily to losses in the re-establishment* of the fully-developed straight pipe flow in the downstream tangent. The calculation of these contributions to $K_{s,G}$ and the factors which influence them are considered in the following Sections. A summary of the equations representing the data is given in Section 11.

5.1 Long Circular-arc Bends

The data of Derivation 37, for pressure losses in coils†, can be used for all long circular-arc bends. Since the majority of the pressure loss is attributed to the fully-developed curved flow, upstream and downstream tangent effects can be neglected and K_d taken as zero. The gross pressure loss coefficient, $K_{s,G}$, reduces to $(K_b + K_c)$ and can be taken as directly proportional to the bend angle, θ .

Figures 3a, 3b and 3c represent the appropriate data from Derivation 37; Figure 3b is derived from Equations (5.4) to (5.6) of Derivation 37 which were based on data for nominally smooth, circular-sectioned long bends and coils. For rectangular ducts, in the absence of data, it is suggested that D is replaced by D_E in R/D and Re . The few data for rough-surfaced long bends do not adequately define the effects of surface roughness. In laminar flow, roughness does not affect pressure losses but it may influence the transition to turbulent flow. For turbulent flow, the correction factor ϕ_4 in Figure 3c is tentatively based on the assumption that the effect of roughness on fully-developed curved flow is the same as on fully-developed straight pipe flow. This is expected to overestimate the effect of roughness, especially at high Reynolds number.

For flows at Reynolds numbers between 2000 and 1.5×10^4 , the pressure losses are uncertain. Over part of this range laminar flow may be established in a long bend even though the upstream pipe flow is turbulent. In the transition region, shown on Figure 3a, values of $K_{s,G}$ probably lie within the limits given by the value for laminar flow at Re_B and the value for turbulent flow at $Re = 1.5 \times 10^4$.

The data for long circular-arc bends are represented by Equations (11.4) to (11.8) in Section 11.

5.2 Short Circular-arc Bends in Pipes of Circular Cross Section

For short circular-arc bends all three pressure loss components in Equation (5.1) are of importance. Some interaction between the curvature and downstream tangent components will occur but data were selected so as to minimise this interaction when correlating the data.

5.2.1 Pressure loss coefficients for laminar flow ($Re \leq 2000$)

Derivations 21 and 25 give experimental data on pressure losses for laminar flow through 90° , short circular-arc bends. The values from the two sources differ considerably, the difference increasing as Reynolds number decreases. A tentative correlation was derived by assuming that the effects of curvature are given by the correlation from Derivation 37 for coils, giving values of $K_b + K_c$, and attributing the

* For short downstream tangents, $L_d/D_E \leq 50$, re-establishment of fully-developed flow is incomplete.

† Reference 13 and Derivation 37 deal more comprehensively with pressure loss and heat transfer for coils.

remainder of the experimental loss to K_d . For high Dean number, $Dn \geq 400$, K_d tends to unity and for $Dn \leq 13.5$ it should tend to zero since curvature does not significantly distort the flow from its straight pipe condition.

No data were found for laminar flow in bends with short downstream tangents but, except for tangents of less than 2 or 3 diameters, the value with a long tangent is unlikely to underestimate the pressure loss.

Equations (11.10) to (11.12) represent the proposed correlation for laminar flow in short circular-arc bends. Figure 4b presents the data for 90° bends only. In view of the uncertainties some simplification of the equations from Derivation 37 has been made.

5.2.2 Pressure loss coefficients for turbulent flow ($Re \geq 4000$)

For short circular-arc bends with turbulent flow, the correlation for K_c (Equation (11.14) and Figure 5b) is based on Equation (5.6) of Derivation 37 for the friction factor in smooth coils, with the addition of a correction term for the effects of surface roughness. The correlation was derived from experimental data for pressure loss in smooth and rough surfaced bends from, principally, Derivations 14, 19, 20, 26, 35 and 38.

The correlation for K_c is valid strictly only if fully-developed flow exists upstream of the bend, *i.e.* generally if $L_u/D \geq 50$. For shorter tangents, fully-developed flow may not be achieved. Data in Derivation 26 suggest that the effects are small if $L_u/D > 50$ and the velocity profile is symmetrical as, for example, in a naturally-developing flow or several diameters downstream of an orifice plate or change of section. Data in Derivation 35 suggest that the effects are more noticeable for small R/D values. Reference 12 gives information on the effects of another bend close upstream on the overall pressure loss coefficient and Derivation 35 considers diffuser/bend interactions. Derivation 17 gives examples of the effect of a peak in the velocity profile near the outer or inner wall.

The downstream tangent loss, K_d (Equations (11.15) and (11.16) and Figure 5c), was derived from experimental data from Derivations 19, 26, 27, 31 and 33. For short tangents, $L_d/D \leq 50$, the flow may not reach a fully-developed state in the tangent. For $L_d/D \geq 5$, the effect is to reduce the pressure loss but, as can be seen from the correction factor*, ϕ_2 , plotted in Figure 5d, the reduction is less than 5 per cent for $30 \leq L_d/D \leq 50$. Figure 5d was derived principally from data for square-sectioned bends^{31,33} discharging to atmosphere[†] and for circular-sectioned bends²⁶ with additional pipe downstream of the measuring point. It can be considered to apply generally for $L_d/D \geq 10$ but for shorter tangents may be affected by the nature of the next downstream component. Reference 12 considers the effect on the overall pressure loss of another bend downstream and Derivation 35 the effect of a diffuser but no data for other fittings are available. In addition to the effect on the bend loss the downstream component will also be affected. For $L_d/D \leq 5$, the static pressure distribution across the duct section may be non-uniform and the effects of terminating the downstream tangent in this region are more marked; very close to the bend an increased pressure loss may be incurred.

The data for short circular-arc bends in turbulent flows are represented by Equations (11.13) to (11.19) in Section 11. Figure 4b shows the gross pressure loss coefficient $K_{s,G}$ for 90° bends in smooth pipes ($\epsilon/D \leq 10^{-5}$). The data for $R/D \leq 1$ are based on few data and are less well fitted by the equations. Figures 5a to 5d provide a graphical method for evaluation of Equations (11.13) to (11.19).

* See Equations (11.15) and (11.16) for application of this factor.

† An additional loss due to the sudden enlargement should be added to values calculated from this Item to obtain the overall system loss in this case.

5.2.3 Pressure loss coefficients for transitional flow ($2000 \leq Re \leq 4000$)

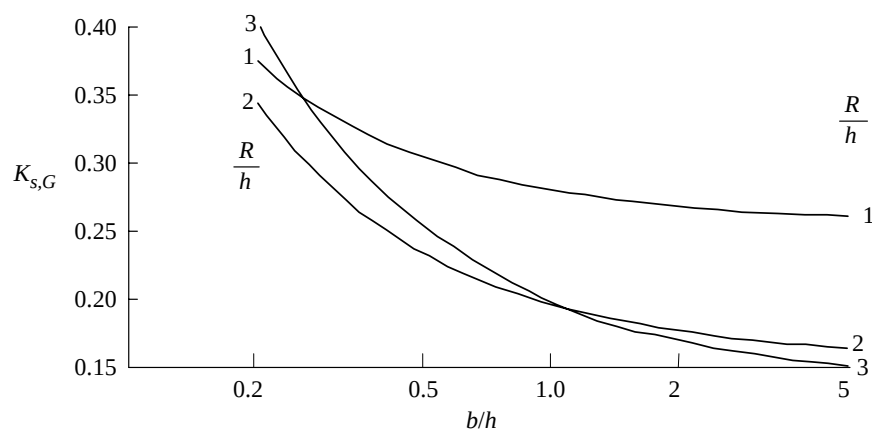
The few data for flow in this region suggest that transition between laminar and turbulent flow in bends is similar in nature to that in naturally rough pipes. The value of $K_{s,G}$ may be obtained by interpolation between the appropriate values of $K_{s,G}$ at $Re=1000$ and $Re=4000$.

5.3 Short Circular-arc Bends in Ducts of Non-circular Cross Section

The available data for bends in ducts of non-circular section are widely scattered. Variations in tangent lengths and measuring locations contribute to the scatter.

For rectangular sections, the equations for a circular section, modified as in Equations (11.21) to (11.27), predict the experimental data from Derivations 31, 33 and 35 to within about ± 25 per cent. Sketch 5.1 shows how the predicted pressure loss coefficient varies with aspect ratio, b/h , for three radius ratios, R/h . For $b/h \leq 1$, the general trend is to increase the pressure loss and values of $b/h \leq \frac{1}{2}$ should be avoided.

For ducts of other non-circular cross sections it is suggested that Equations (11.21) to (11.27) are applied although there are few data to confirm this approach.



Sketch 5.1 $\theta = 90^\circ$, $Re = 10^5$, $\epsilon/D_E = 10^{-6}$, $L_d/D_E \geq 50$

When the straight pipes are bent to form circular-arc bends the cross section is often flattened and this will usually result in an increase in pressure loss, even though the aspect ratio is favourable, because of area changes through the bend.